

**Borkovits Tamás**  
Astronomical Observatory of Szeged University



**Then and now:**

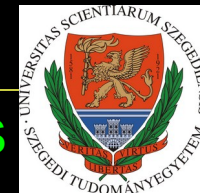
**A new look on the ETVs of hierarchical triple candidates in the primordial Kepler-field revisited by TESS**

**Collaborators: S. Rappaport (M.I.T. Kavli Institute), E. Forgács-Dajka, T. Mitnyan, I.B. Bíró, I. Csányi (Baja), A. Pál (Konkoly), T. Hajdu, J. Sztakovics (ELTE, Astron. Dept.) et al.**

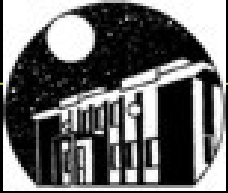


Tamás Borkovits – Then and now:

Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



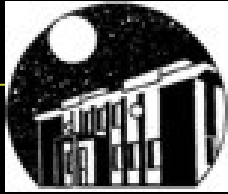
**THEN AND NOW**



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Tamás Borkovits – Then and now:

## Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



THEN

Monthly Notices

of the  
ROYAL ASTRONOMICAL SOCIETY



MNRAS **448**, 946–993 (2015)

doi:10.1093/mnras/stv015

### Eclipse timing variation analyses of eccentric binaries with close tertiaries in the *Kepler* field

T. Borkovits,<sup>1,2★</sup> S. Rappaport,<sup>3</sup> T. Hajdu<sup>4</sup> and J. Sztakovics<sup>4</sup>

<sup>1</sup>Baja Astronomical Observatory, H-6500 Baja, Szegedi út, Kt. 766, Hungary

<sup>2</sup>ELTE Gothard-Lendület Research Group, H-9700 Szombathely, Szent Imre herceg út 112, Hungary

<sup>3</sup>M.I.T. Department of Physics and Kavli Institute for Astrophysics and Space Research, 70 Vassar St, Cambridge, MA 02139, USA

<sup>4</sup>Astronomical Department of Eötvös University, H-1118 Pázmány Péter stny. 1/A, Budapest, Hungary

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MNRAS **455**, 4136–4165 (2016)

doi:10.1093/mnras/stv2530

### A comprehensive study of the *Kepler* triples via eclipse timing

T. Borkovits,<sup>1,2★</sup> T. Hajdu,<sup>3</sup> J. Sztakovics,<sup>3</sup> S. Rappaport,<sup>4★</sup> A. Levine,<sup>5</sup> I. B. Bíró<sup>1</sup>  
and P. Klagyivik<sup>6,7</sup>

<sup>1</sup>Baja Astronomical Observatory of Szeged University, H-6500 Baja, Szegedi út, Kt. 766, Hungary

<sup>2</sup>ELTE Gothard-Lendület Research Group, H-9700 Szombathely, Szent Imre herceg út 112, Hungary

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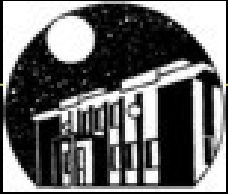
<sup>4</sup>Department of Physics, and Kavli Institute for Astrophysics and Space Research, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

<sup>5</sup>Kavli Institute for Astrophysics and Space Research, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

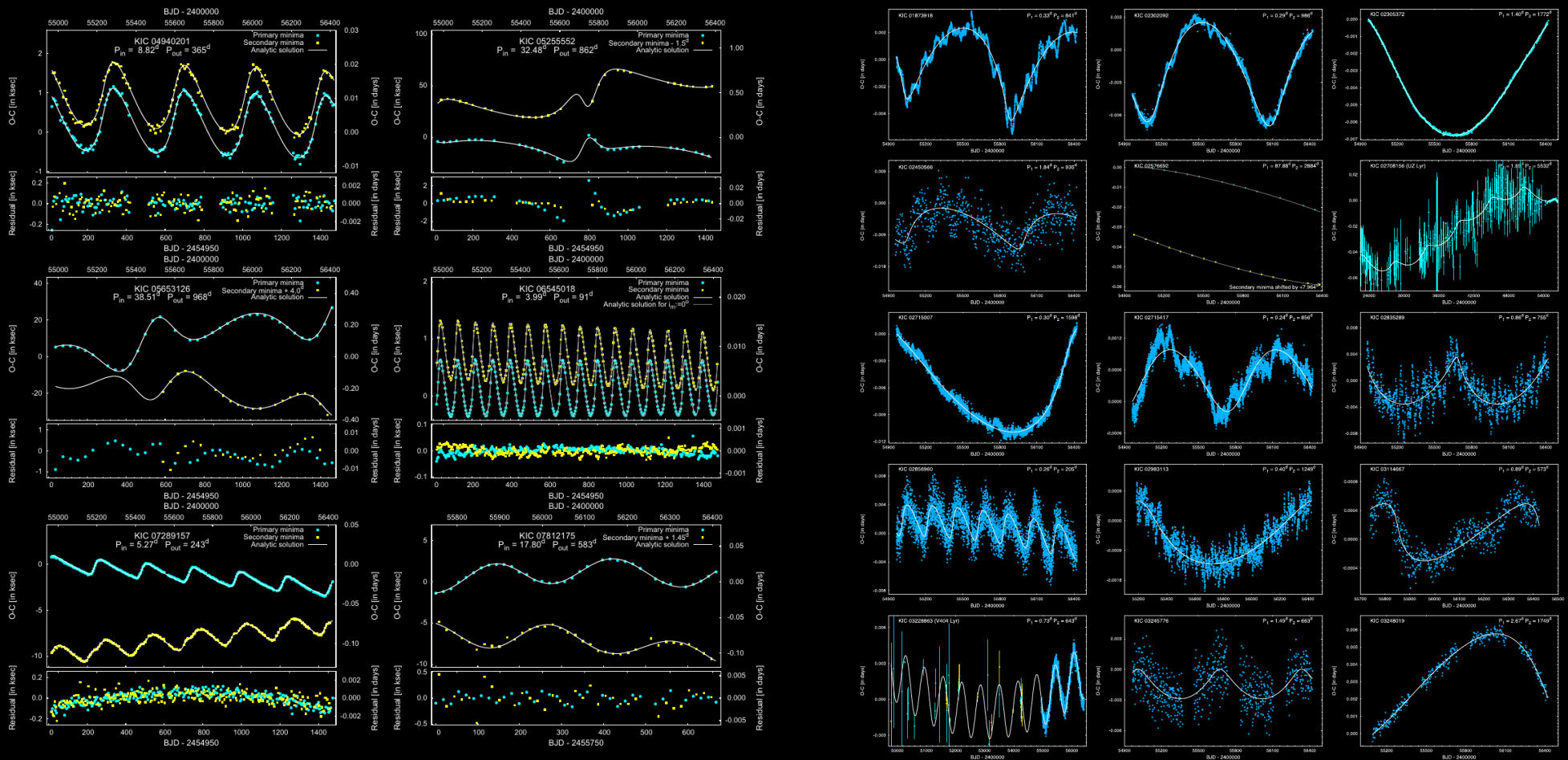
<sup>6</sup>Instituto de Astrofísica de Canarias, E-38205 La Laguna, Tenerife, Spain

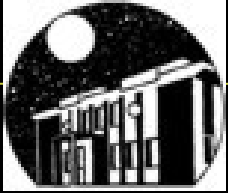
<sup>7</sup>Departamento de Astrofísica, Universidad de La Laguna, E-38206 La Laguna, Tenerife, Spain

Accepted 2015 October 27. Received 2015 October 27; in original form 2015 September 14



THEN



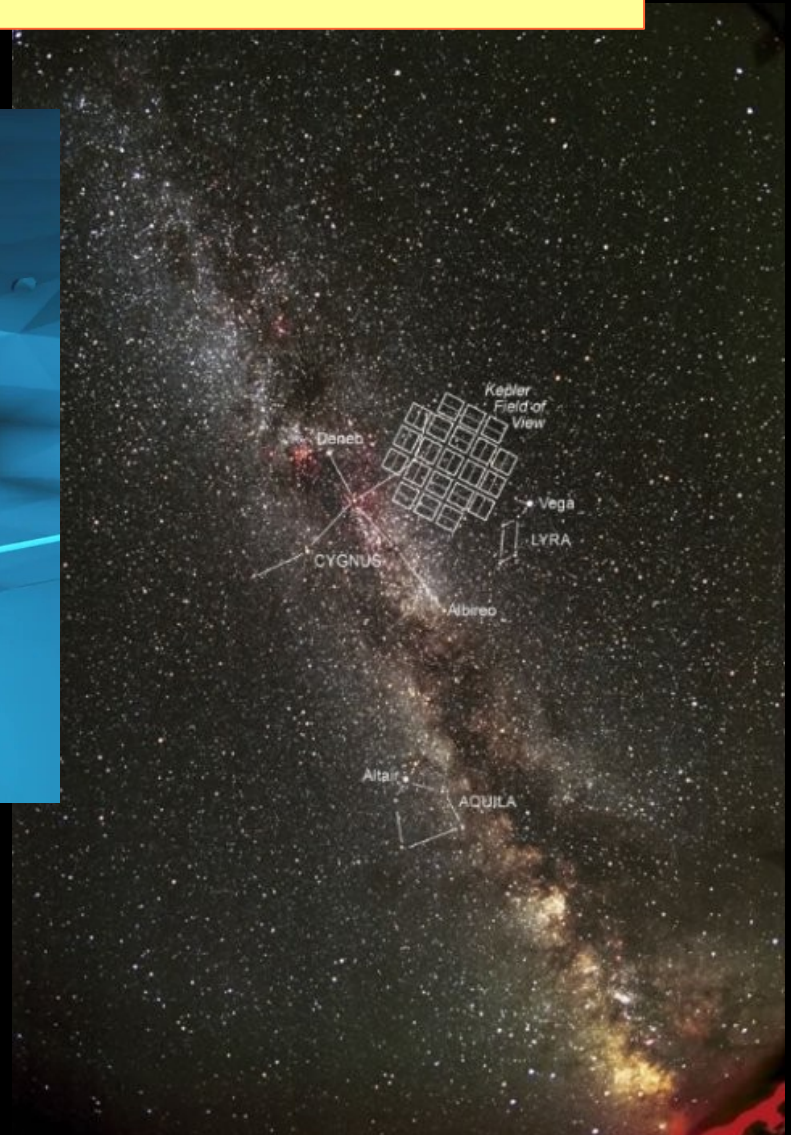


Tamás Borkovits – Then and now:

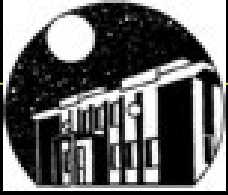
# Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



Since then ...



Kepler has gone

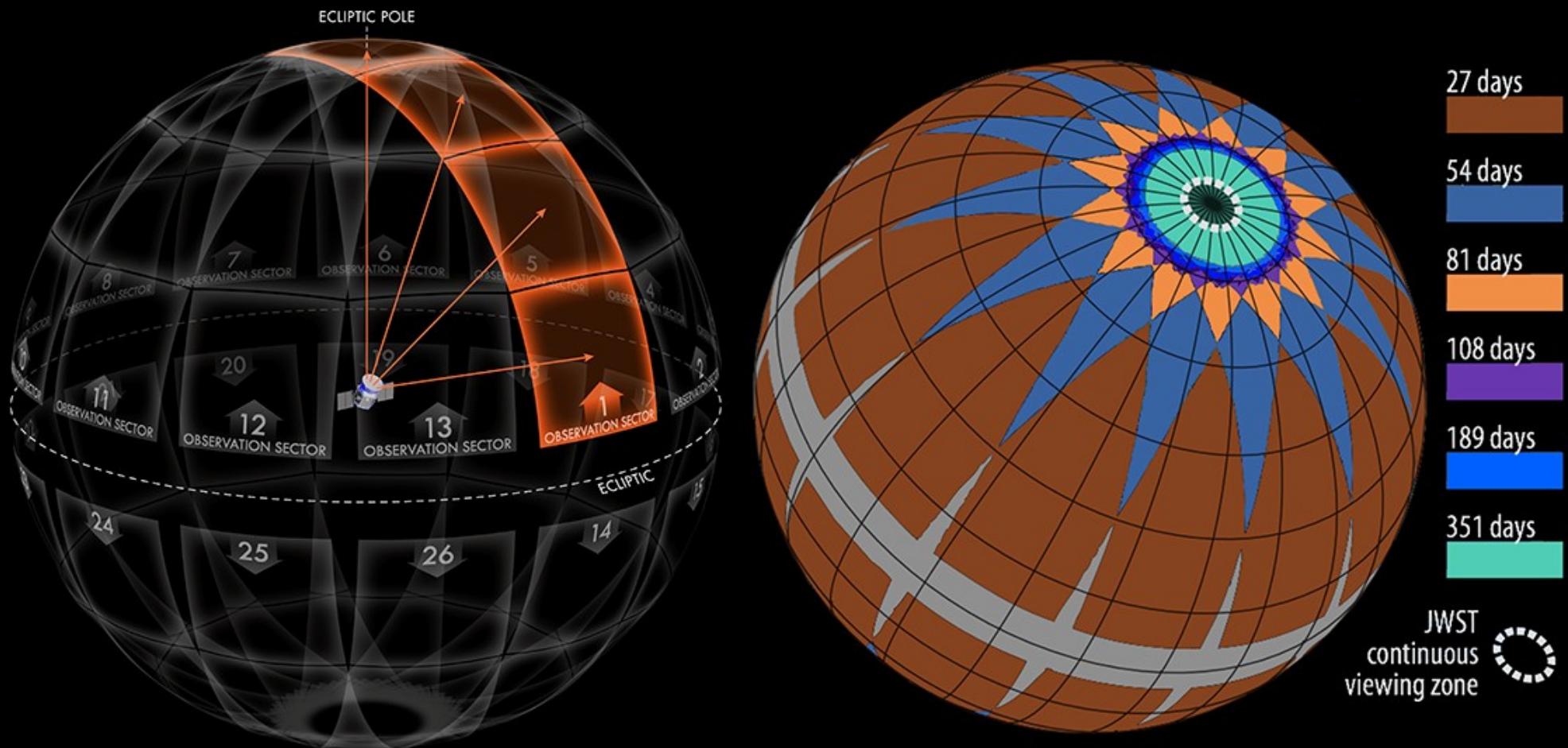


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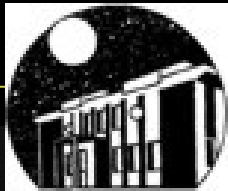
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Since then ...

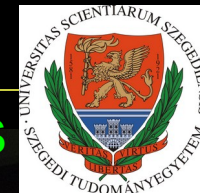


TESS has arrived

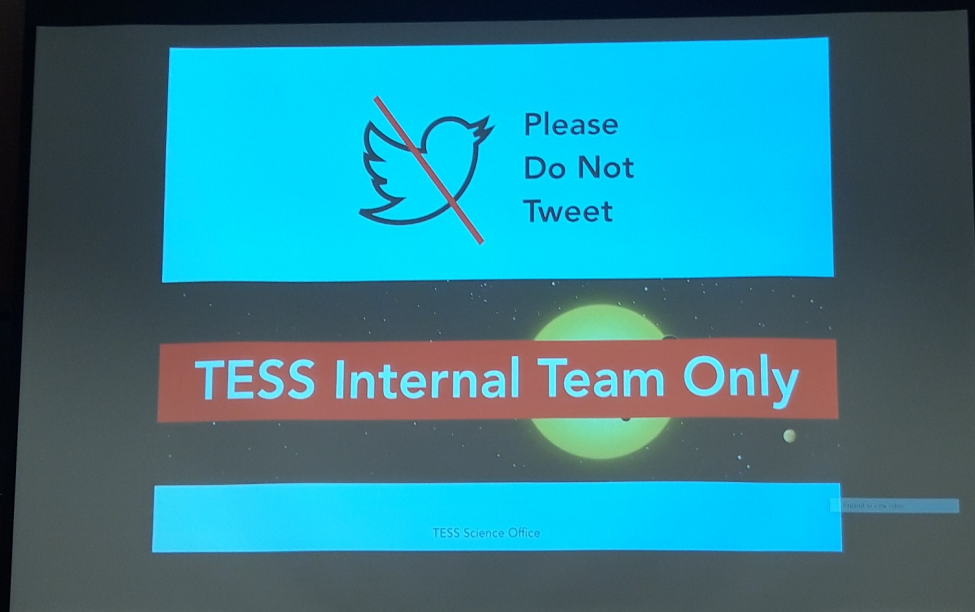


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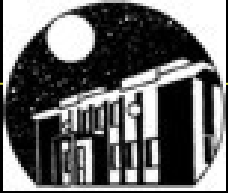


Since then ...



M.I.T. – 2018. 08. 06. 9:53





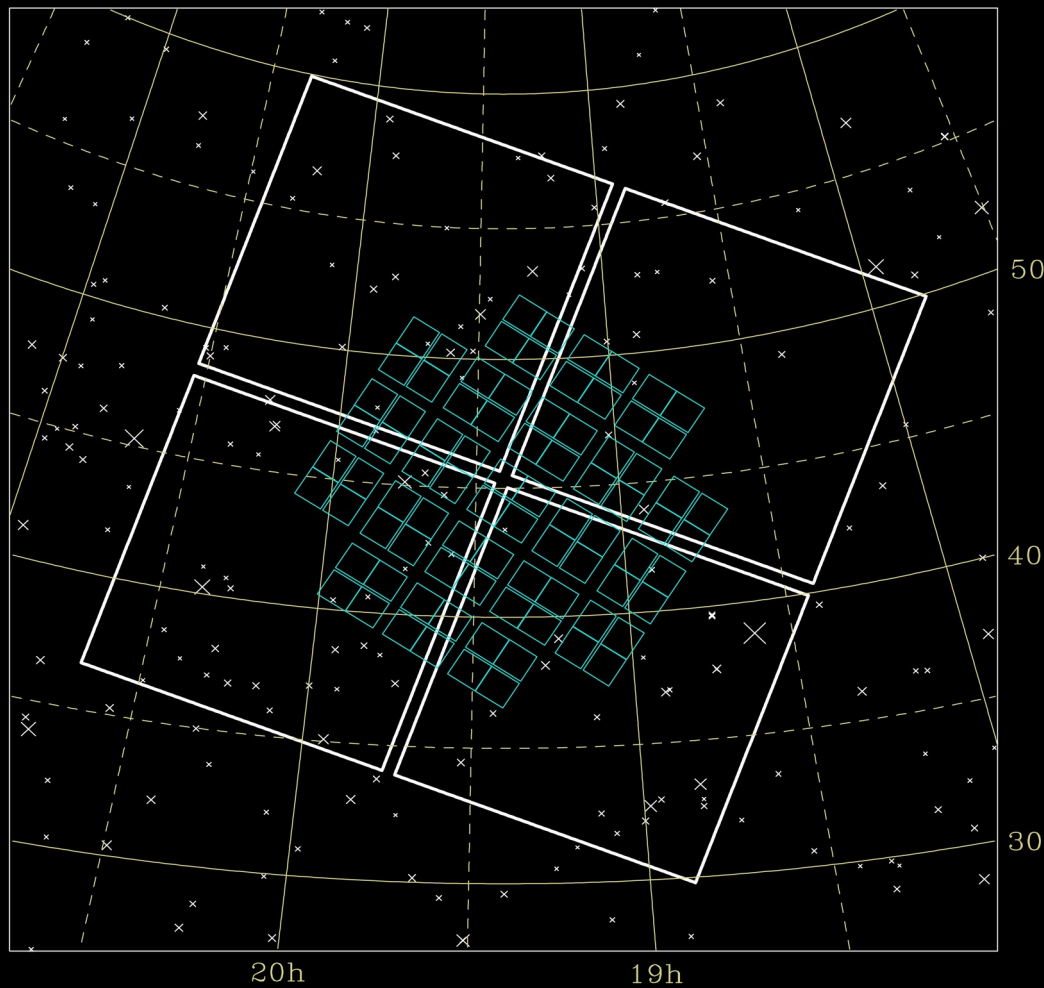
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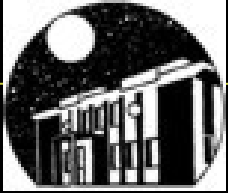


Since then ...

Sector 14 Camera 3



One more year ...



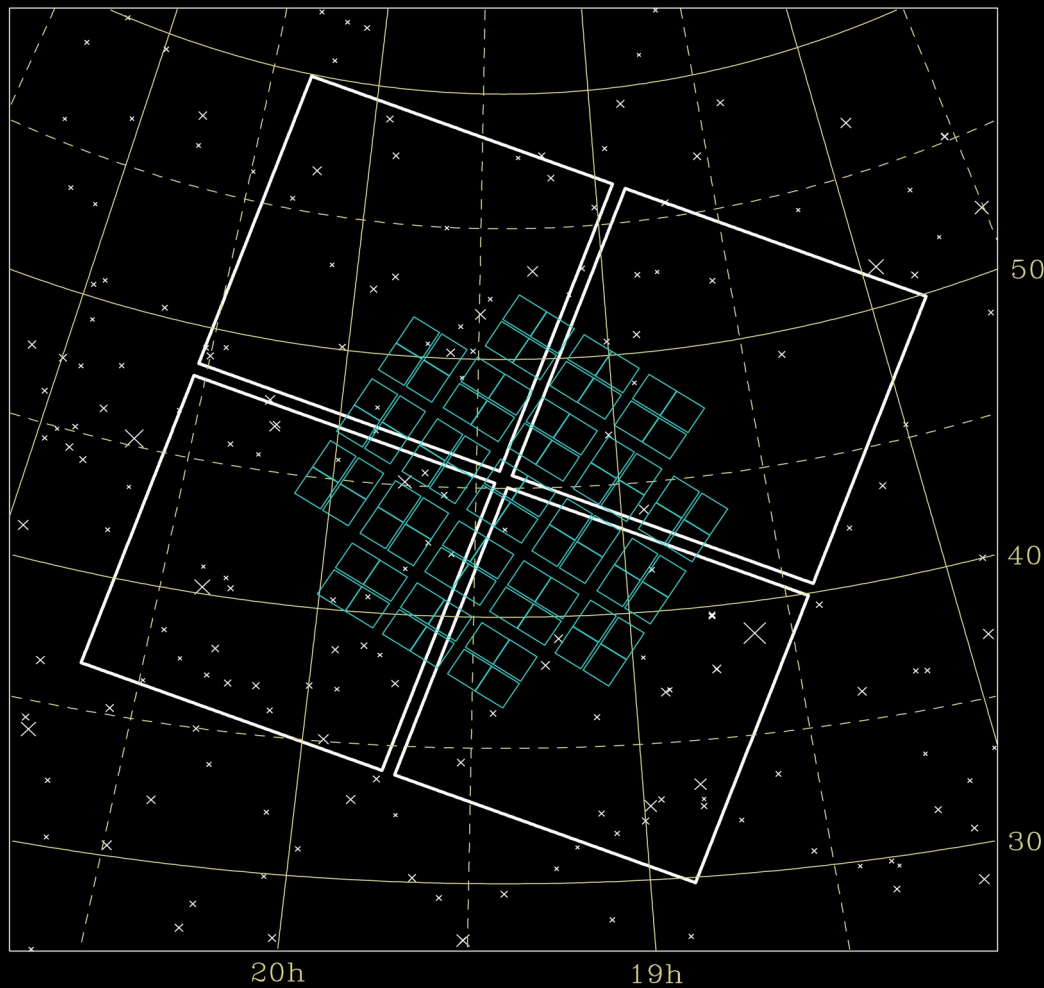
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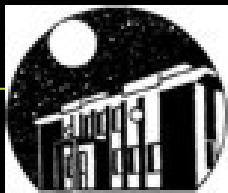
Since then ...

Sector 14 Camera 3



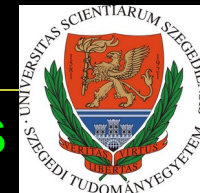
One more year ...

... and, summer  
(of 2019) is coming!

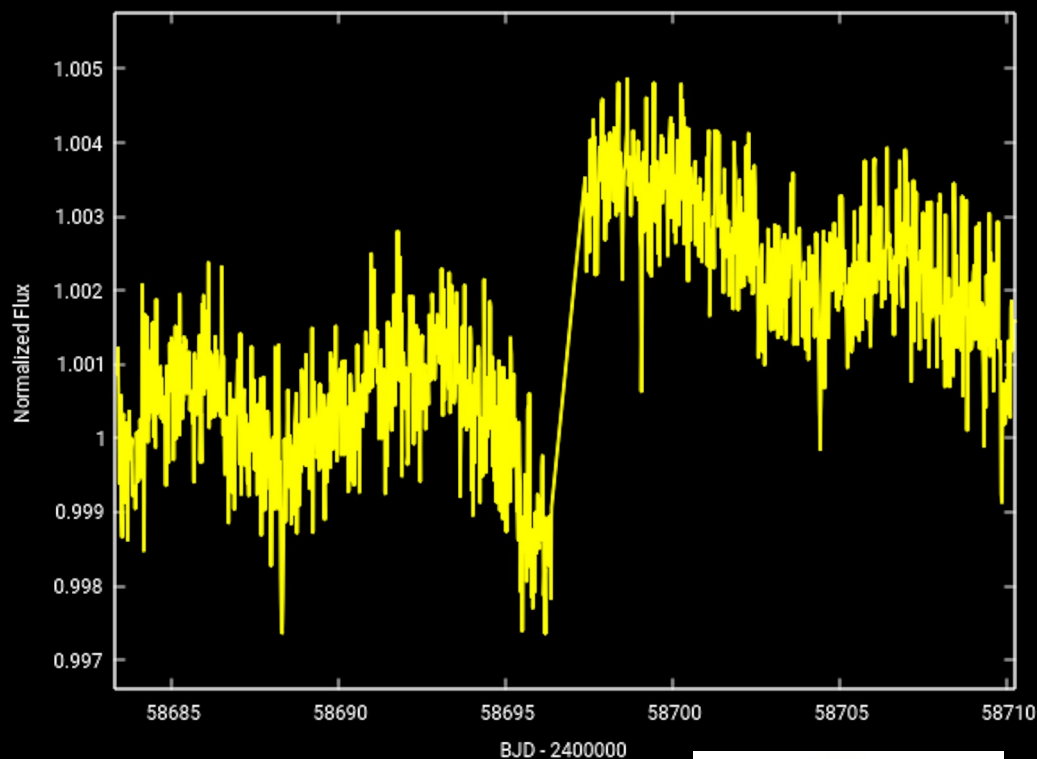
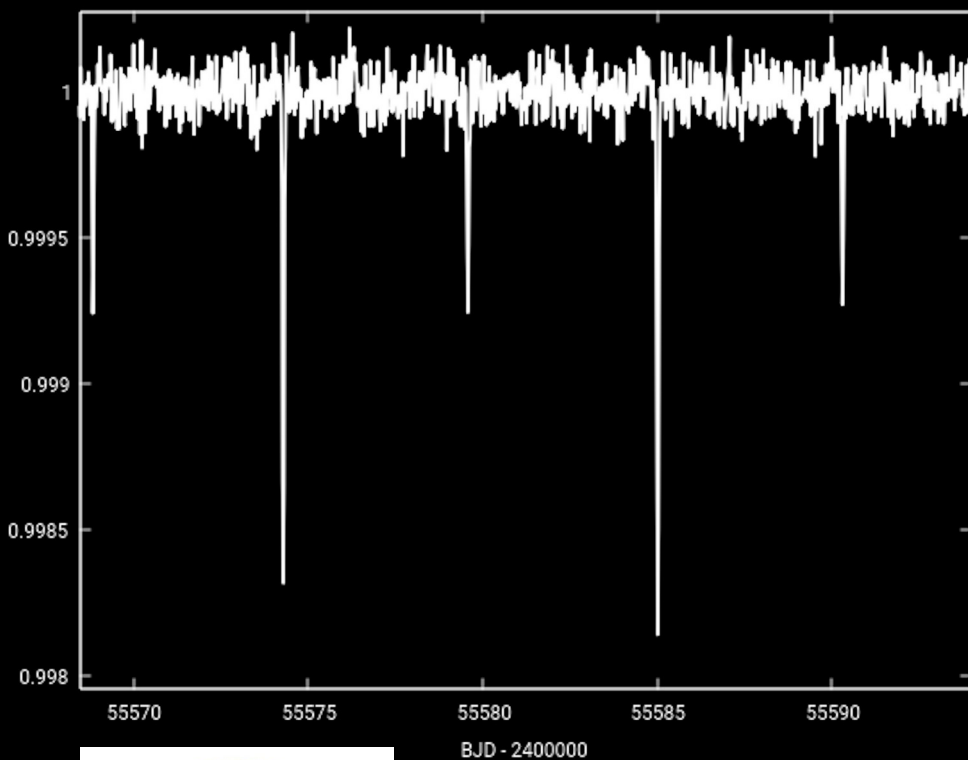


Tamás Borkovits – Then and now:

# Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



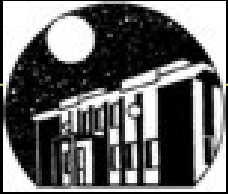
Since then ...



Then ...

... Now



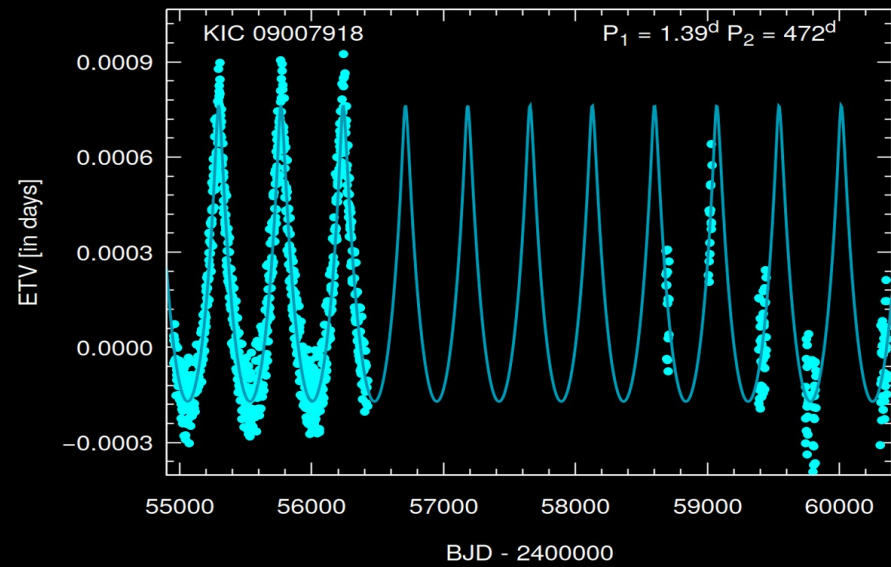
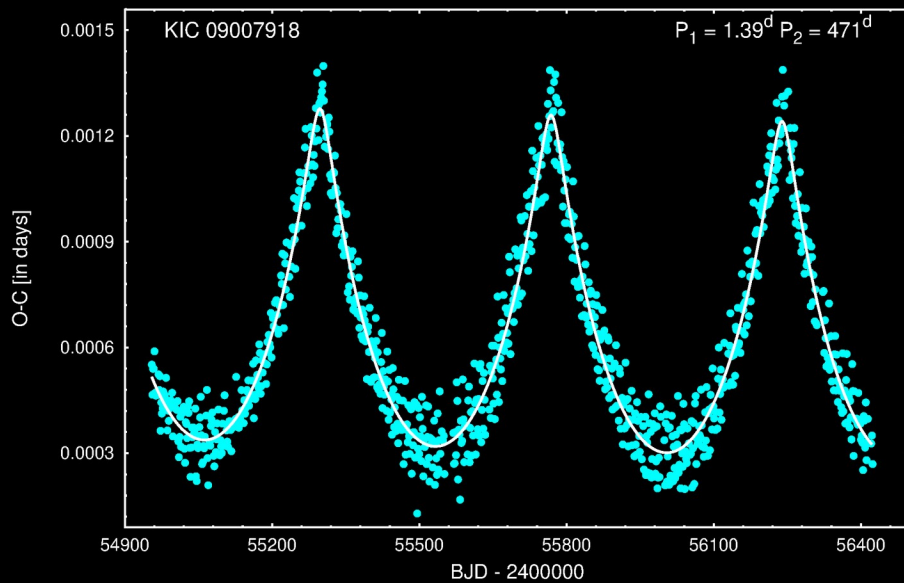
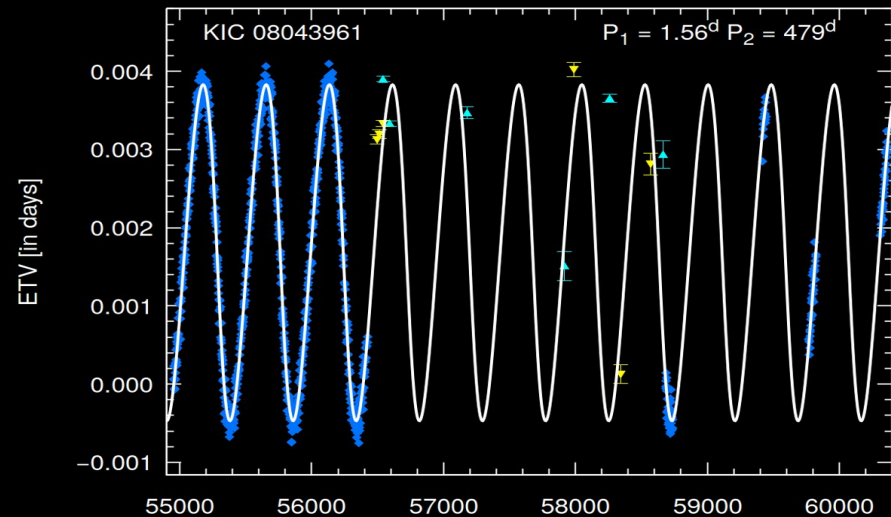
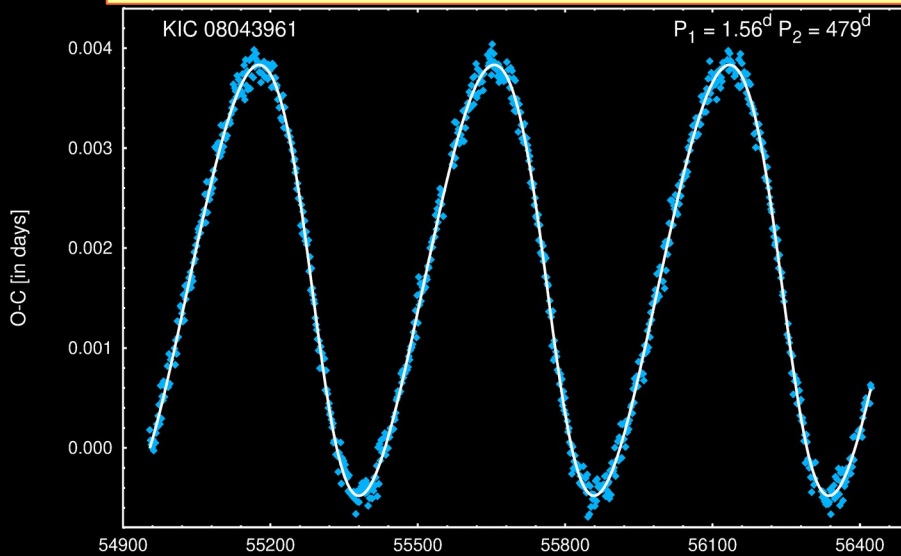


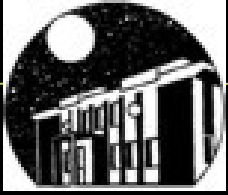
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Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



LTTE systems periods shorter than half of the prime *Kepler* mission



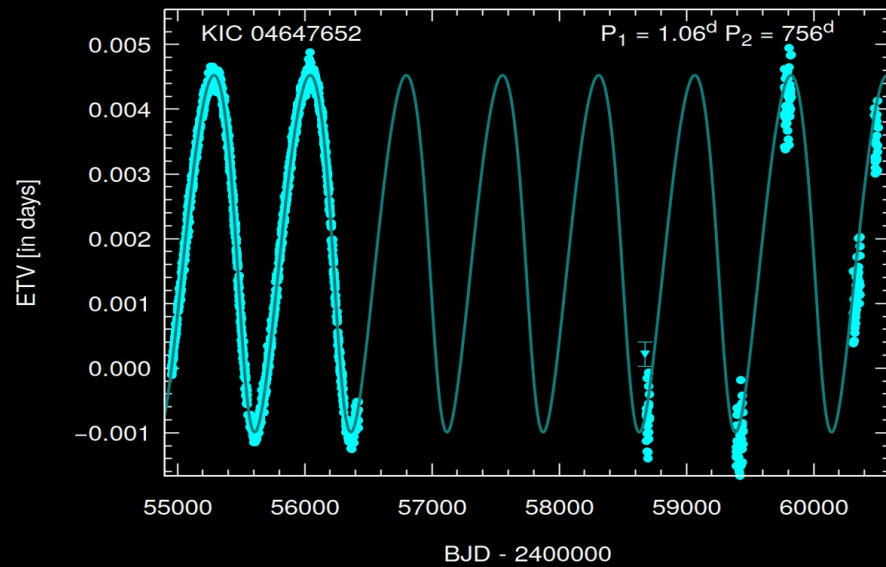
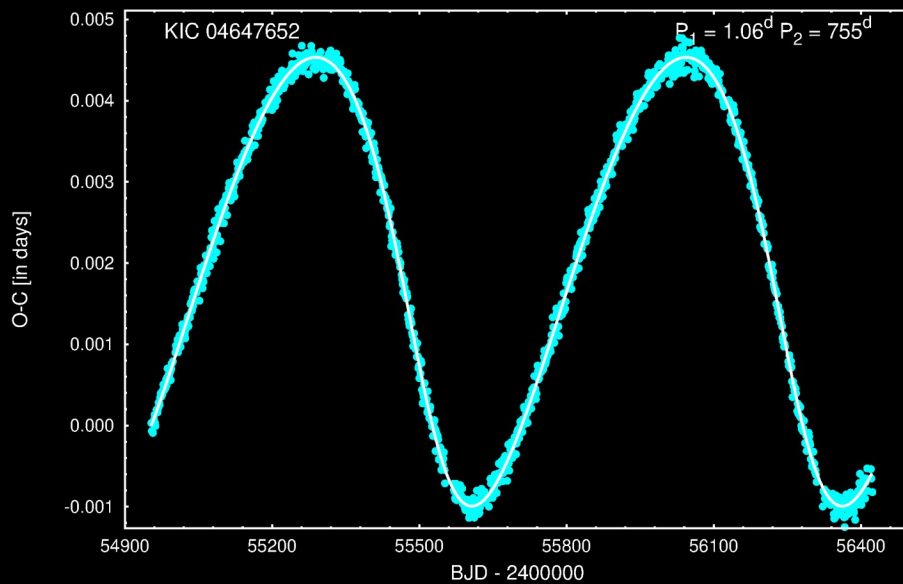
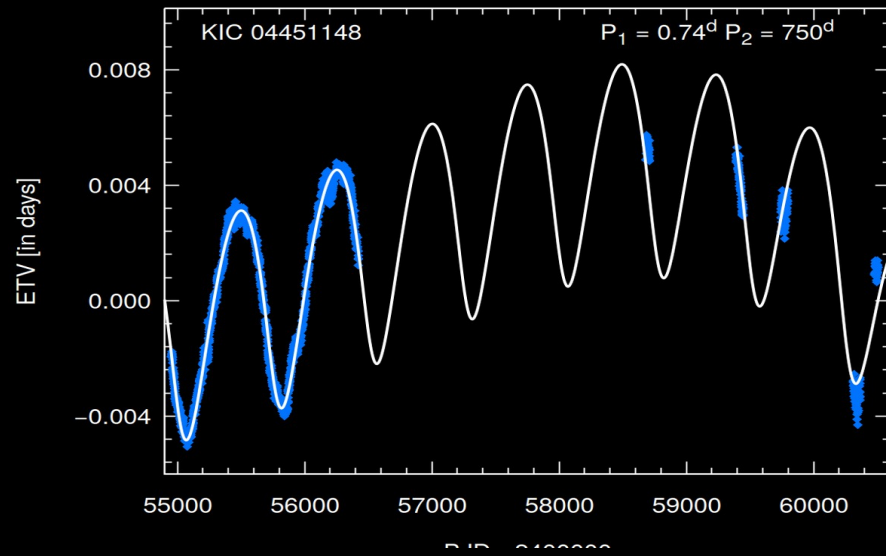
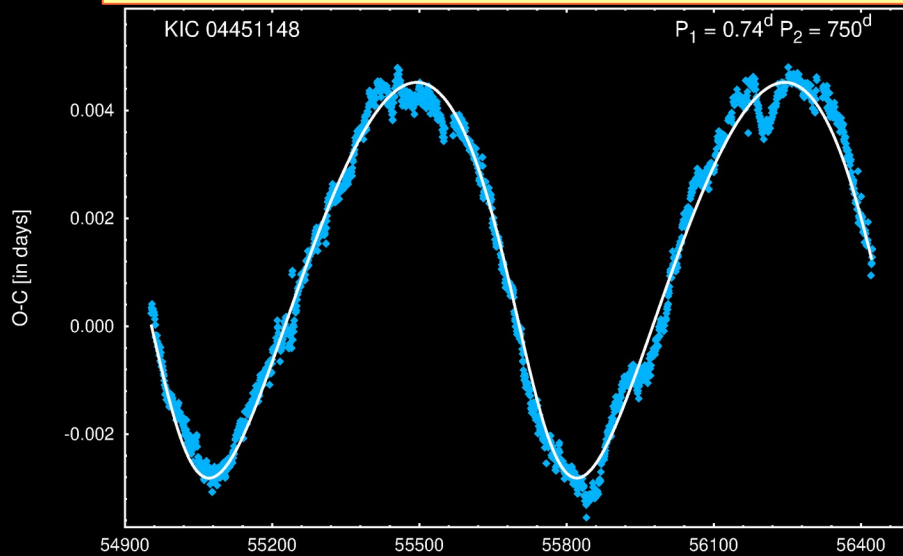


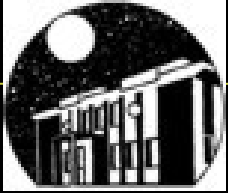
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Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



LTTE systems periods longer than half of the prime *Kepler* mission



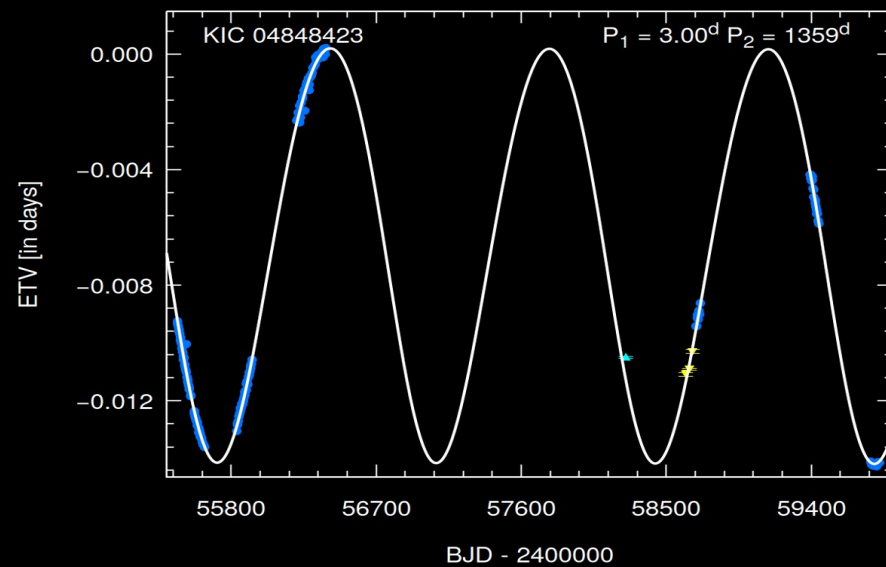
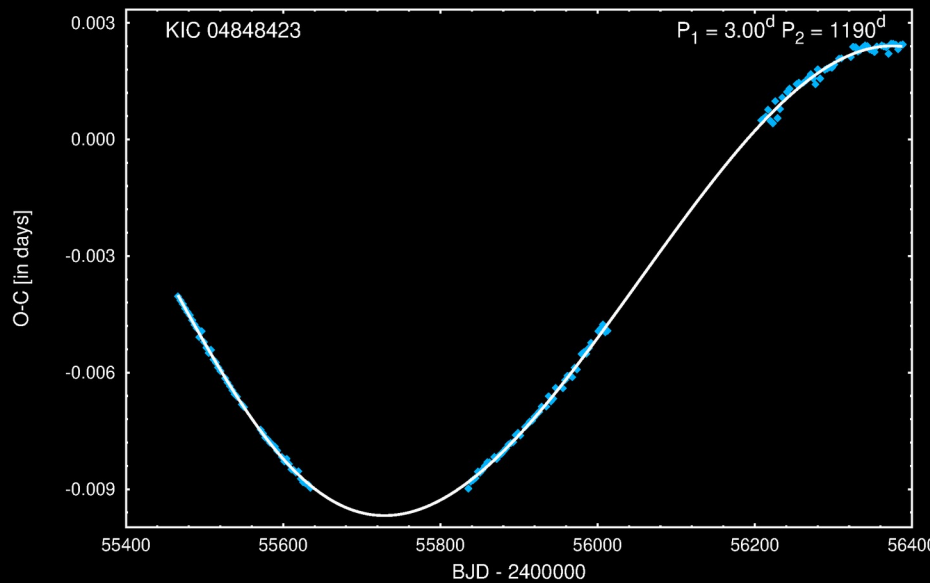
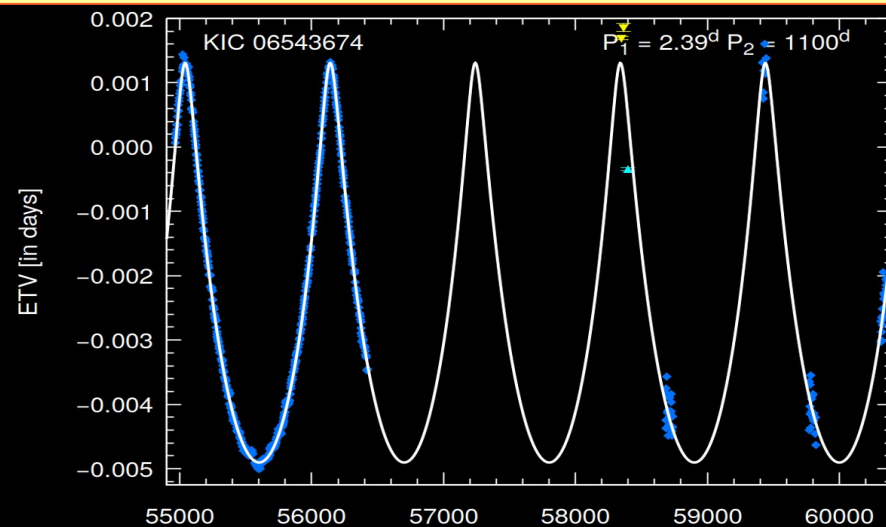
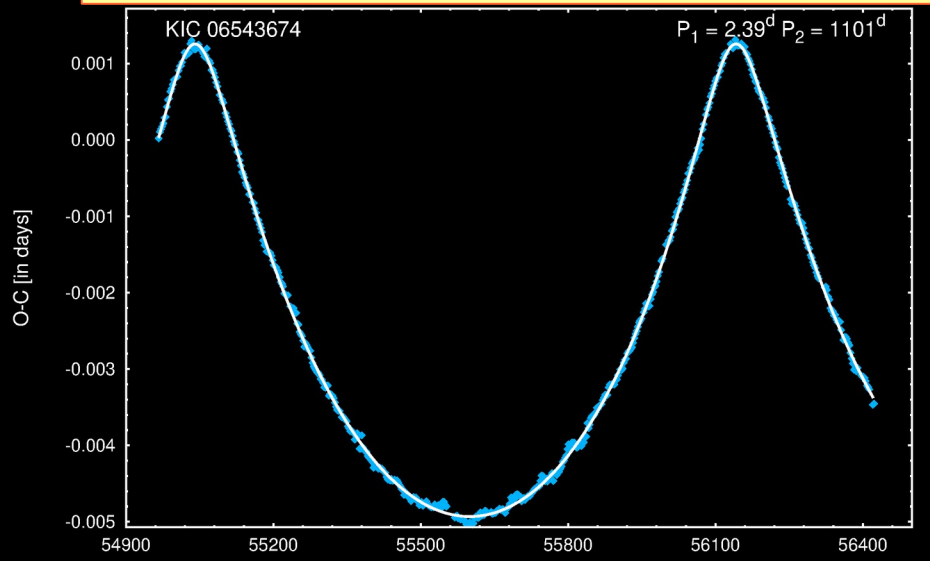


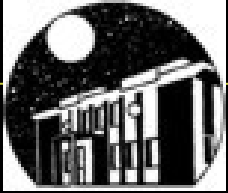
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LTTE systems periods close to the length of the prime *Kepler* mission



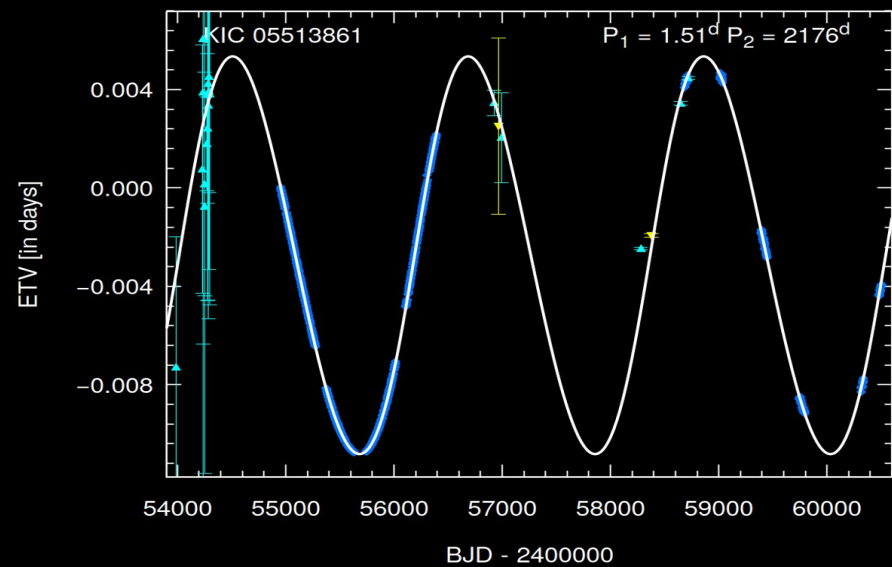
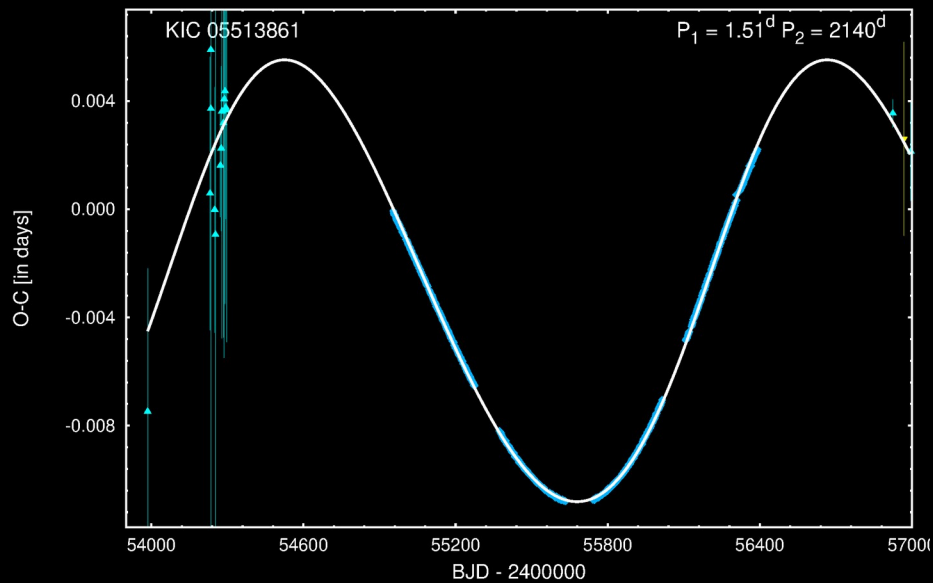
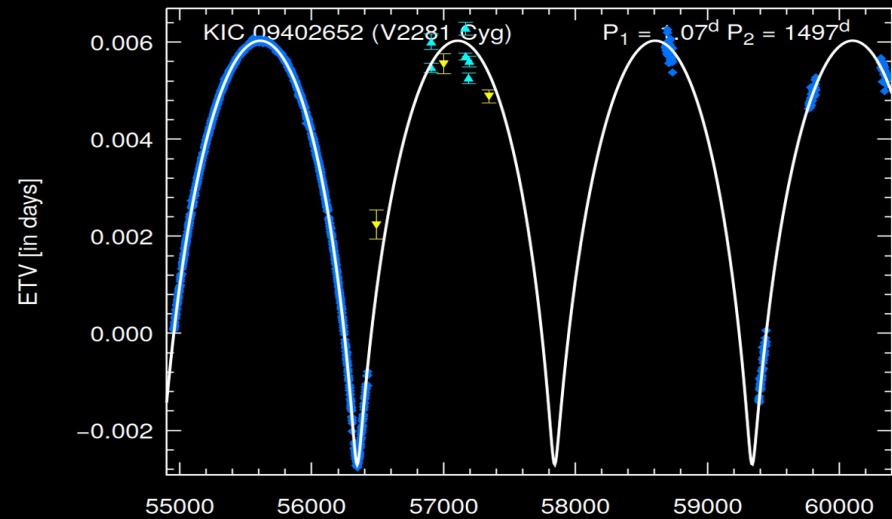
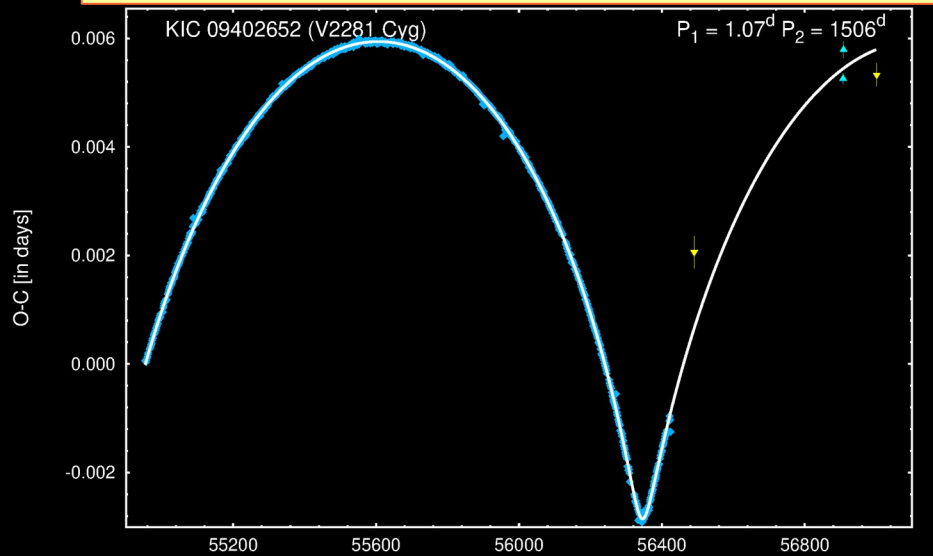


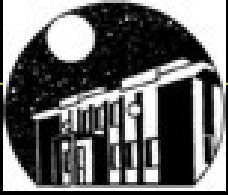
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Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



LTTE systems periods longer than the length of the prime *Kepler* mission



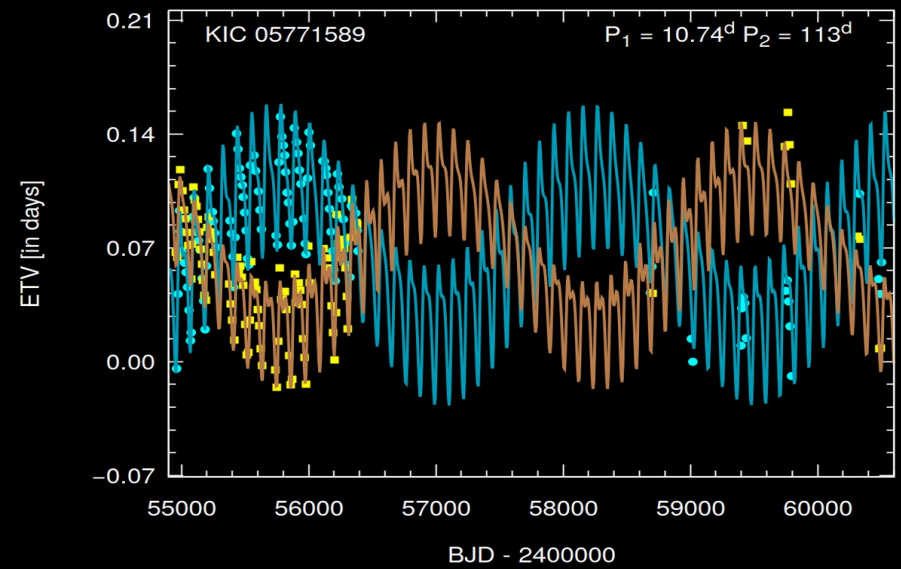
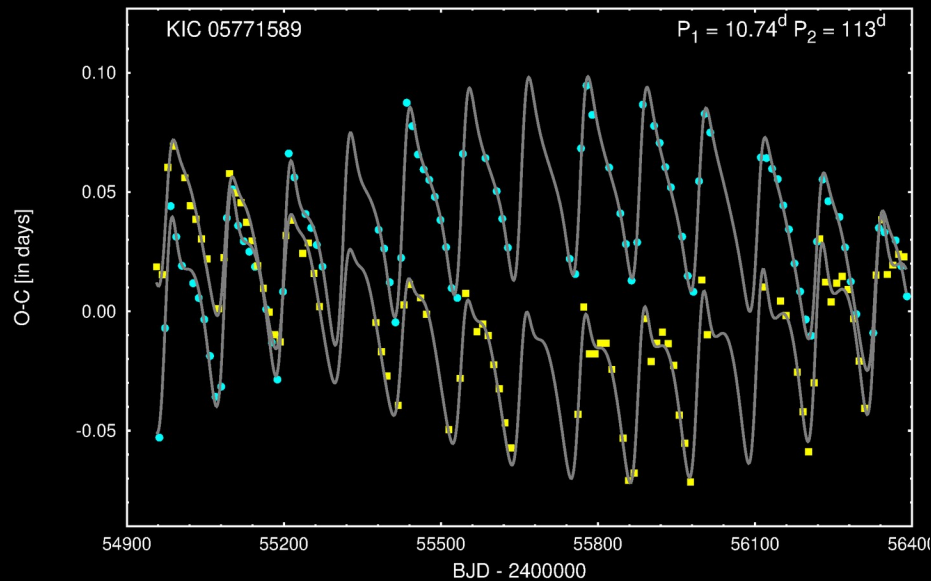
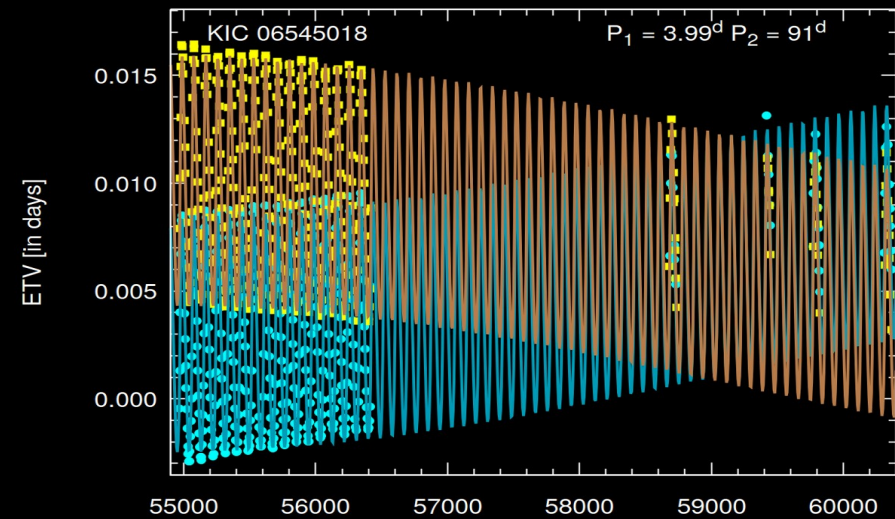
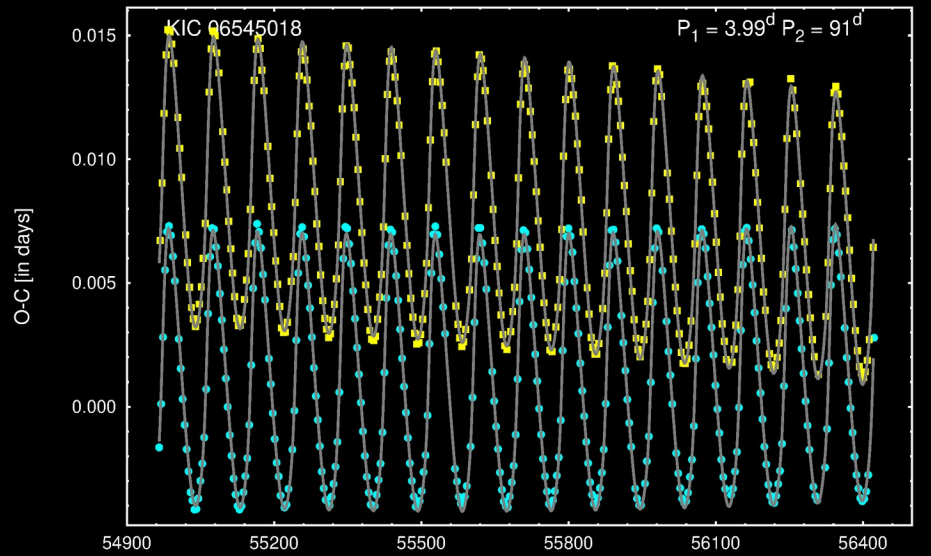


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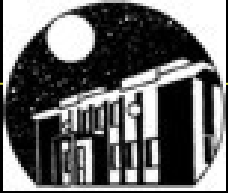
Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



## Extreme tight and compact systems – **HARDCORE** triples





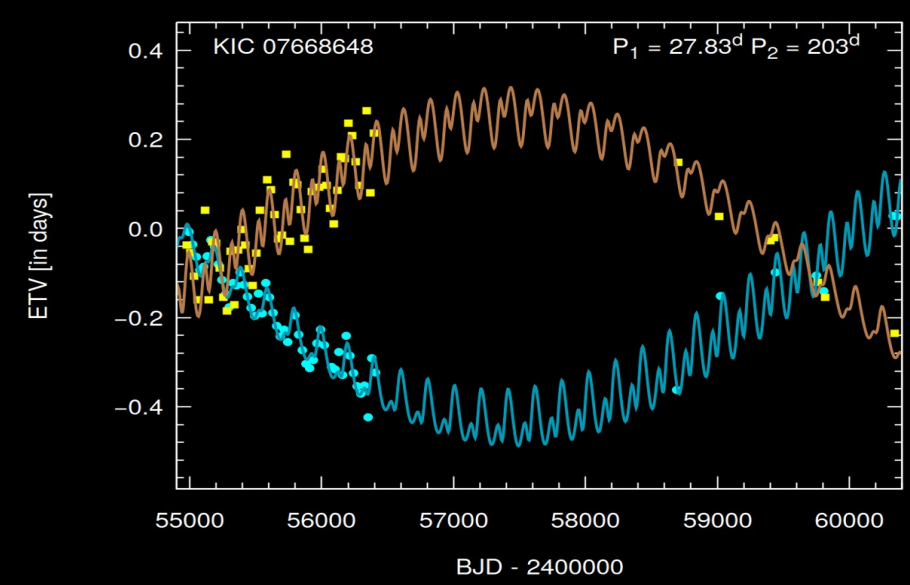
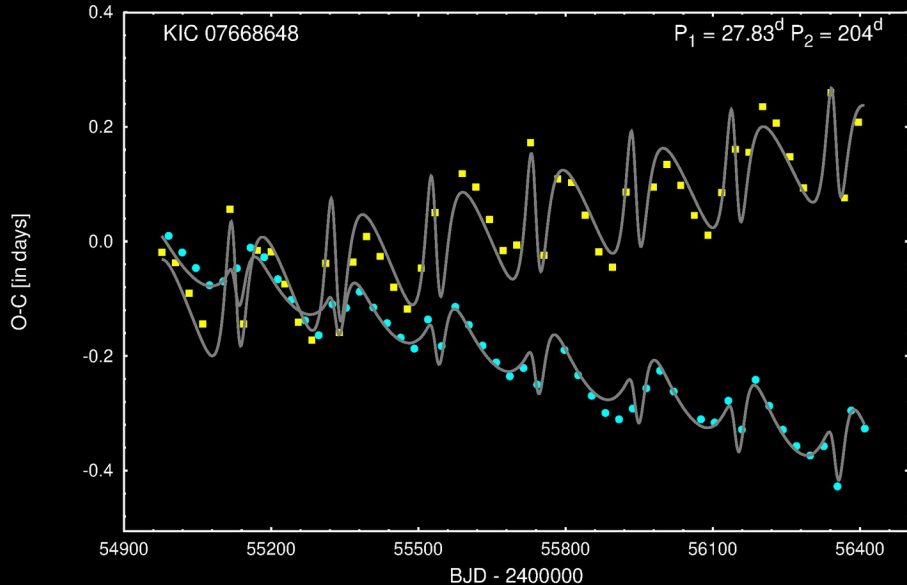
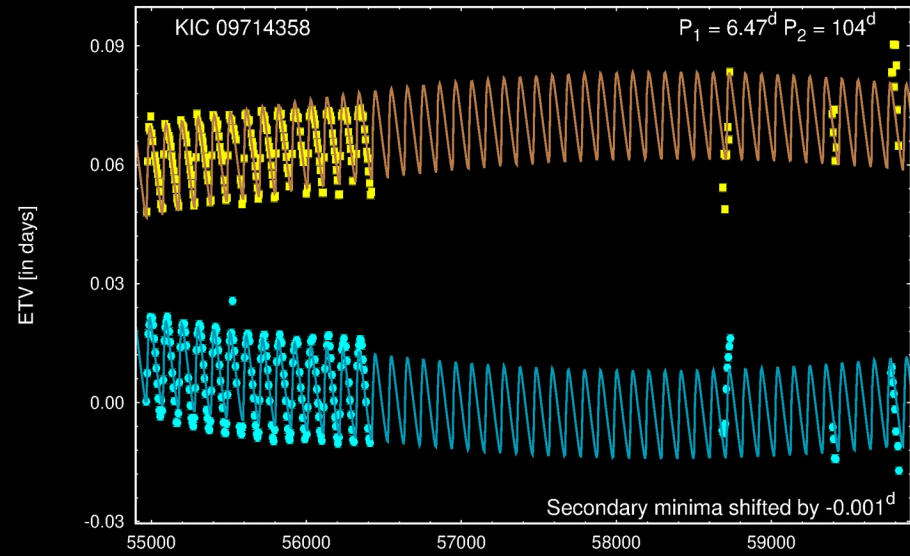
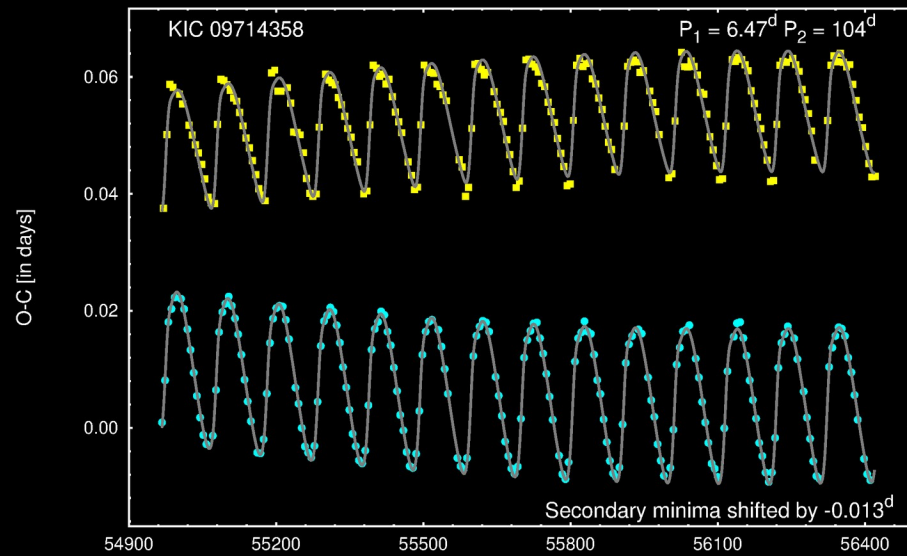


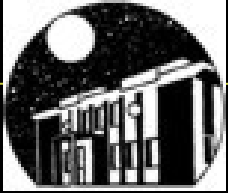
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Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



## Extreme tight and compact systems – HARDCORE triples



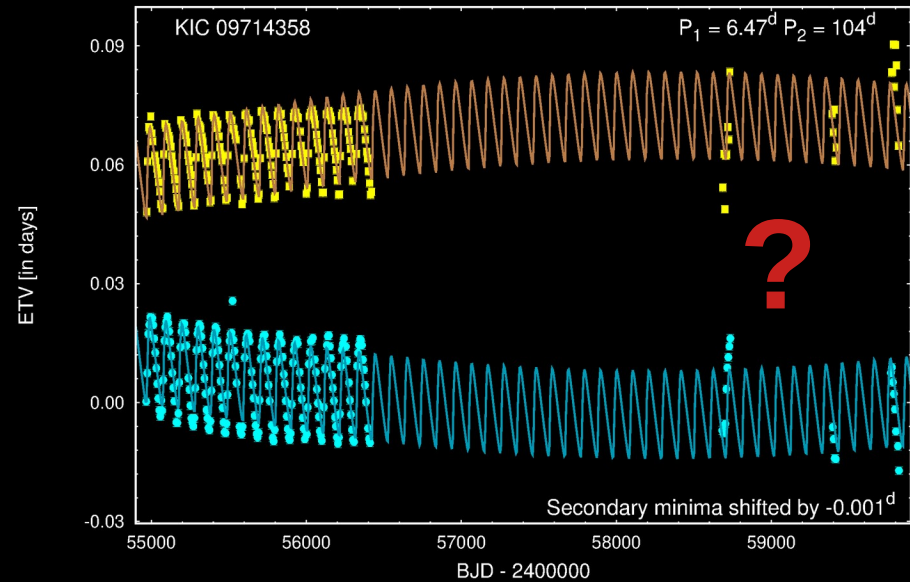
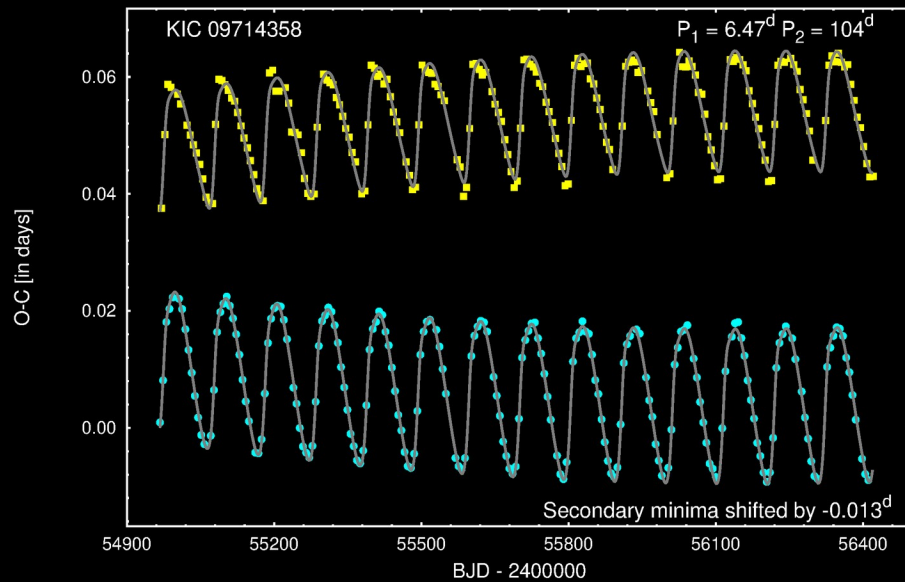


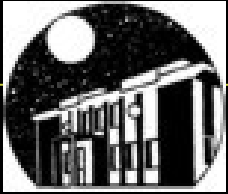
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# Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



## Extreme tight and compact systems – HARDCORE triples



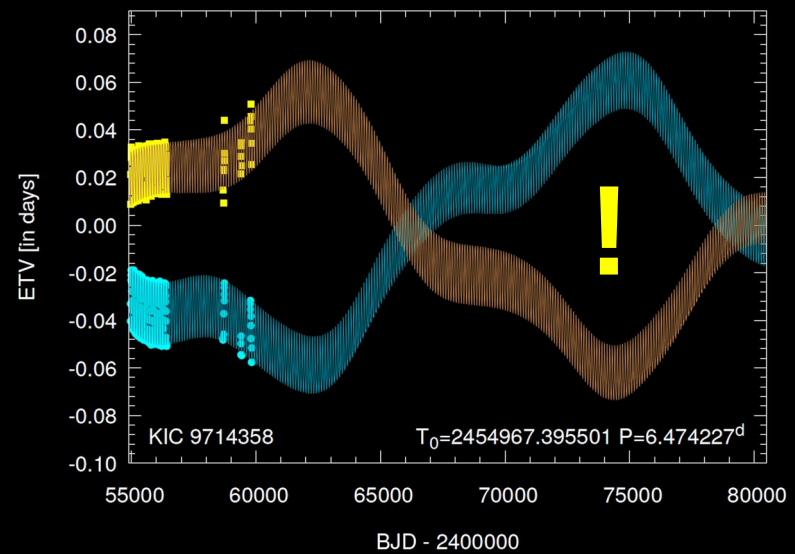
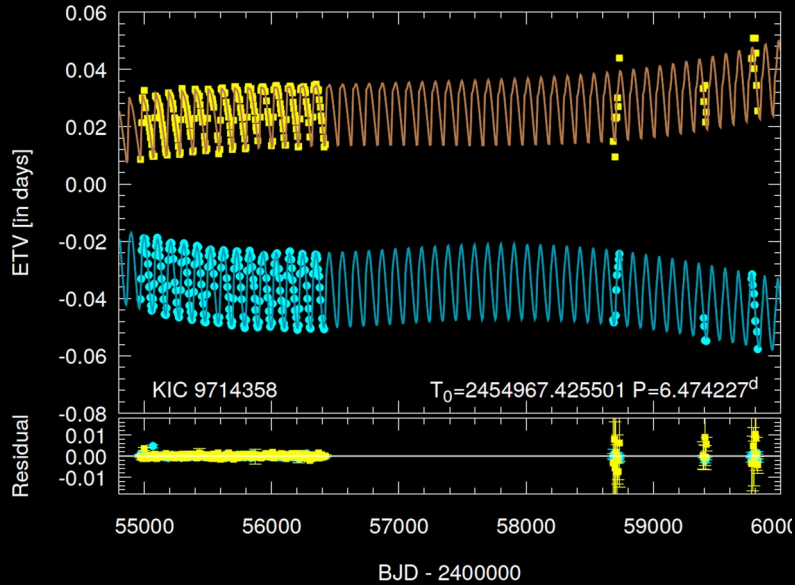
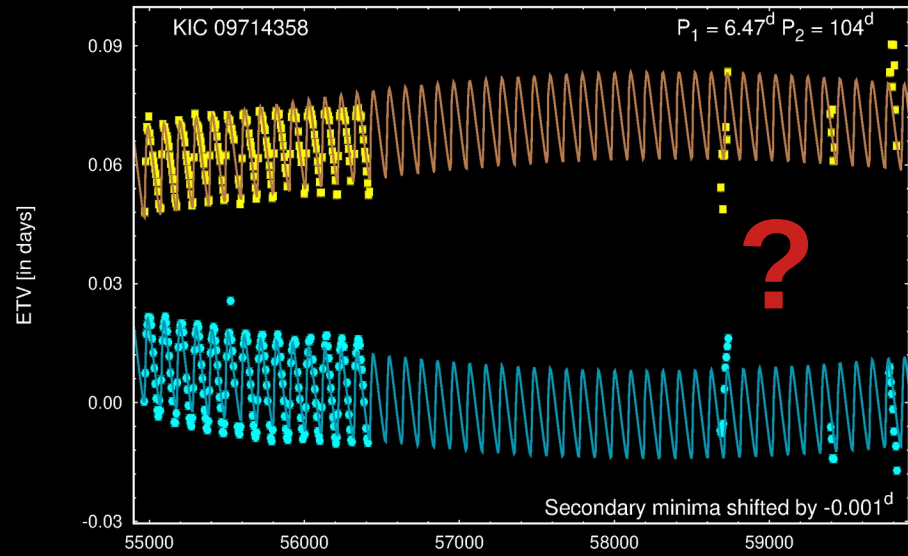
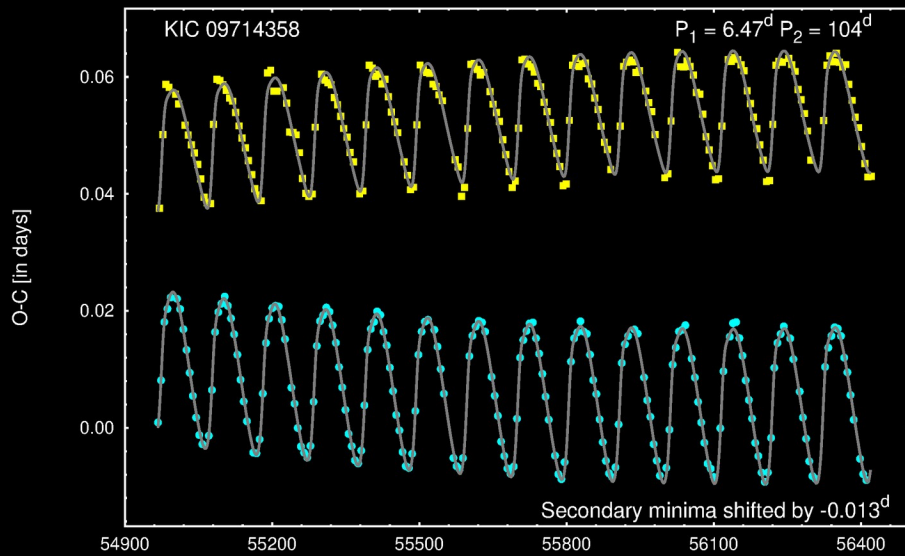


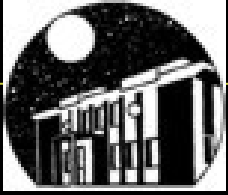
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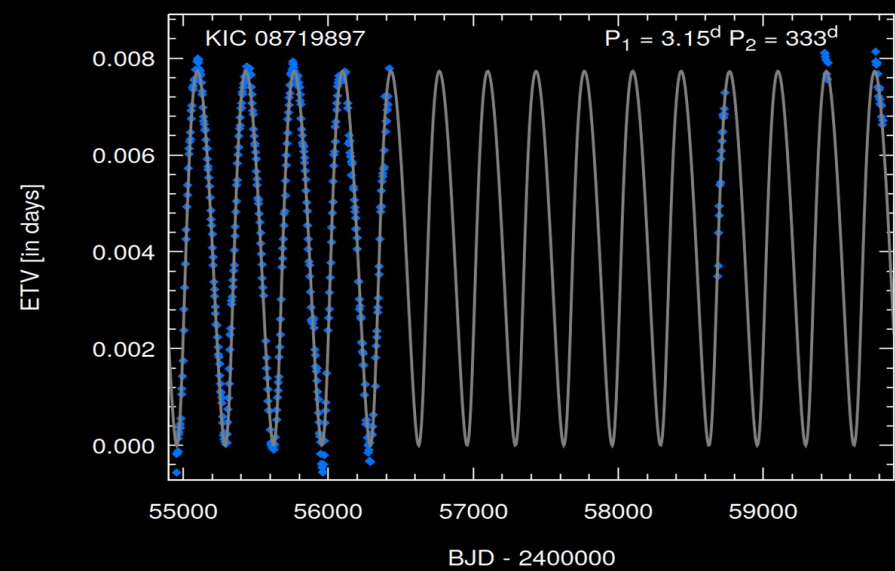
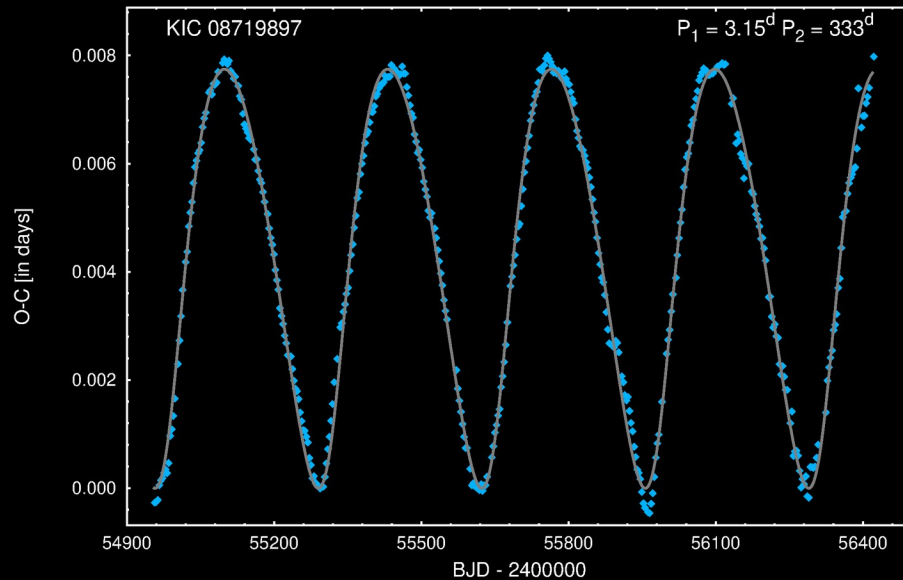
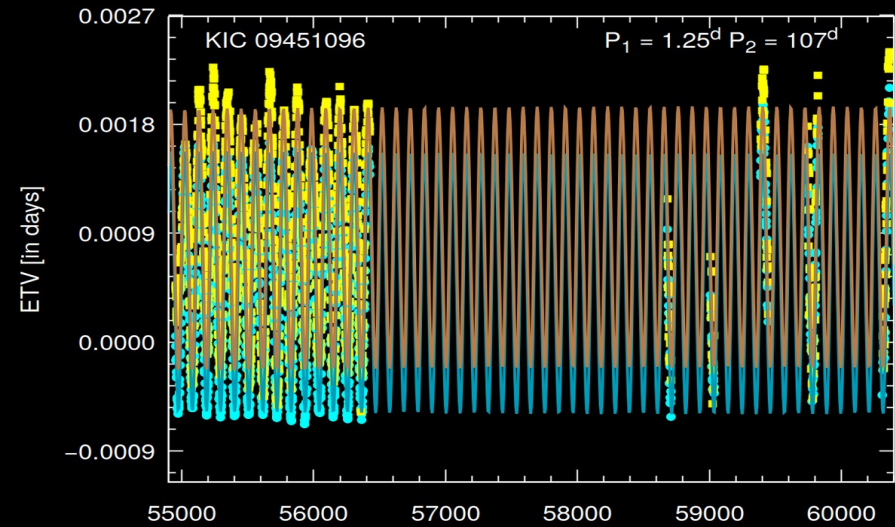
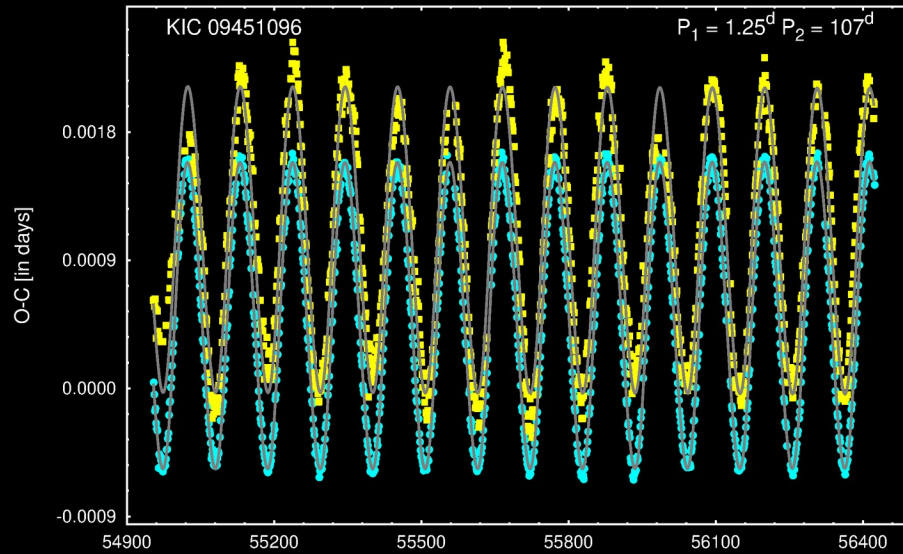


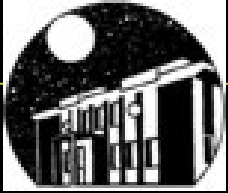
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Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



Not so tight, but compact systems



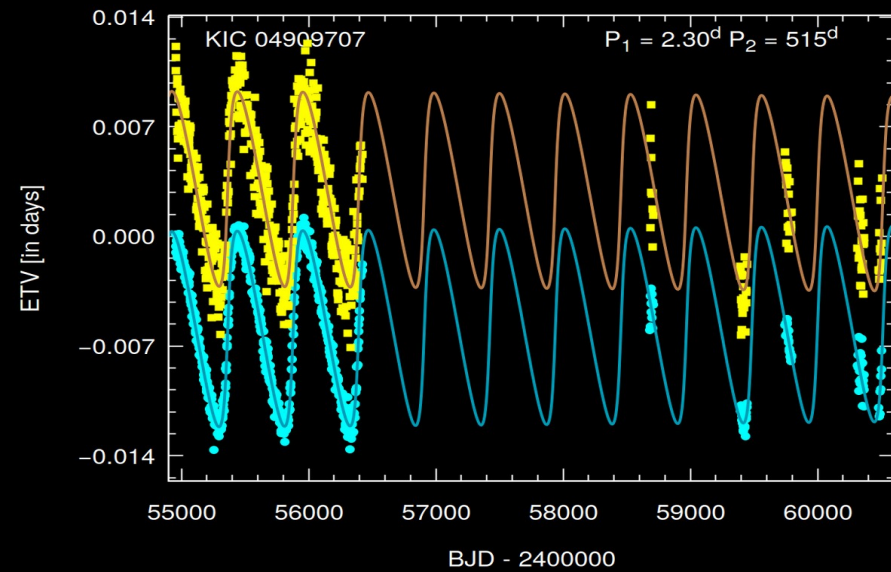
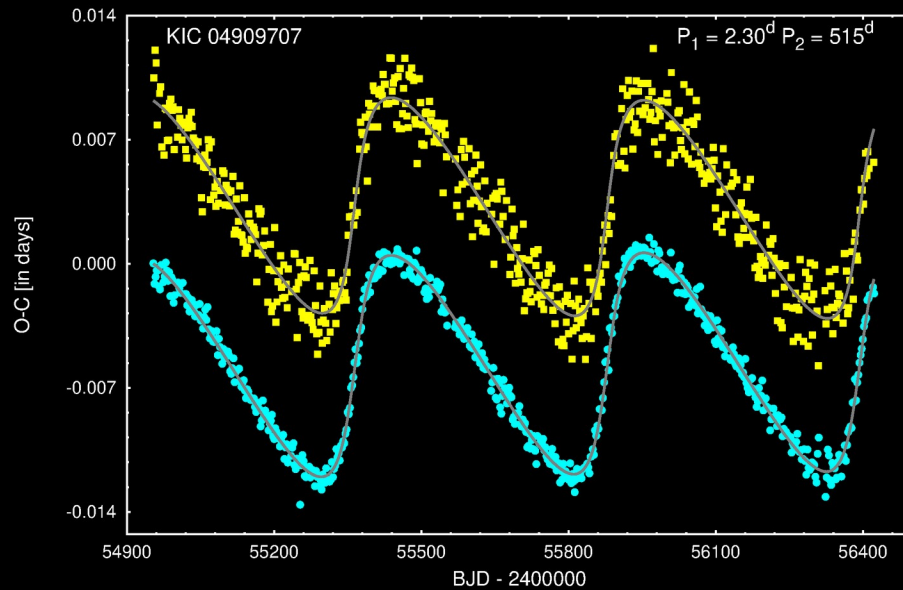
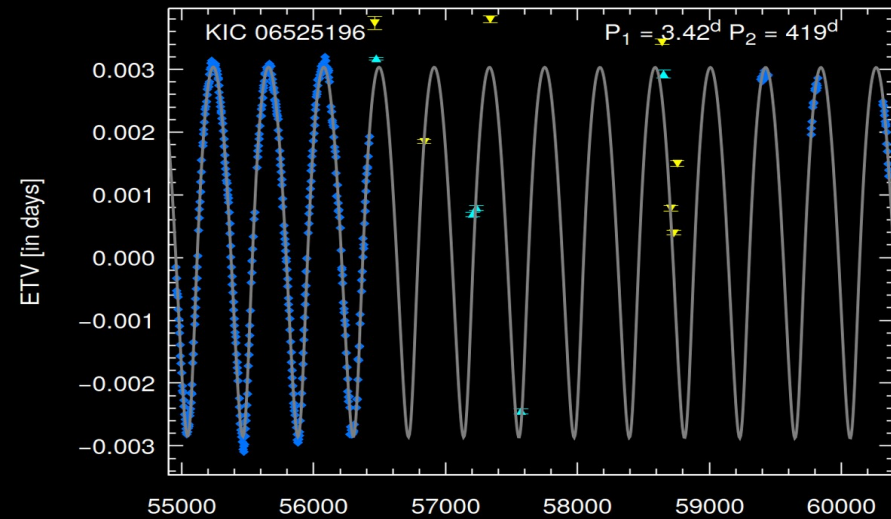
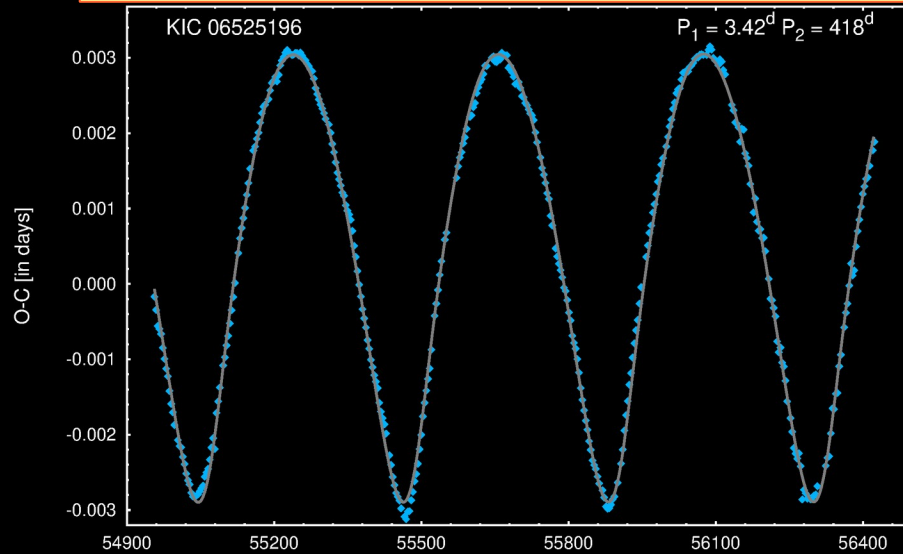


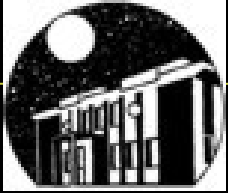
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## Further short-period triples with 3-rd body perturbations



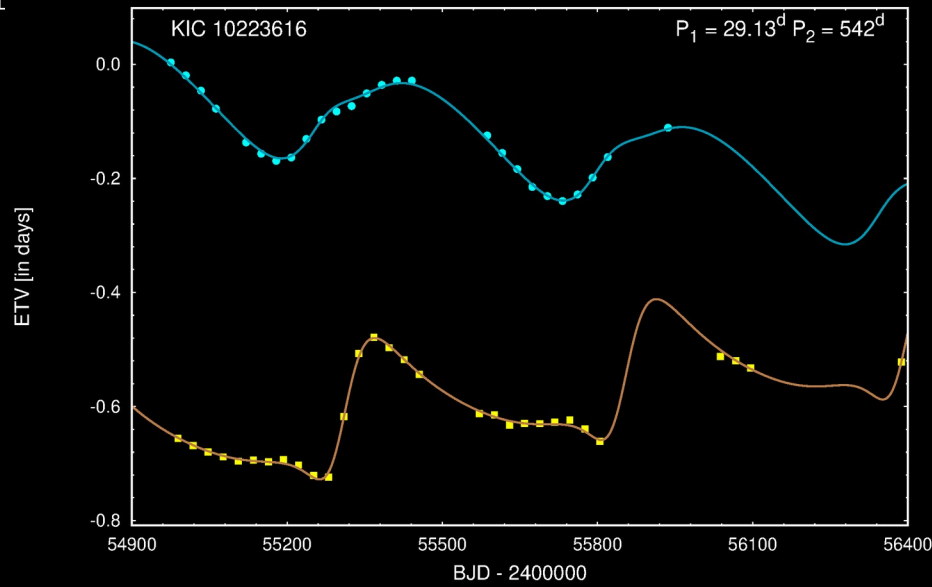
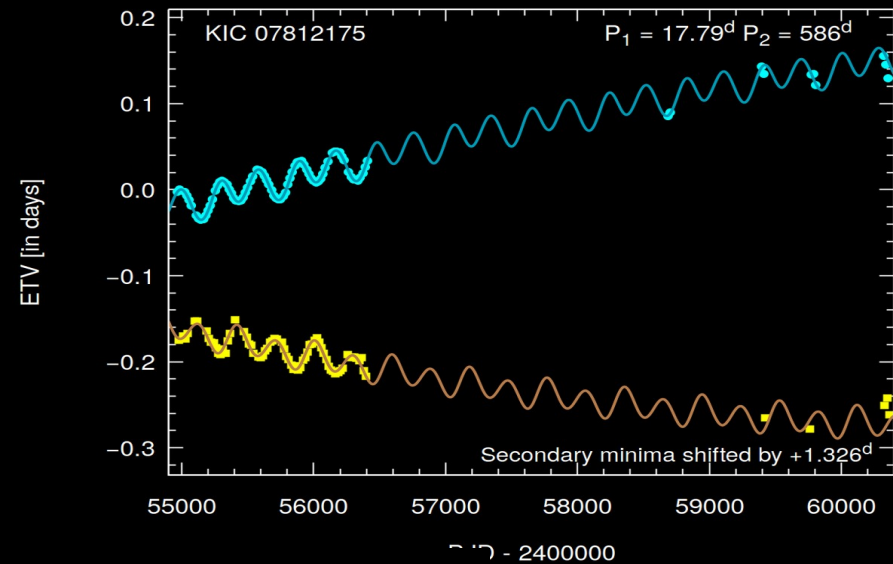
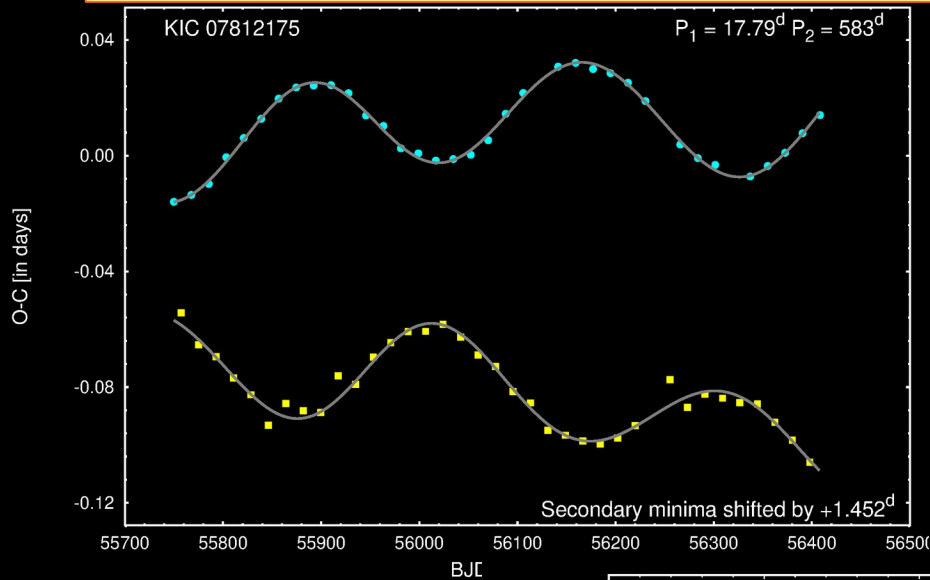


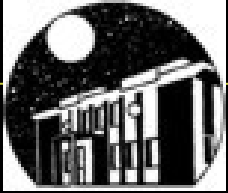
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Further shorter period triples with 3-rd body perturbations



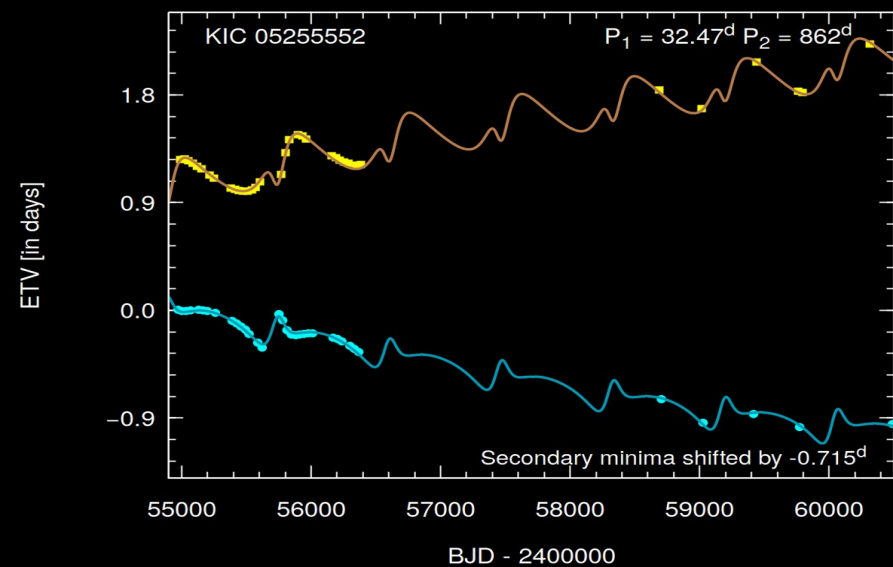
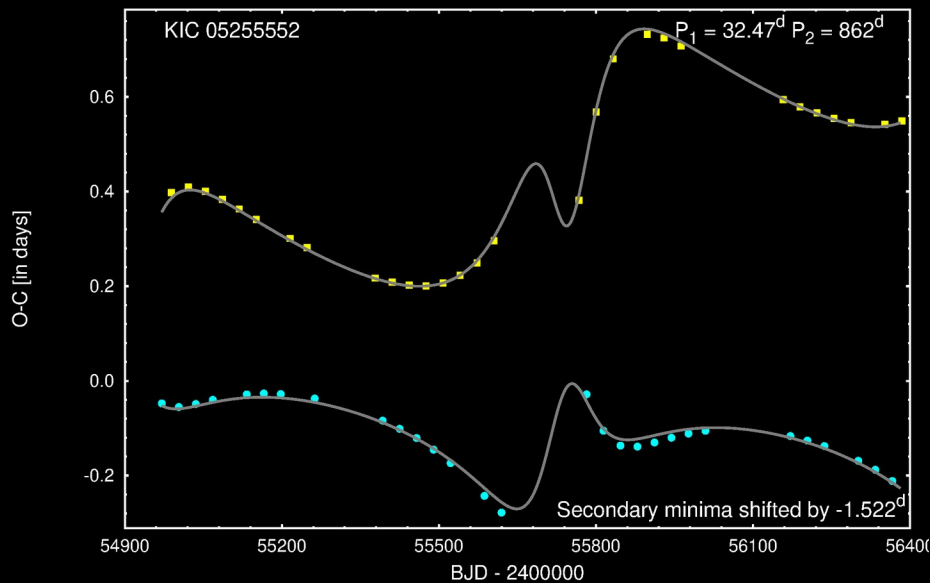
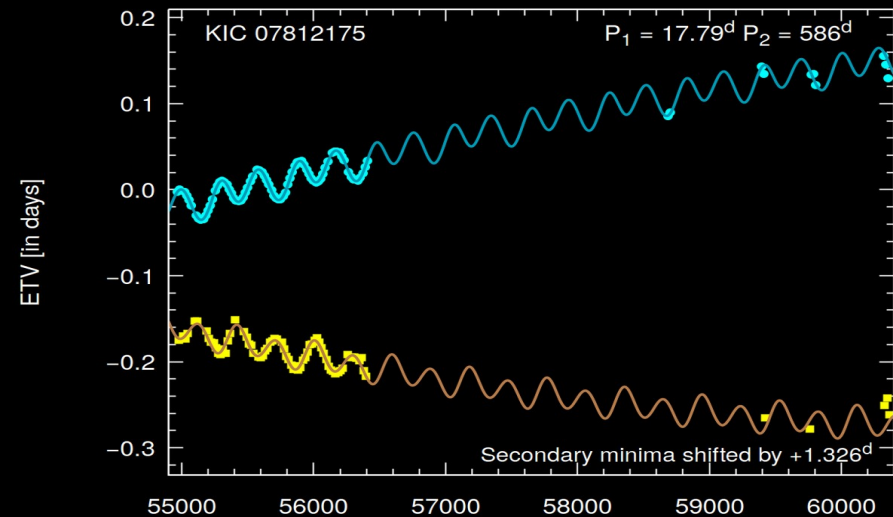
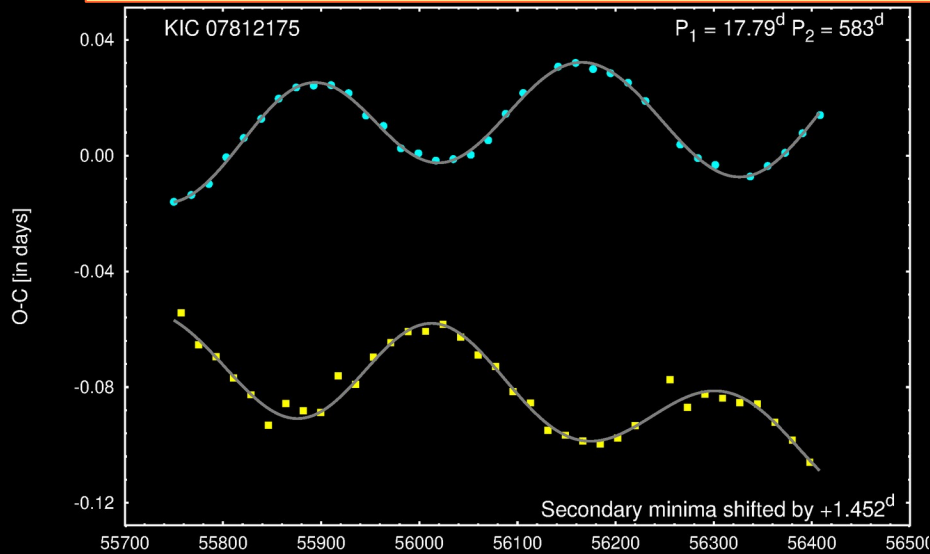


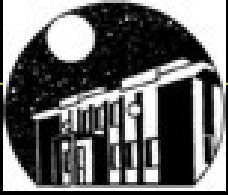
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Further shorter period triples with 3-rd body perturbations



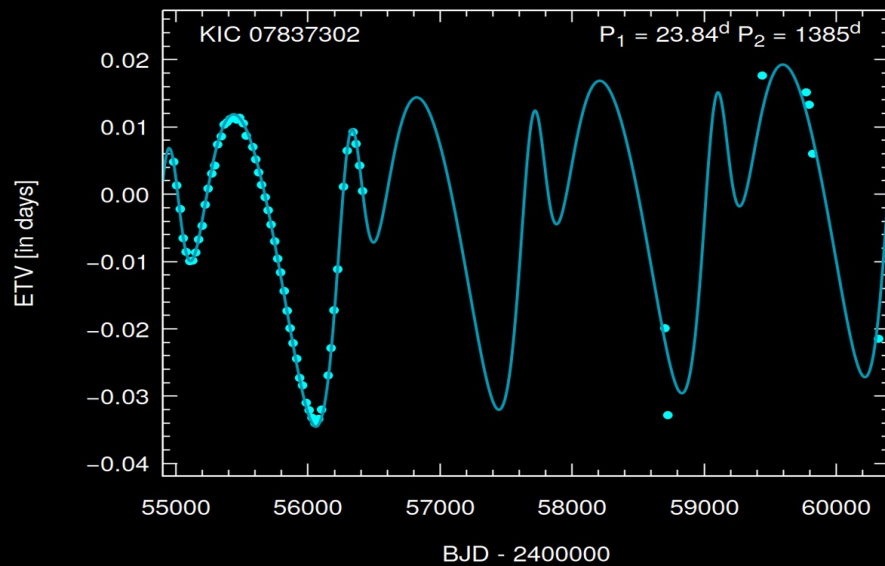
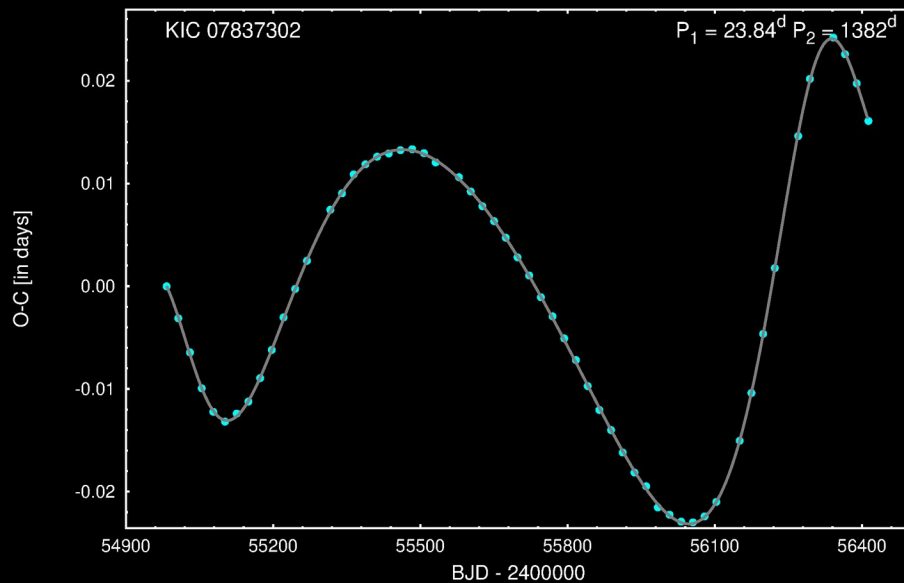
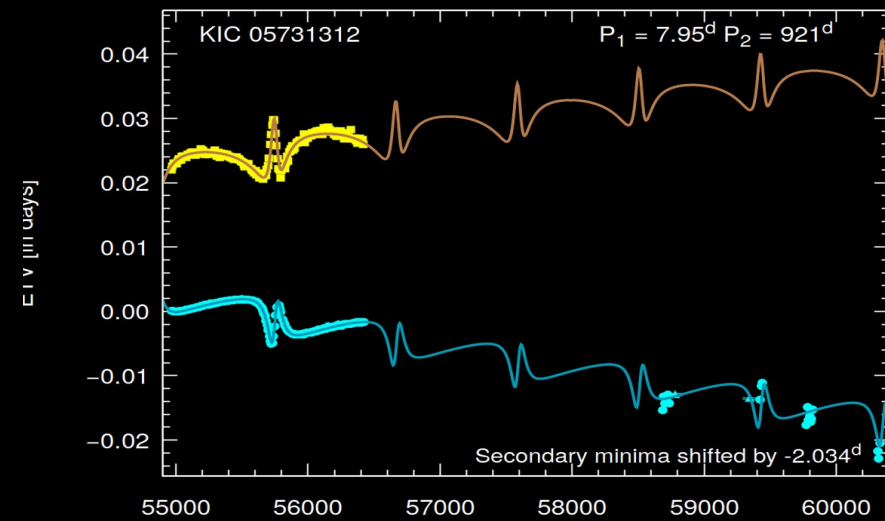
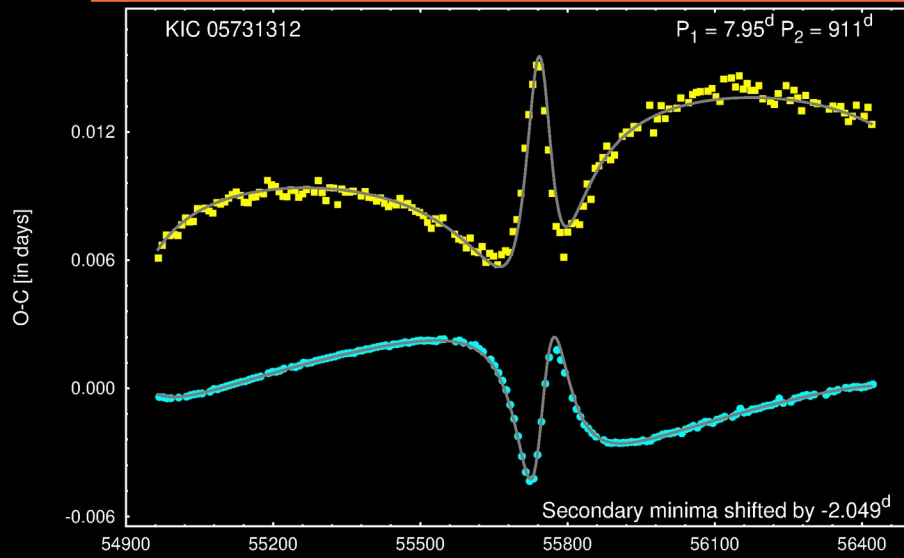


Tamás Borkovits – Then and now:

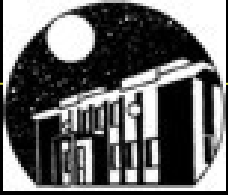
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## Longer period triples with 3-rd body perturbations





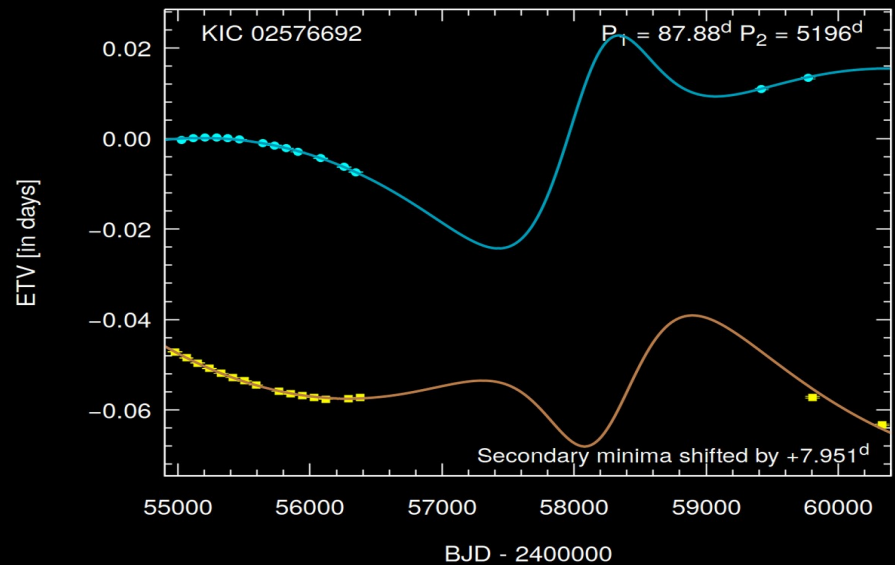
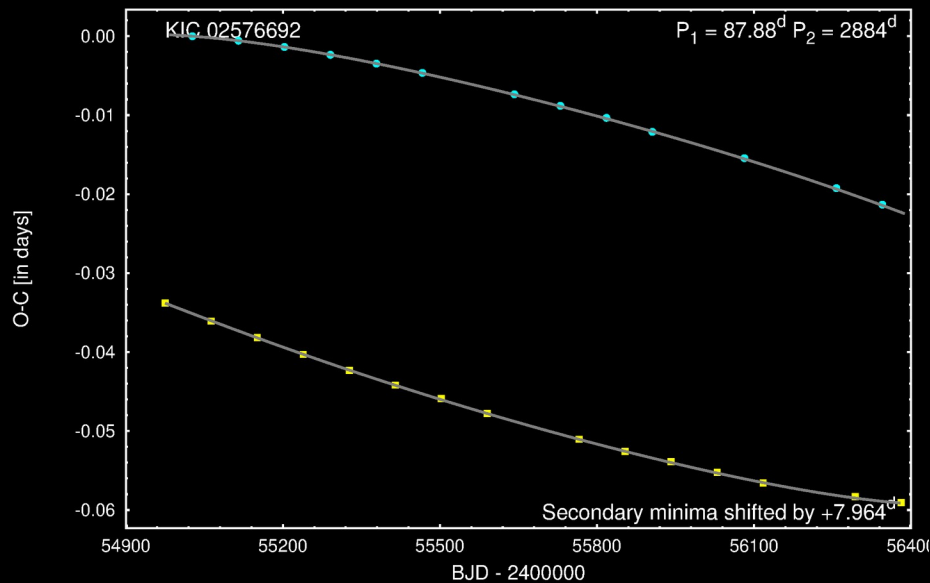
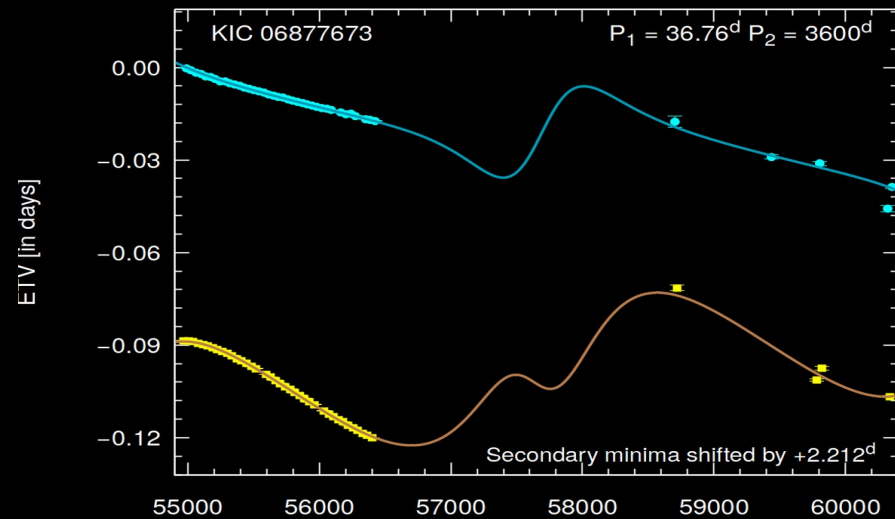
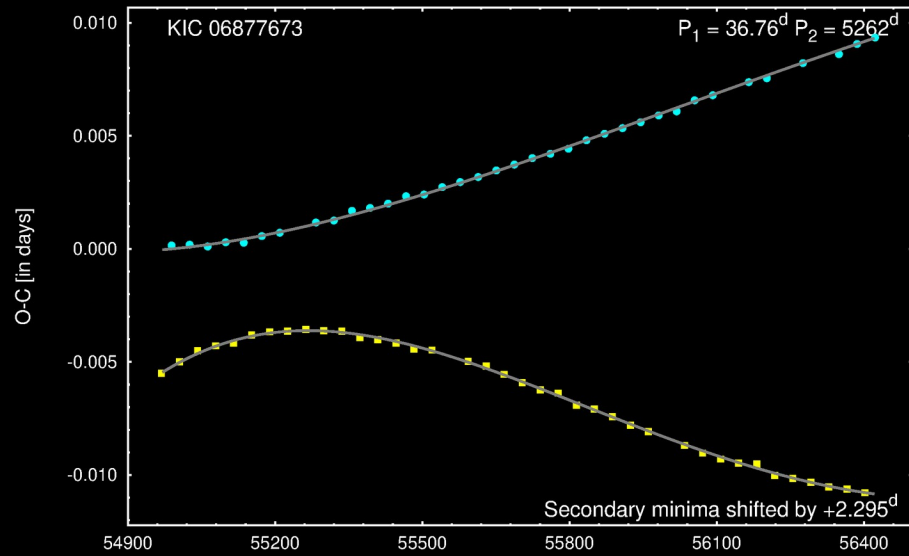


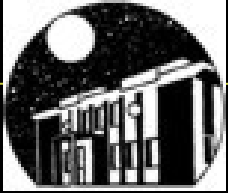
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Longer period triples with 3-rd body perturbations (category: JOKE)



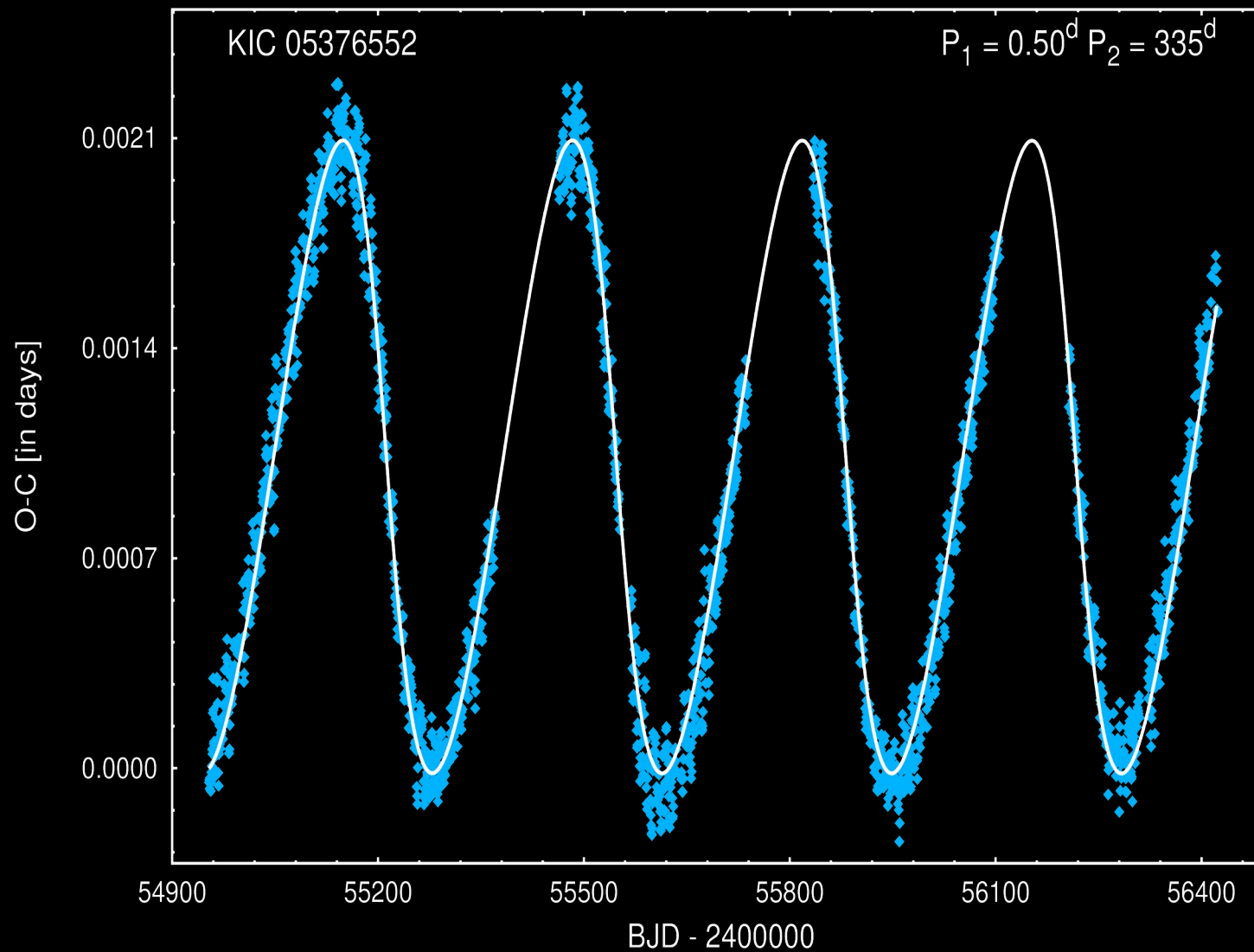


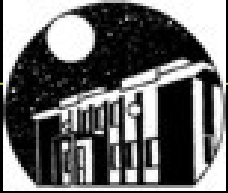
Tamás Borkovits – Then and now:

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### Other funny ETVs



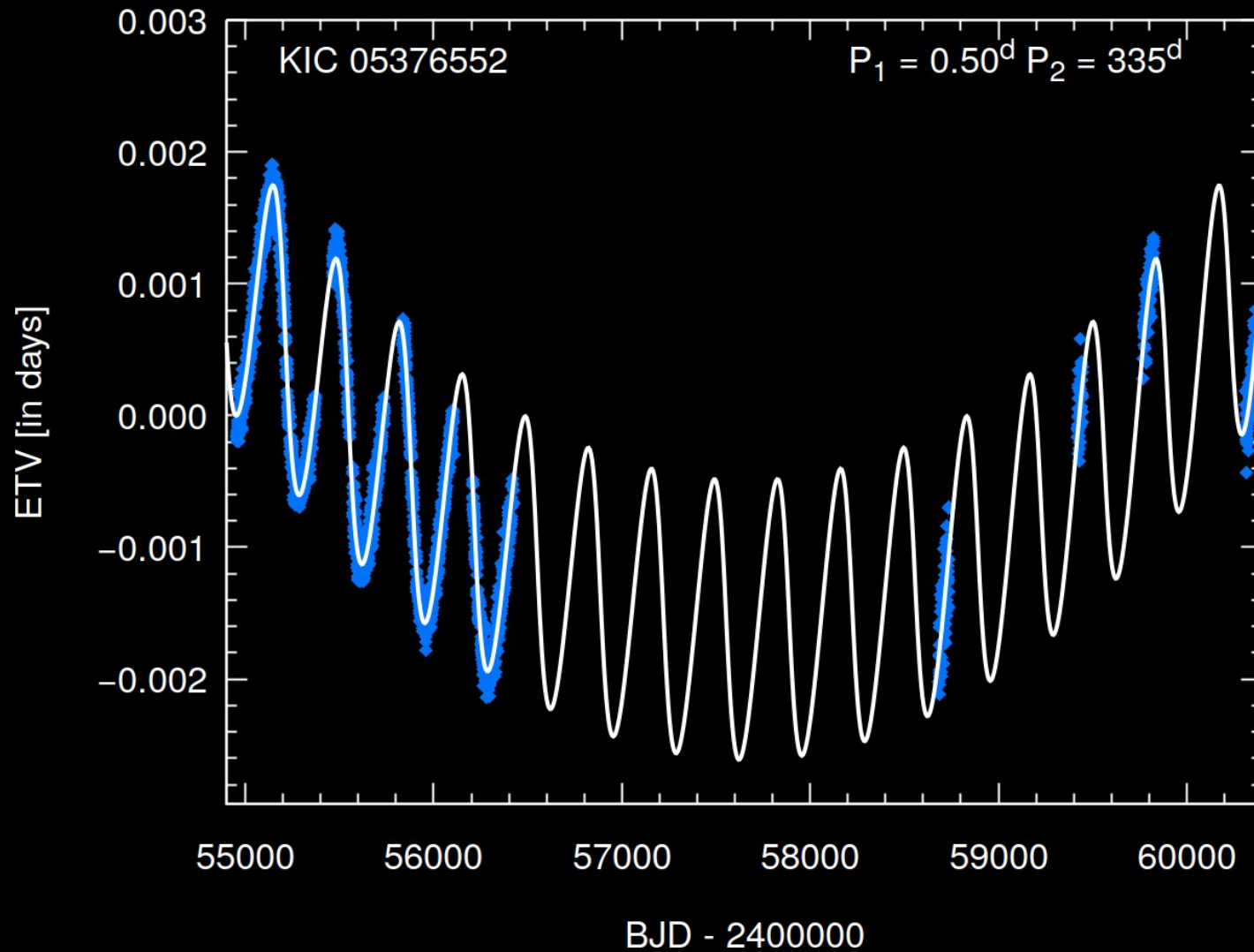


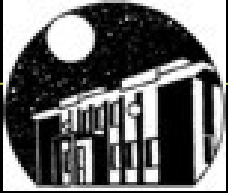
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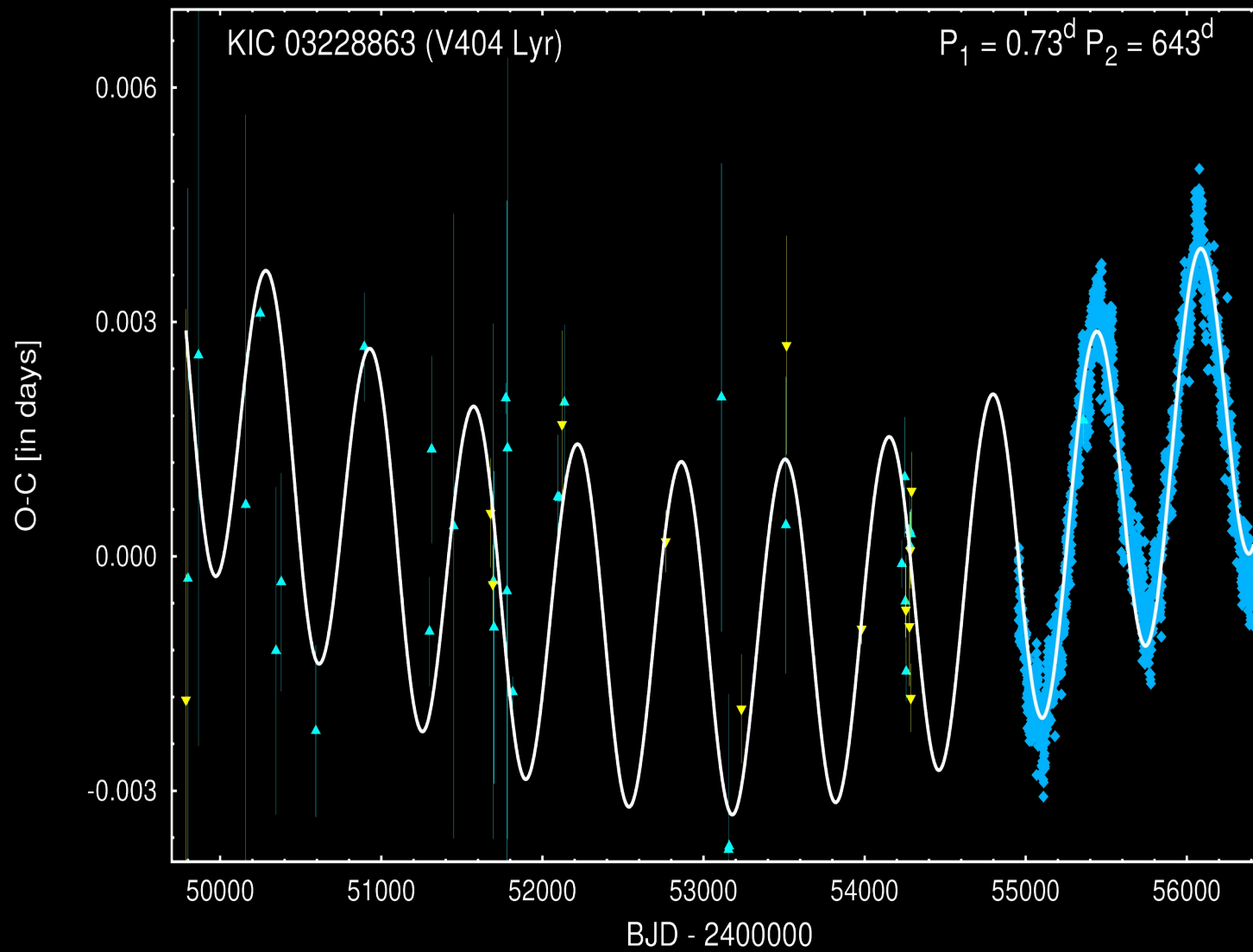


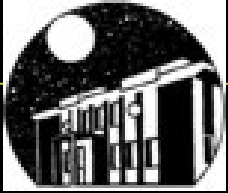
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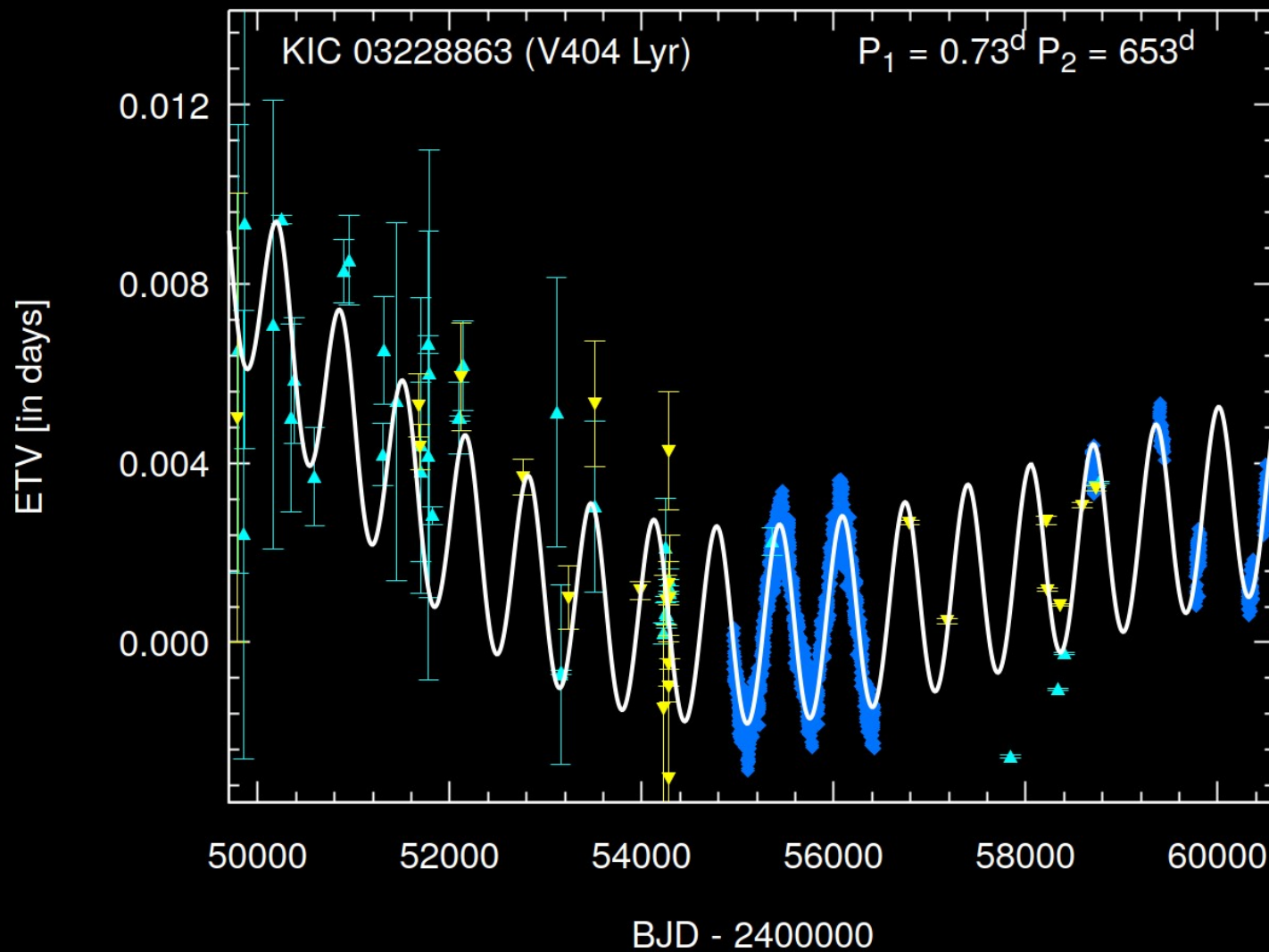


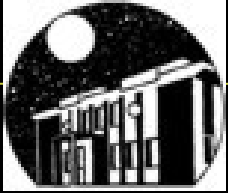
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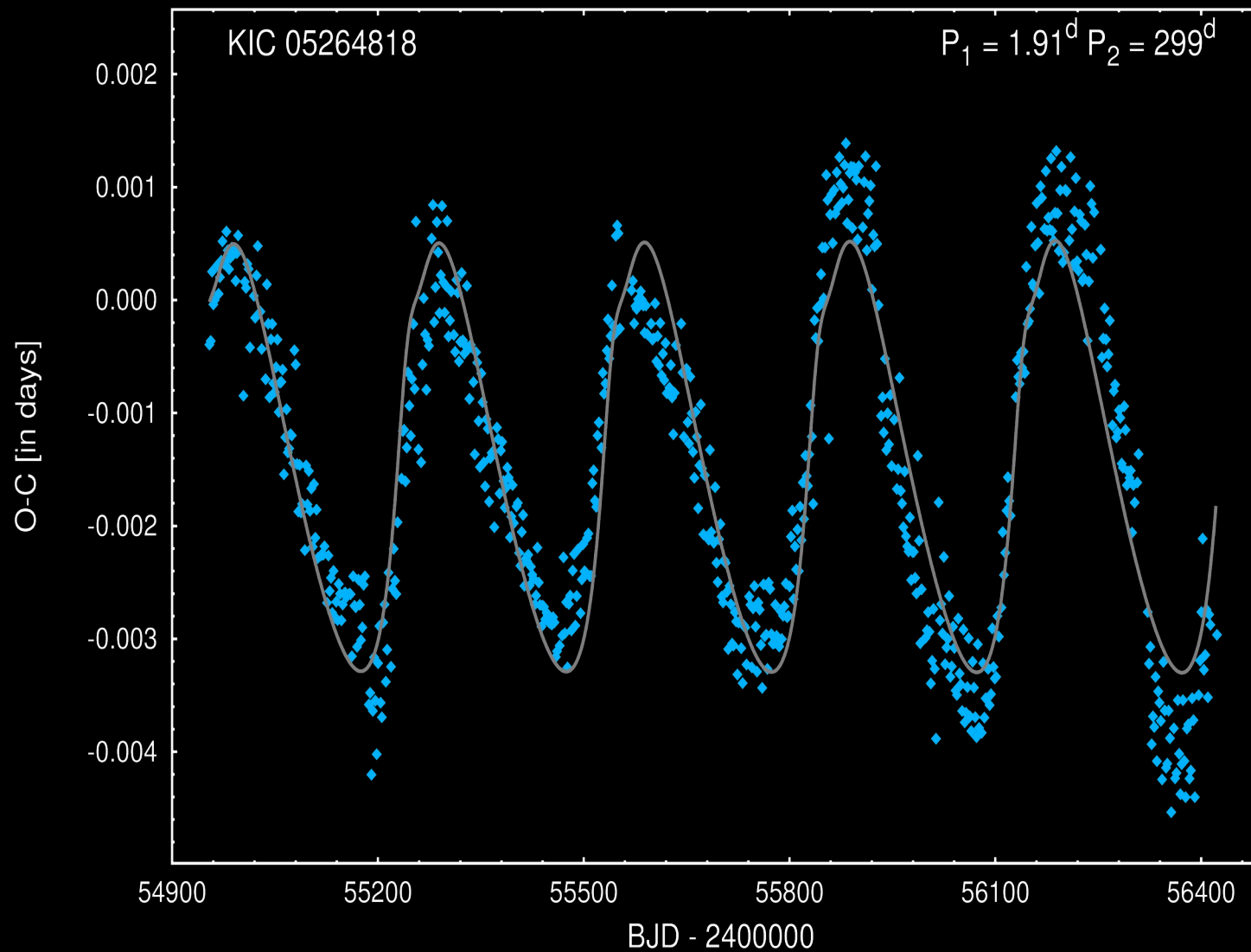


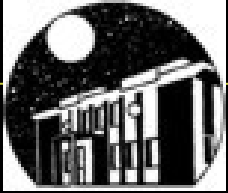
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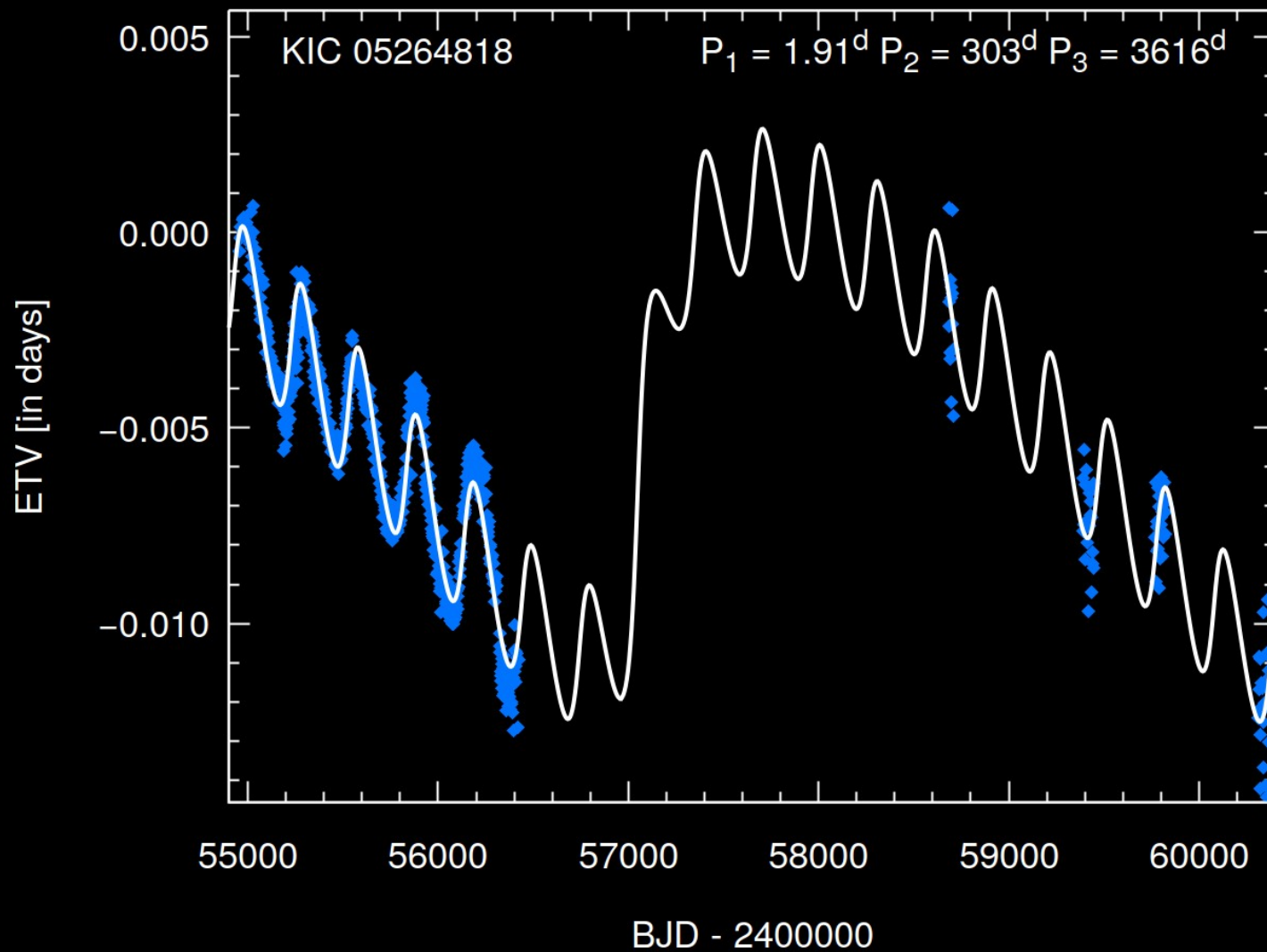


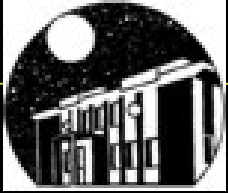
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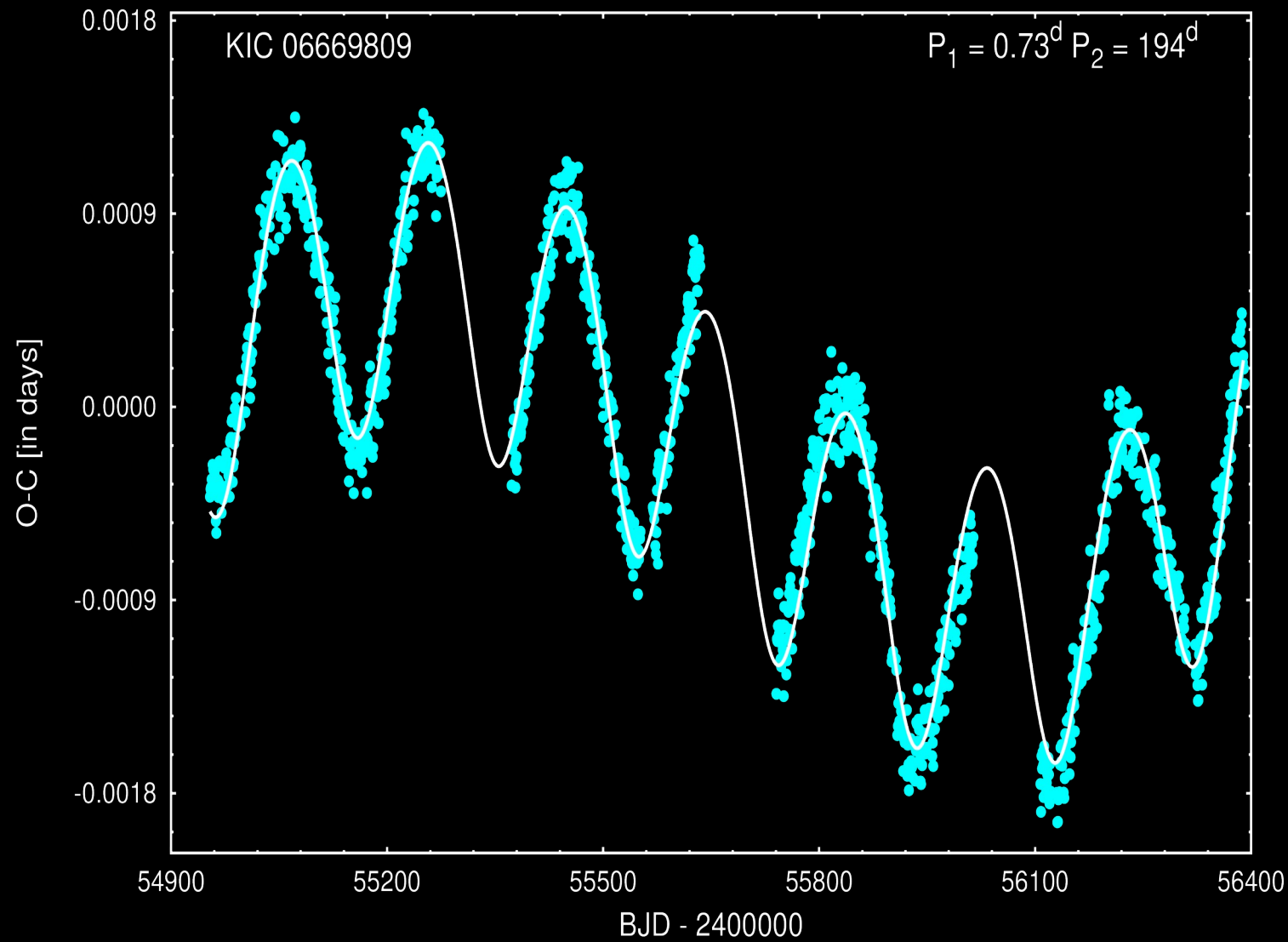


Tamás Borkovits – Then and now:

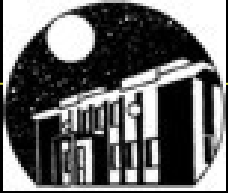
Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



### Other funny ETVs





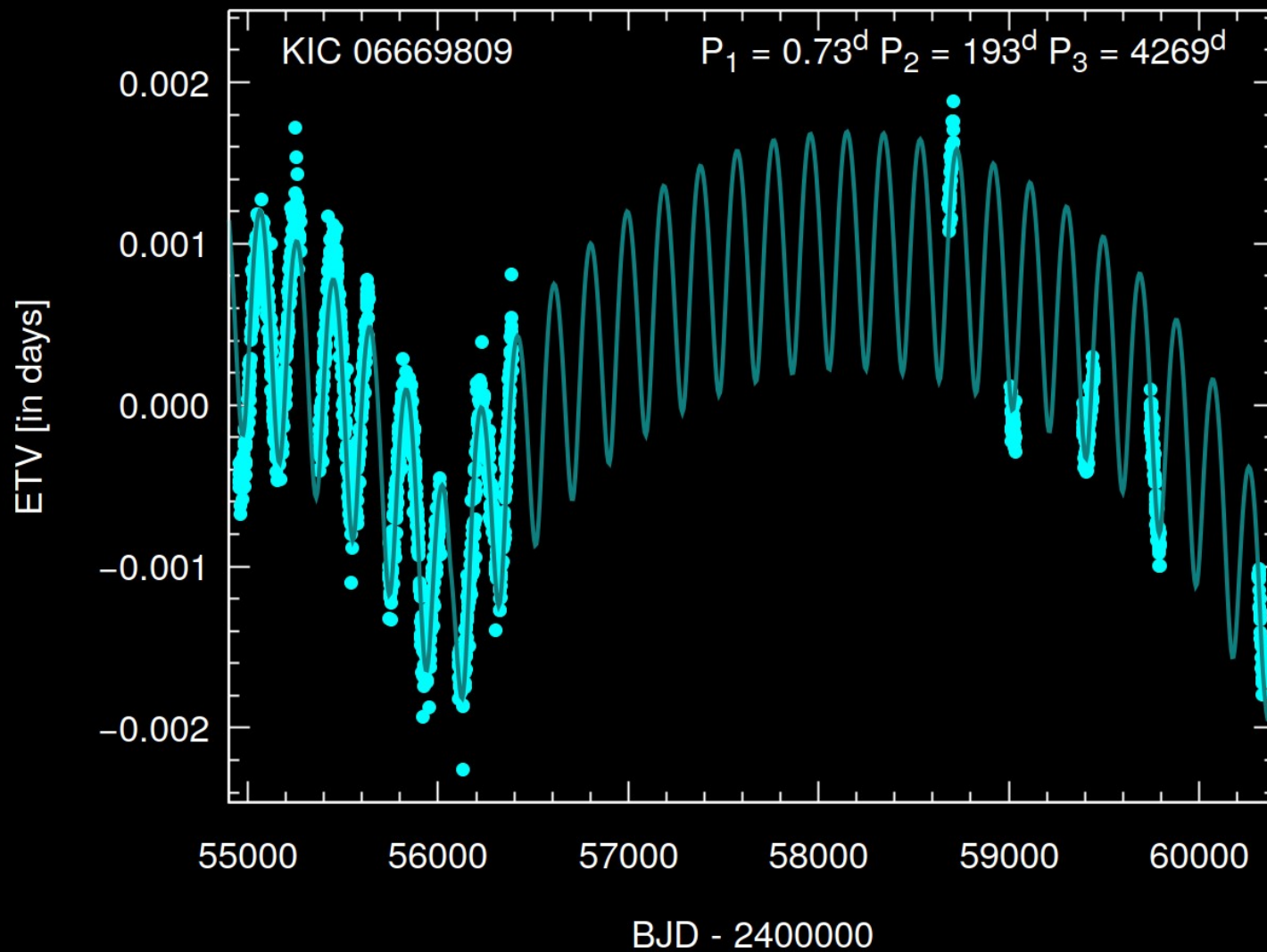


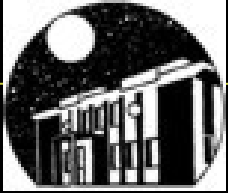
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### Other funny ETVs



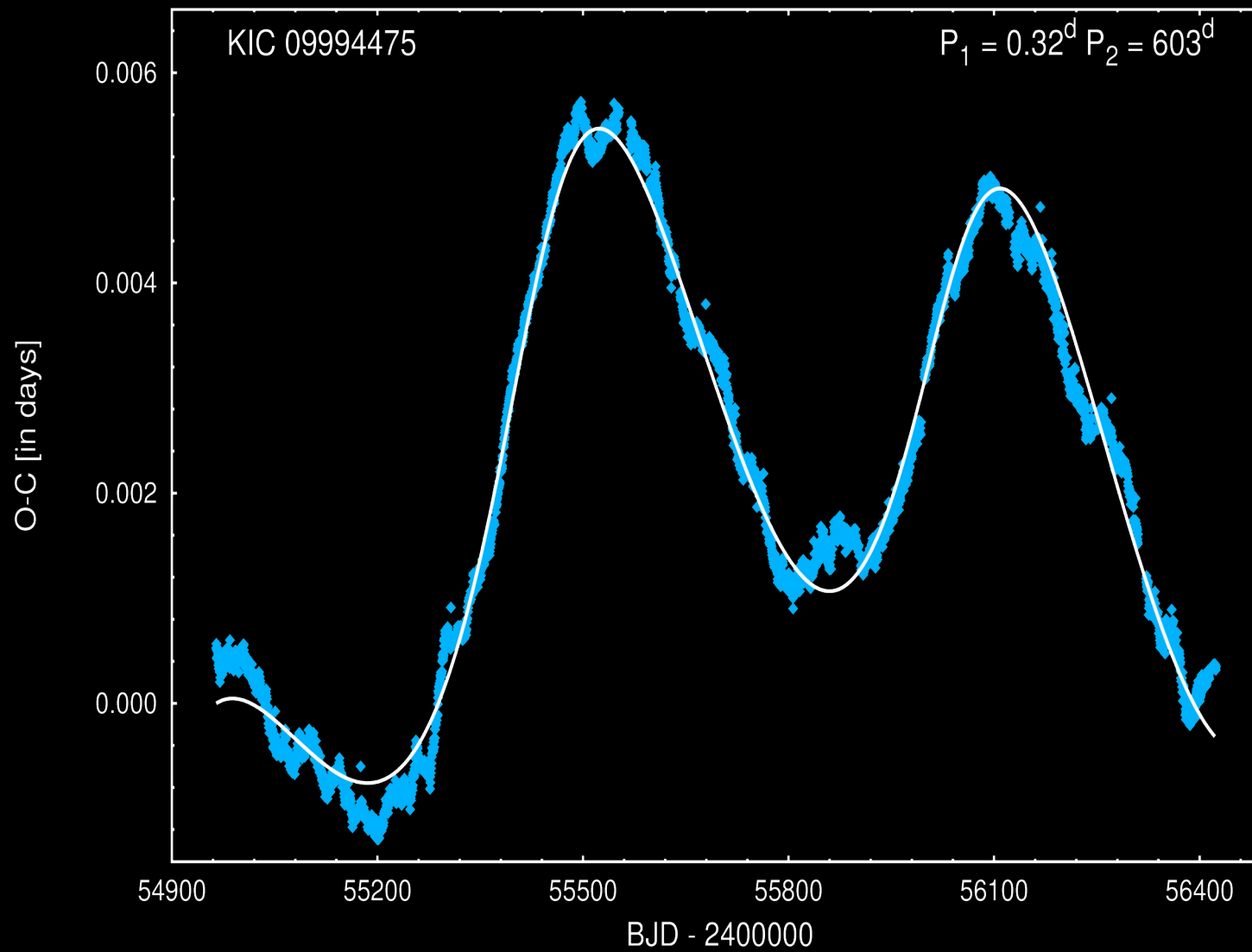


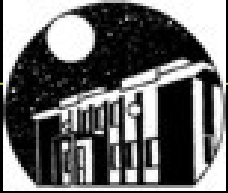
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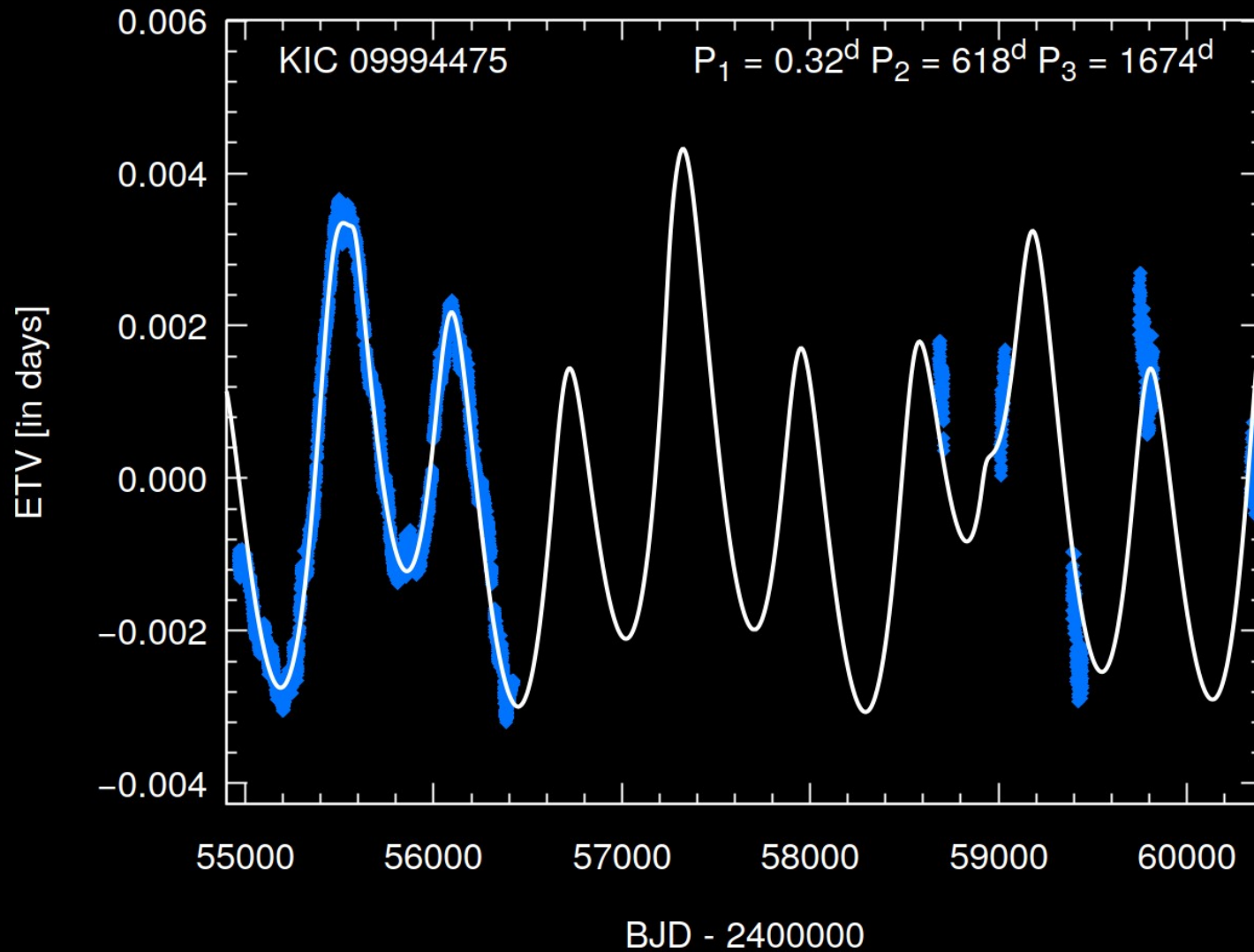


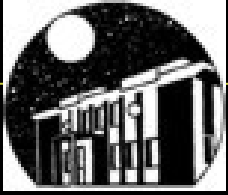
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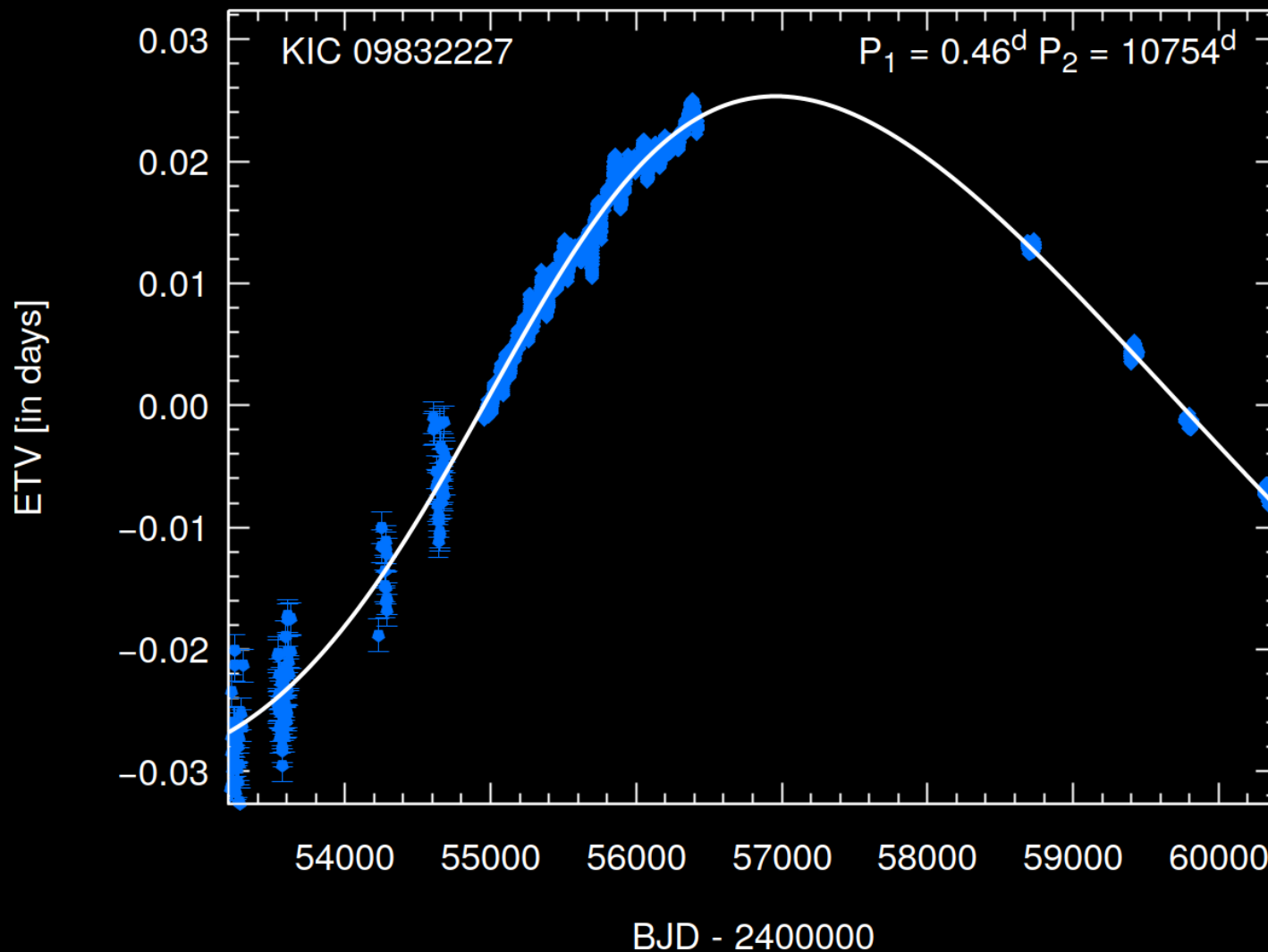


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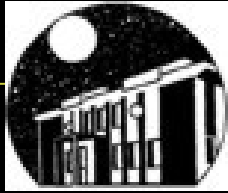
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An ordinary (non funny) ETV – related to Ondřej's talk



Yes, this is the formerly expected red nova!



Tamás Borkovits – Then and now:

Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



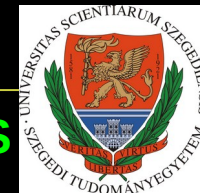
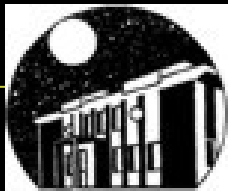
## The Royal Road: Eclipse Timing Variation Analysis

### The effects of the third body:

- Light-Travel Time Effect (LITE, LTTE – Rømer-delay)

$$\Delta_{\text{LTTE}} = -\frac{a_{\text{AB}} \sin i_2 (1 - e_2^2) \sin(v_2 + \omega_2)}{c (1 + e_2 \cos v_2)}$$

$a_{\text{AB}}$  – semi-major axis of the EB's orbit around the CM of the triple  
 $e_2, i_2, \omega_2, v_2$  – eccentricity, inclination, argument of periastron and true anomaly of the relative orbit of the **third** body  
 $c$  – speed of light



The Royal Road: Eclipse Timing Variation Analysis

The effects of the third body:

- Light-Travel Time Effect (LITE, LTTE – Rømer-delay)

Changing to eccentric anomaly ( $\mathcal{E}_2$ ):

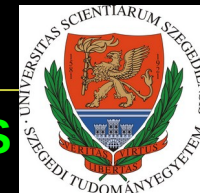
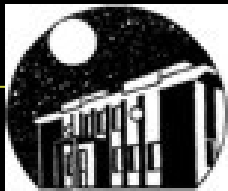
$$\begin{aligned}\Delta_{\text{LTTE}} &= -\frac{a_{\text{AB}} \sin i_2}{c} \left[ \sqrt{1 - e_2^2} \sin \mathcal{E}_2 \cos \omega_2 + (\cos \mathcal{E}_2 - e_2) \sin \omega_2 \right] \\ &= -\frac{a_{\text{AB}} \sin i_2}{c} \left[ \sqrt{1 - e_2^2 \cos^2 \omega_2} \sin(\mathcal{E}_2 + \phi) - e_2 \sin \omega_2 \right],\end{aligned}$$

Amplitude:

$$\mathcal{A}_{\text{LTTE}} = \frac{a_{\text{AB}} \sin i_2}{c} \sqrt{1 - e_2^2 \cos^2 \omega_2}$$

Phase:

$$\phi = \tan^{-1} \left( \frac{\sin \omega_2}{\sqrt{1 - e_2^2 \cos^2 \omega_2}} \right)$$



## The Royal Road: Eclipse Timing Variation Analysis

### The effects of the third body:

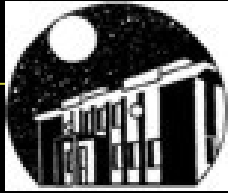
- Light-Travel Time Effect (LITE, LTTE – Rømer-delay)

Introducing the mass function:

$$f(m_C) = \frac{m_C^3 \sin^3 i_2}{m_{ABC}^2} = \frac{4\pi^2 a_{AB}^3 \sin^3 i_2}{G P_2^2}$$

and thus, the amplitude of LTTE can be written as

$$\begin{aligned} \mathcal{A}_{LTTE} &= \frac{G^{1/3}}{c} \left( \frac{P_2}{2\pi} \right)^{2/3} f(m_C)^{1/3} \sqrt{1 - e_2^2 \cos^2 \omega_2} \\ &\approx 1.1 \times 10^{-4} \frac{m_C \sin i_2}{m_{ABC}^{2/3}} P_2^{2/3} \sqrt{1 - e_2^2 \cos^2 \omega_2}, \end{aligned}$$



Tamás Borkovits – Then and now:

Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



## The Royal Road: Eclipse Timing Variation Analysis

### The effects of the third body:

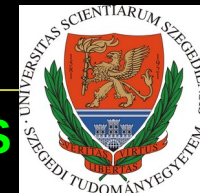
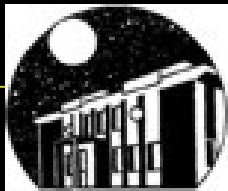
- Dynamical perturbations of a third body

Three different time-scales

Classifications of periodic perturbations	Period	Relative amplitude
Short period perturbations	$\sim P_1$	$\sim (P_1/P_2)^2$
Medium period perturbations	$\sim P_2$	$\sim P_1/P_2$
Long period perturbations	$\sim P_2^2/P_1$	1

**Note: This is the classification introduced by Brown, 1936 for his Lunar-theory. Classification and nomenclature based on the planetary theory departs!**





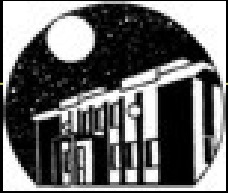
The Royal Road: Eclipse Timing Variation Analysis

The effects of the third body:

- Medium-period perturbations of a third body

Quadrupole-terms (Borkovits, Csizmadia, Forgács-Dajka, Hegedüs, 2011)

$$\begin{aligned}(O - C)_{v_2} = & \frac{P_1 A_L}{2\pi} \left\{ (1 - e_1^2)^{1/2} \left\{ \left[ \frac{4}{5} f_1(e_1) + \frac{6}{5} K_1(e_1, \omega_1) \right] \left[ \left( I^2 - \frac{1}{3} \right) \mathcal{M} + \frac{1}{2} (1 - I^2) \mathcal{S}(2v_2 + 2g_2) \right] \right. \right. \\ & + \left[ \frac{51}{20} e_1^2 f_2(e_1) \cos 2g_1 + 2K_2(e_1, \omega_1, g_1) + \frac{1}{8} e_1^2 K_4(e_1, \omega_1, g_1) \right] \left[ (1 - I^2) \mathcal{M} + \frac{1}{2} (1 + I^2) \mathcal{S}(2v_2 + 2g_2) \right] \\ & \left. \left. - \frac{1}{2} \left[ \frac{51}{20} e_1^2 f_2(e_1) \sin 2g_1 + 2K_3(e_1, \omega_1, g_1) + \frac{1}{8} e_1^2 K_5(e_1, \omega_1, g_1) \right] 2IC(2v_2 + 2g_2) \right\} \right. \\ & + \frac{\sin i_m \cot i_1}{(1 - e_1^2)^{1/2}} \left\{ \left[ -\frac{2}{5} \left( 1 + \frac{3}{2} e_1^2 \right) \cos u_{m1} + e_1^2 \cos(2g_1 + u_{m1}) \right] [1 + 2K_1(e_1, \omega_1)] I \left[ \mathcal{M} - \frac{1}{2} \mathcal{S}(2v_2 + 2g_2) \right] \right. \\ & \left. \left. + \frac{1}{2} \left[ \frac{2}{5} \left( 1 + \frac{3}{2} e_1^2 \right) \sin u_{m1} + e_1^2 \sin(2g_1 + u_{m1}) \right] [1 + 2K_1(e_1, \omega_1)] C(2v_2 + 2g_2) \right\} \right\},\end{aligned}$$



The Royal Road: Eclipse Timing Variation Analysis

The effects of the third body:

- Medium-period perturbations of a third body – special cases

Circular inner orbit (Borkovits, Érdi, Forgács-Dajka, Kovács T., 2003)

$$\Delta_{L10} = \frac{P_1}{2\pi} \frac{m_C}{m_{ABC}} \frac{P_1}{P_2} \left[ \left( 1 - \frac{3}{2} \sin^2 i_m \right) \mathcal{M} + \frac{3}{4} \sin^2 i_m \mathcal{S} \right] + \dots$$

Both orbits circular

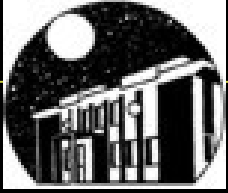
$$\Delta_{L10}^{e_2=0} = \frac{P_1}{2\pi} \frac{m_C}{m_{ABC}} \frac{P_1}{P_2} \frac{3}{4} \sin^2 i_m \sin(2u_2 - 2n_2)$$

Circular inner orbit – coplanar orbits (Agol et al., 2005)

$$\Delta_{L10}^{\text{copl}} = \frac{P_1}{2\pi} \frac{m_C}{m_{ABC}} \frac{P_1}{P_2} \left( 3e_2 \sin v_2 - \frac{3}{4} e_2^2 \sin 2v_2 + \frac{1}{3} e_2^3 \sin 3v_2 \right) + O(e_2^4)$$

Two circular & coplanar orbits (Trinity)

$$\Delta_{L10} = 0.$$



The Royal Road: Eclipse Timing Variation Analysis

The effects of the third body:

- Medium-period perturbations of a third body
- Comparison of the amplitudes with the LTTE--terms:

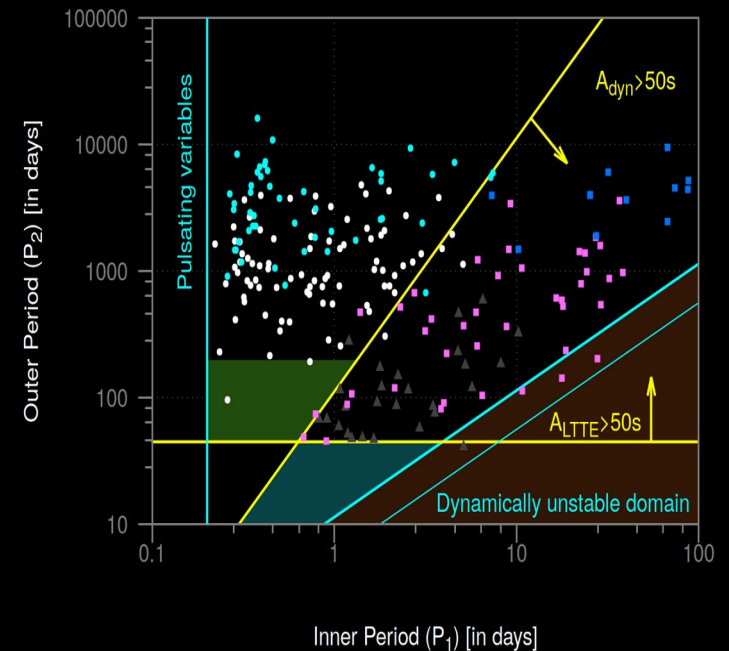
$$\frac{A_{\text{dyn}}}{A_{\text{LTTE}}} = \frac{c}{(2\pi G m_{\text{ABC}})^{1/3} \sin i_2} \mathcal{E}(e_2, \omega_2) \left(\frac{P_1}{P_2}\right)^2 P_2^{1/3}, \quad (13)$$

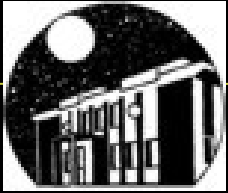
where

$$\mathcal{E}(e_2, \omega_2) = (1 - e_2^2)^{-3/2} (1 - e_2^2 \cos^2 \omega_2)^{-1/2} \quad (14)$$

and therefore, for a given total mass

$$\begin{aligned} \frac{A_{\text{dyn}}}{A_{\text{LTTE}}} &\geq \frac{c}{(2\pi G m_{\text{ABC}})^{1/3}} \left(\frac{P_1}{P_2}\right)^2 P_2^{1/3} \\ &\geq 1.45 \times 10^3 m_{\text{ABC}}^{-1/3} \frac{P_1^2}{P_2^{5/3}}. \end{aligned}$$





The Royal Road: Eclipse Timing Variation Analysis

The effects of the third body:

- Medium-period perturbations of a third body
- Comparison of the amplitudes with the LTTE--terms:

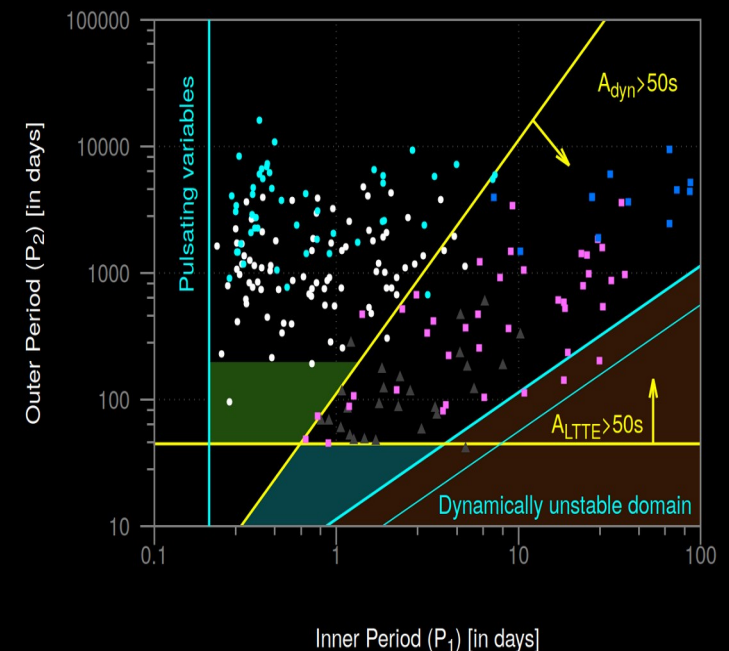
$$\frac{A_{\text{dyn}}}{A_{\text{LTTE}}} = \frac{c}{(2\pi G m_{\text{ABC}})^{1/3} \sin i_2} \mathcal{E}(e_2, \omega_2) \left(\frac{P_1}{P_2}\right)^2 P_2^{1/3}, \quad (13)$$

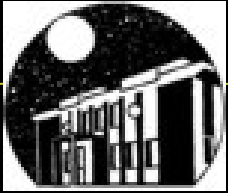
where

$$\mathcal{E}(e_2, \omega_2) = (1 - e_2^2)^{-3/2} (1 - e_2^2 \cos^2 \omega_2)^{-1/2} \quad (14)$$

– For *Kepler*  
(the life-time of the original mission ~ 1470 days):

$$\frac{A_{\text{dyn}}}{A_{\text{LTTE}}} \geq m_{\text{ABC}}^{-1/3} \left(\frac{P_1}{11.46}\right)^2 \left(\frac{1470}{P_2}\right)^{5/3}$$



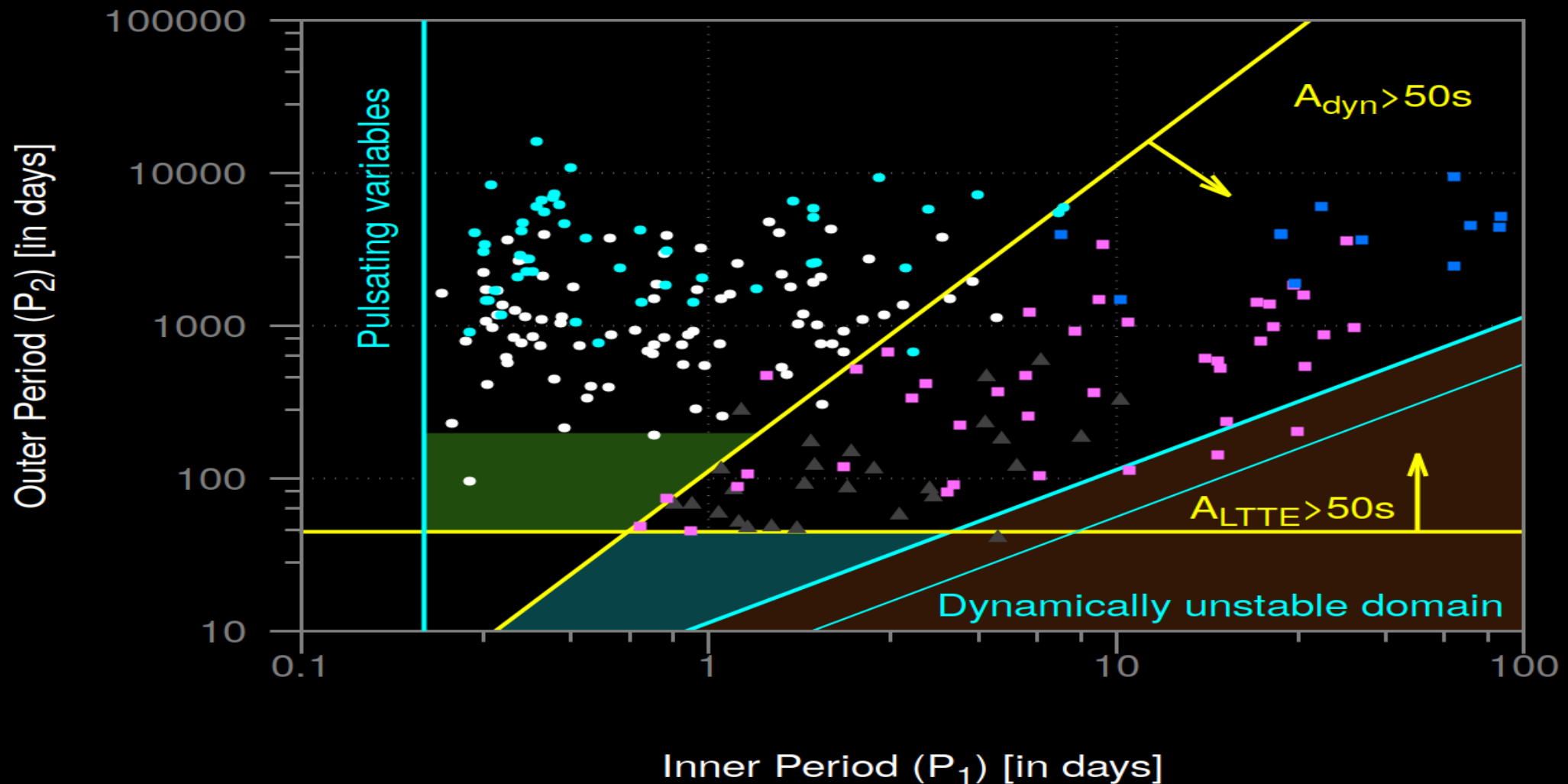


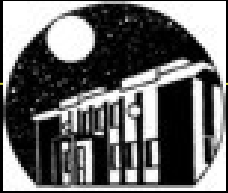
Tamás Borkovits – Then and now:

# Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



## The Royal Road: Eclipse Timing Variation Analysis





Tamás Borkovits – Then and now:



## Eclipse Timing Variation Analyses of *Kepler* triples – after TESS

### The Royal Road: Eclipse Timing Variation Analysis

#### The effects of the third body:

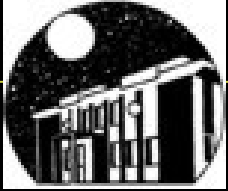
- “Apse-node” timescale perturbations

For the closest *Kepler* triples apsidal motion and nodal regression timescale is shorter than a decade! **These must be taken into account!**

General form of apsidal motion (irrespective on its origin):

$$\Delta_{\text{apse}} = \frac{P_1}{2\pi} \left[ \arctan \left( \frac{\pm e_1 \cos \omega_1}{1 + \sqrt{1 - e_1^2} \mp e_1 \sin \omega_1} \right) \pm \sqrt{1 - e_1^2} \frac{e_1 \cos \omega_1}{1 \mp e_1 \sin \omega_1} \right]$$

**In case of third-body perturbations the variation of  $\omega_1$  no longer linear in time and furthermore,  $e_1$  and  $P_1$  also vary!**



Tamás Borkovits – Then and now:

Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



## The Royal Road: Eclipse Timing Variation Analysis

### The effects of the third body:

- “Apse-node” timescale perturbations

The period of the dynamical apsidal advance (and nodal regression)

$$P_{\text{apse}} \sim \frac{8}{15} \frac{m_{\text{ABC}}}{m_{\text{C}}} \frac{P_2^2}{P_1} f(e_1, e_2, i_m, g_1, g_2)$$

It can be calculated from the long-term+LTTE third body solution, therefore, it is constrained – helps to resolve some degeneracies

(see Borkovits et al. 2015 for details, especially Appendix C)



Tamás Borkovits – Then and now:

Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



## The Royal Road: Eclipse Timing Variation Analysis

### The effects of the third body:

Why are these good for us?

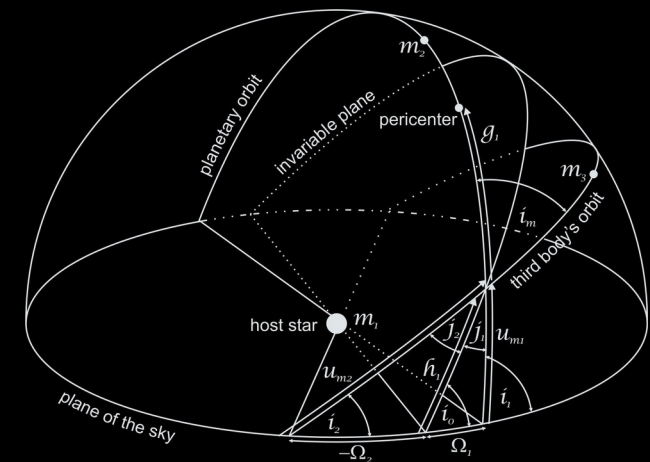
- LTTE + dynamical perturbations:

– light-time effect:  $P_2$ ,  $a_{AB} \sin i_2$  [in km],  $e_2$ ,  $\omega_2$ ,  $f(m_C)$

– grav. perturbations:

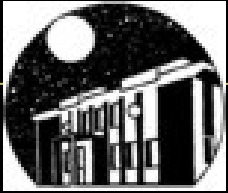
– ( $P_2$  time-scale):  $P_2$ ,  $m_C/M$ ,  $e_2$ ,  $(\Omega_1 - \Omega_2)$ ,  $\dot{i}_{mut}$ ,  $i_0$ ,  $g_2$   
 $e_1$ ,  $\omega_1$ ,  $g_1$ ,  $h$ ,  $j_1$ ,  $j_2$

– ( $P_2^2/P_1$  time-scale):  $e_1$ ,  $\omega_1$ ,  $e_2$ ,  $\dot{i}_{mut}$ ,  $g_1$ ,  $g_2$ ,  $h$ ,  $m_C/M$   
 (apsidal motion, orbital plane precession)



The yellow quantities were almost completely unknown for compact triples before *Kepler*-era, although they are very important for dynamical evolution studies





Search for hierarchical triples in the *Kepler* sample

## Steps of the analysis

### 3. Search for third-body solution

- Levenberg-Marquardt non-linear LSQ search.

The general form:

$$\Delta = \sum_{i=0}^3 c_i E^i + [\Delta_{\text{LTTE}} + \Delta_{\text{dyn}} + \Delta_{\text{apsid}}]_0^E .$$

where the expected  $\Delta$  itself is

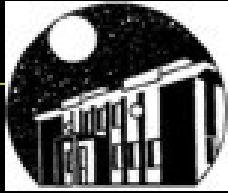
$$\Delta = T(E) - T_0 - P_s E$$

**Always included: linear polynomial + LTTE terms (2+5 parameters)**

**Dynamical term(s) added where the shape and/or the estimated  $A_{\text{dyn}}/A_{\text{LTTE}}$  ratio makes it necessary (9 or less extra parameters – some of them may be constrained!)**

**Apsidal motion terms for eccentric inner orbits (3 parameters included in the above)**

**Quadratic (or, very rarely, cubic polynomial) in a very limited cases (+1 or 2 parameters)**



Search for hierarchical triples in the *Kepler* sample

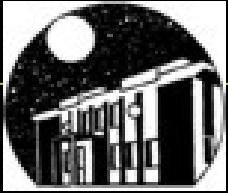
Results from 2024 (2016)

Summary

- A total of 189 (222) third-body ETV solution was found

	LTTE only	LTTE+dyn	Sum
Group I	54 (38)	37 (31)	91 (69)
Group II	31 (64)	4 (14)	35 (78)
Group III	50 (58)	13 (17)	63 (75)
Sum total	135 (160)	54 (62)	189 (222)
False positive	0 (8)	0 (0)	187 (230)

- **Group I:** More than two outer orbital periods are covered (or extra eclipses verify the third body)
- **Group II:** More than one, but less than two outer periods
- **Group III:** Less than one outer orbital period
- **False Positive:** The signal from LTTE but the source is not an EB



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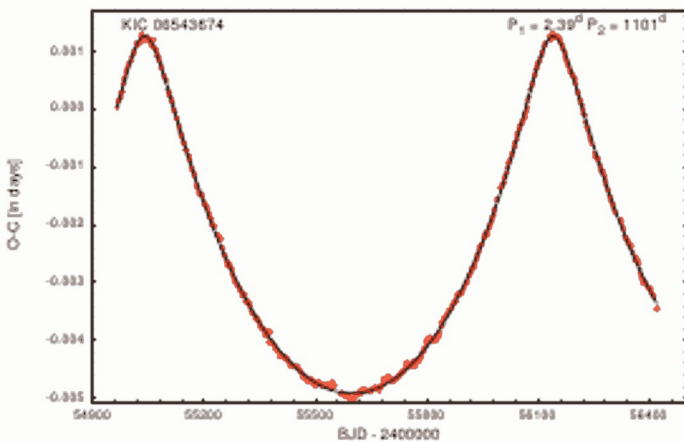
## Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



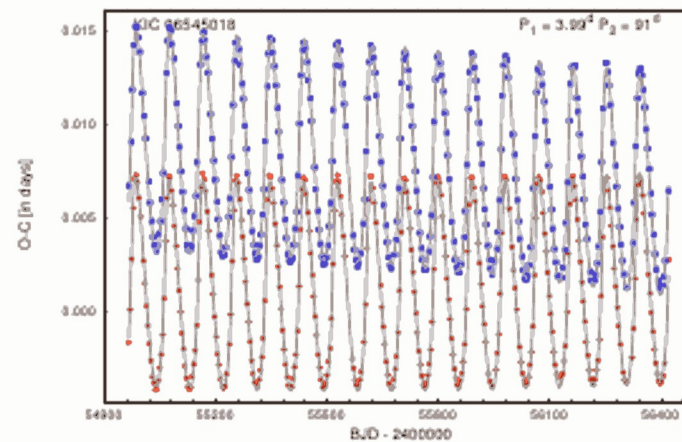
Results on hierarchical triples in the *Kepler* sample

### Group I systems (2024 – not final)

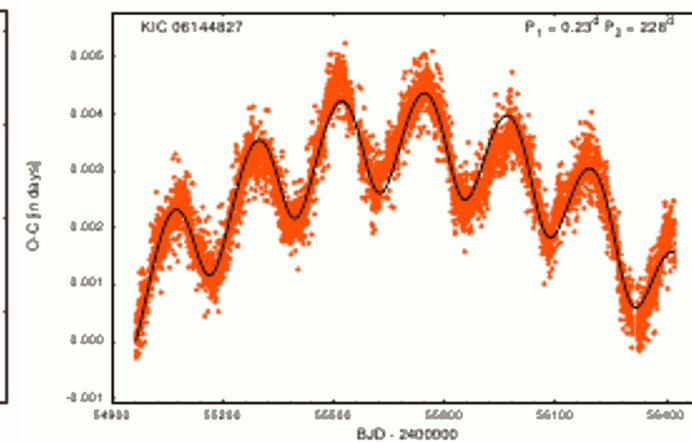
- $96\text{d} < P_2 < 2714\text{d}$  – pure LTTE systems
- $45\text{d} < P_2 < 1581\text{d}$  – LTTE+dyn systems
- 12 systems with extra eclipses – one of them identified in TESS data (extra eclipses in TESS data: 6 EBs)
- These are the most certain ones – although there might be a few false positive amongst them.



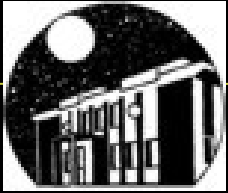
KIC 06543674



KIC 06545018



KIC 06144827



Tamás Borkovits – Then and now:

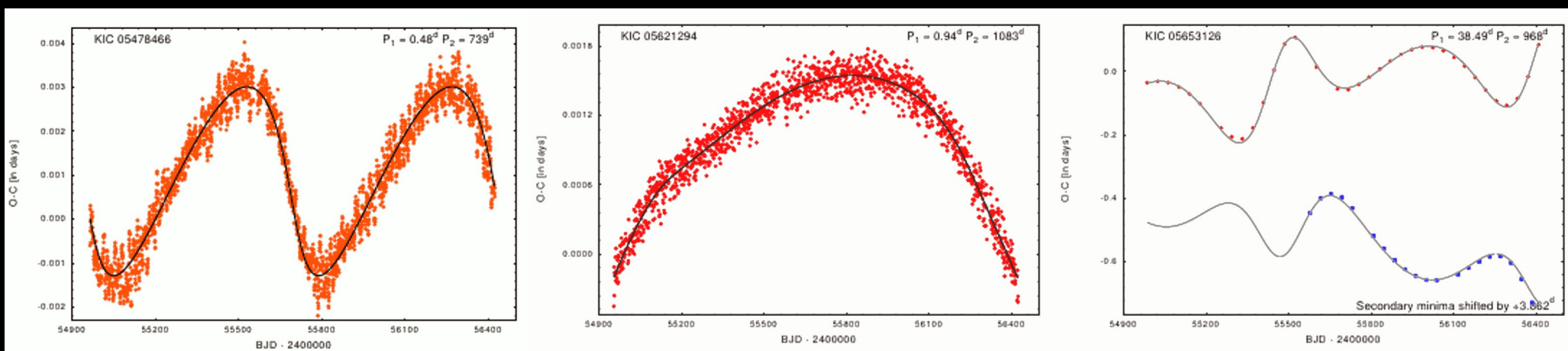
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Results on hierarchical triples in the *Kepler* sample

## Group II systems

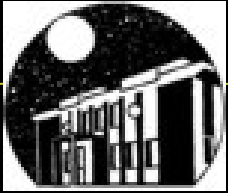
- $618\text{d} < P_2 < 4770\text{d}$  – pure LTTE systems
- $1385\text{d} < P_2 < 3418\text{d}$  – LTTE+dyn systems
- LTTE+dyn systems seems to be certain enough, but for pure LTTE cases there is a greater probability of false 3rd body interpretation



KIC 05478466

KIC 05621294

KIC 05653126



Tamás Borkovits – Then and now:

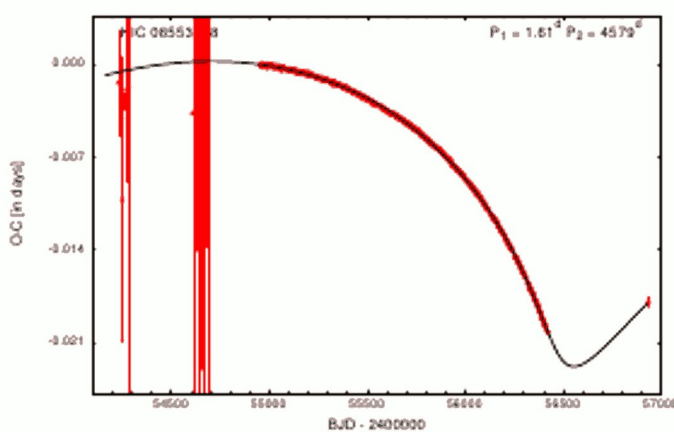
Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



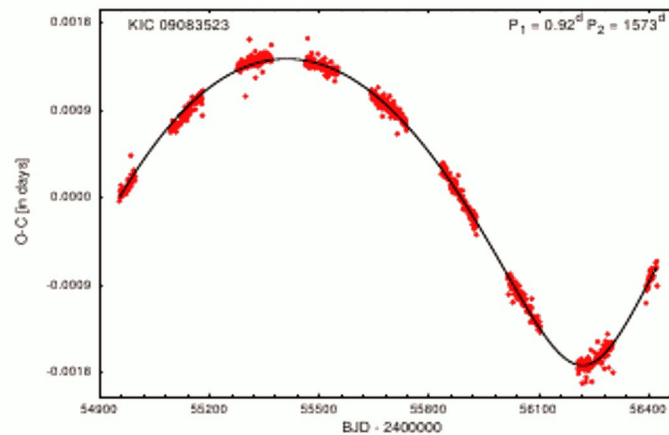
Results on hierarchical triples in the *Kepler* sample

## Group III systems

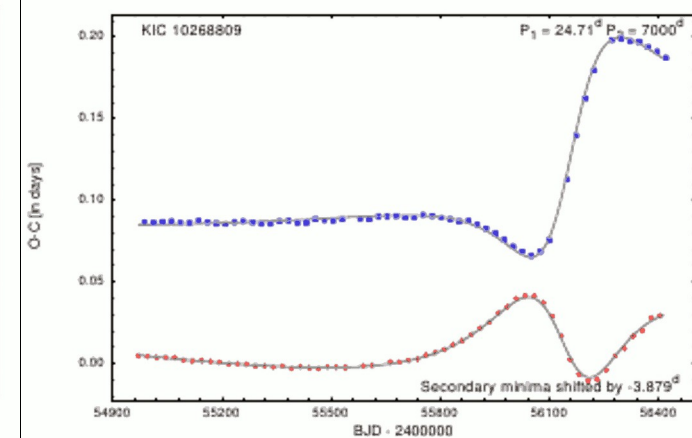
- $765\text{d} < P_2 < 16003\text{d}$  – pure LTTE systems
- $1487 < P_2 < 9402\text{d}$  – LTTE+dyn systems
- Most uncertain cases, but many of them most probably real triple



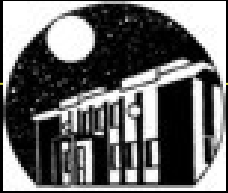
KIC 08553788



KIC 09083523



KIC 10268809



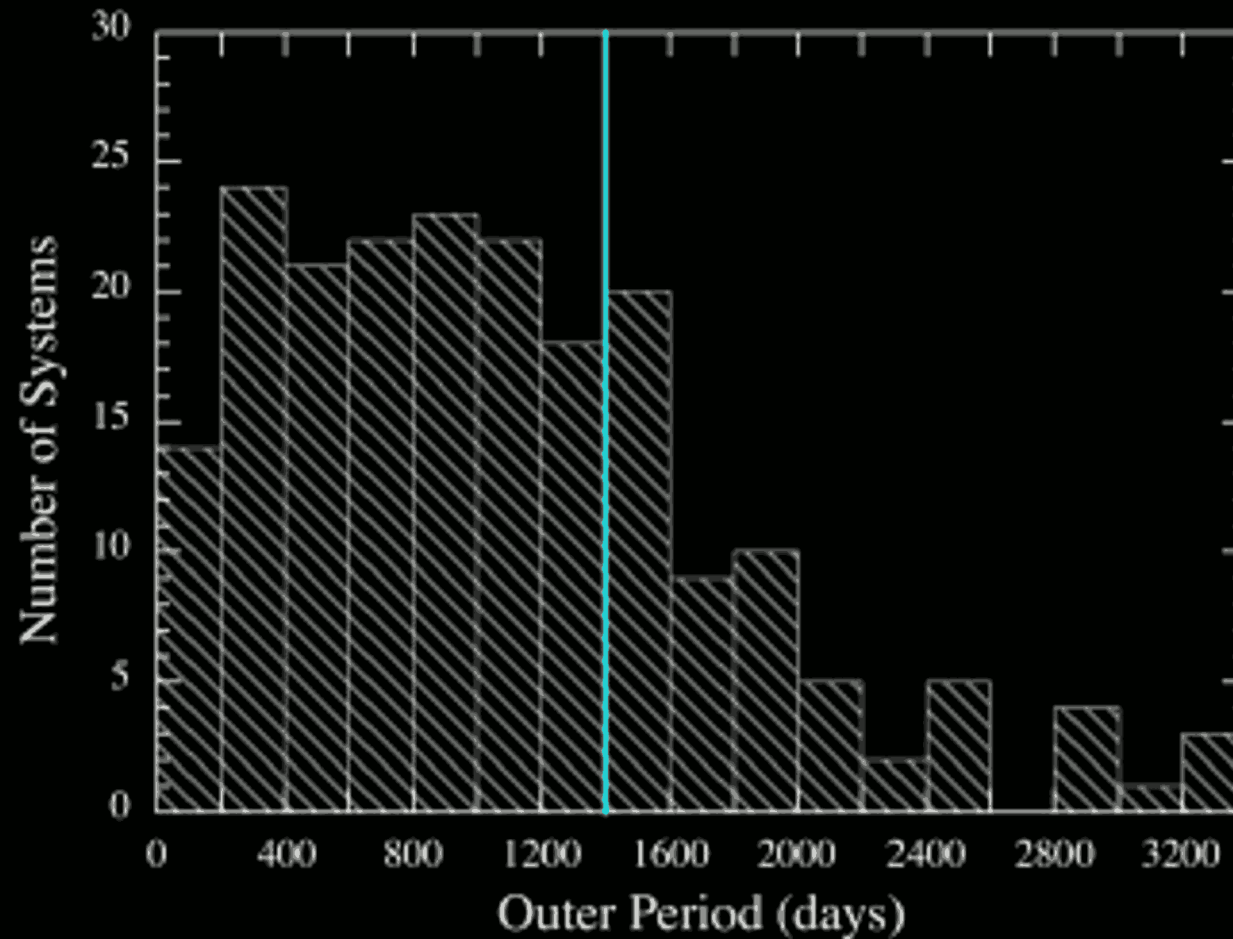
Tamás Borkovits – Then and now:

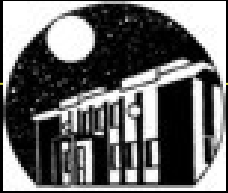
Eclipse Timing Variation Analyses of *Kepler* triples – after TESS



Results on hierarchical triples in the *Kepler* sample

## Statistics





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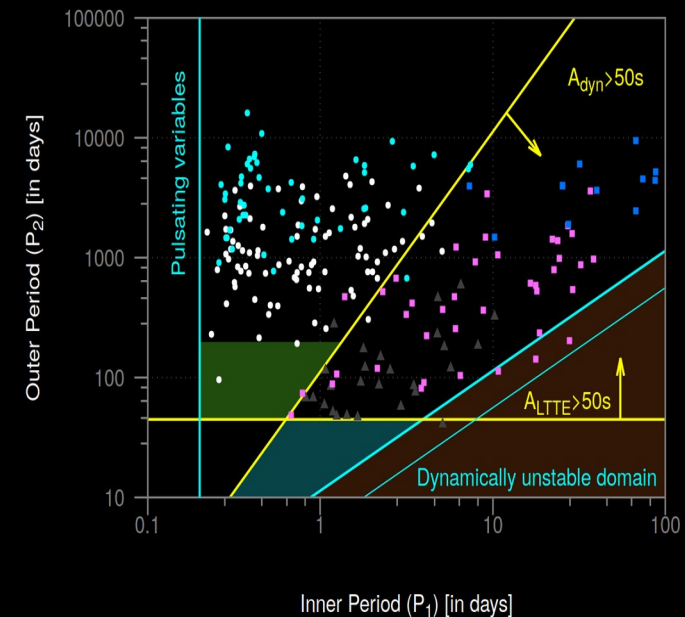


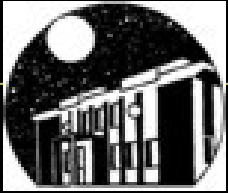
## Eclipse Timing Variation Analyses of *Kepler* triples – after TESS

Results on hierarchical triples in the *Kepler* sample

### Statistics

- **Lower end of outer period distribution:**  
The tightest binaries have no close ternary companions.  
(See the empty yellow region!)
- This might indicate some differences of the evolution (formation) of contact binaries.





Results on hierarchical triples in the *Kepler* sample

Statistics

- **Eccentricity distribution:**  
It is similar to that which was shown in Duchene & Kraus 2013 for different samples of wider field-binaries

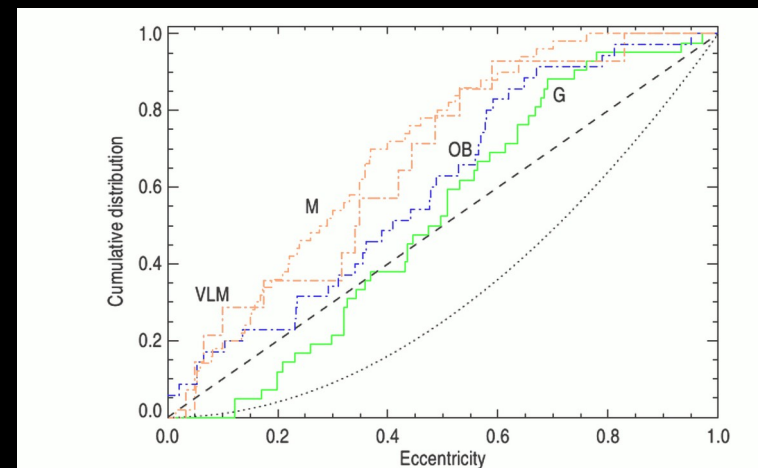
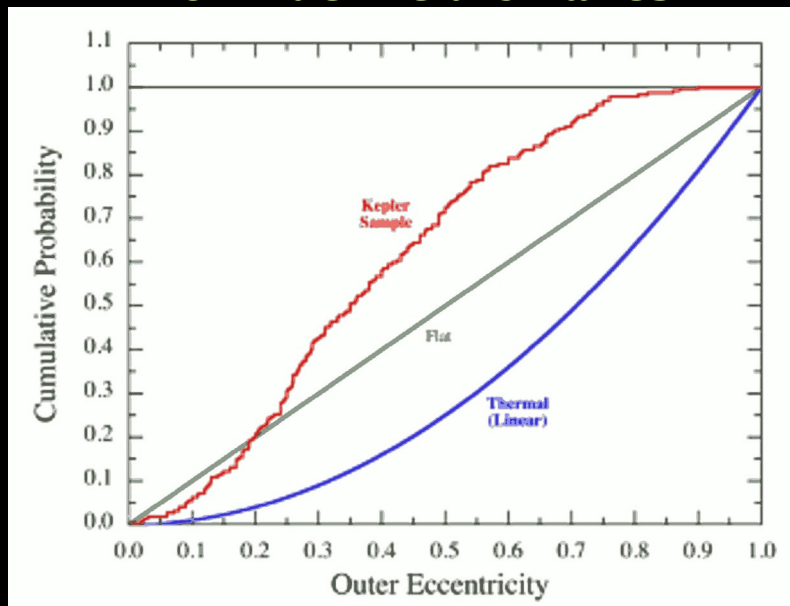
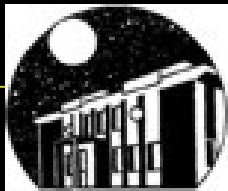


Figure 3:

Cumulative distribution of eccentricities for systems with  $2 \leq \log P \leq 4$  for field multiple systems among solar-type stars (green curve; Raghavan et al. 2010), low-mass stars (orange curve; from the SB9 catalog, Pourbaix et al. 2004), VLM stars and BDs (red curve; Dupuy & Liu 2011), high-mass stars (blue curve; from the SB9 catalog and Abt 2007, Sana et al. 2012a). The dot-dashed curves indicate incomplete samples, for which the eccentricity distribution is potentially biased. The dashed and dotted curves represent the expected distribution for a flat and thermal distribution,  $f(e) = 2e$ , respectively.

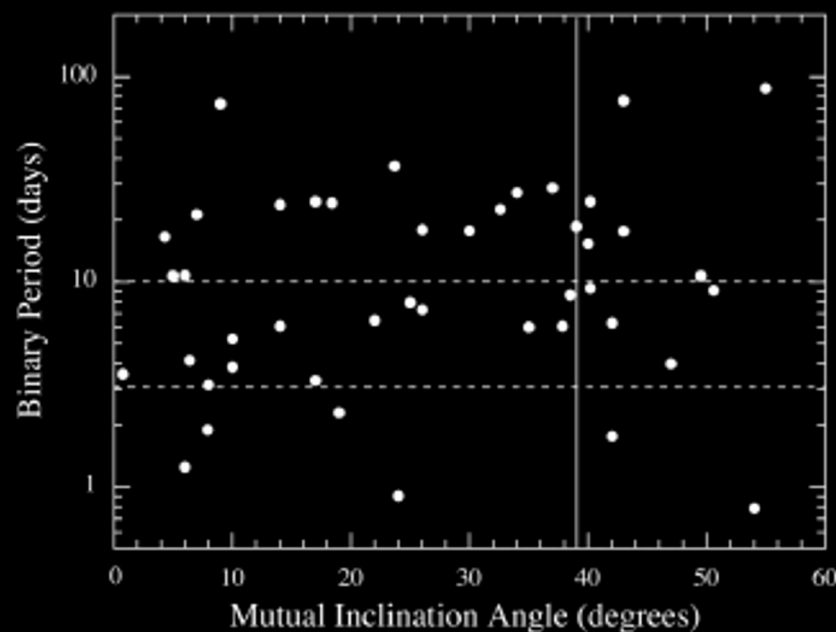
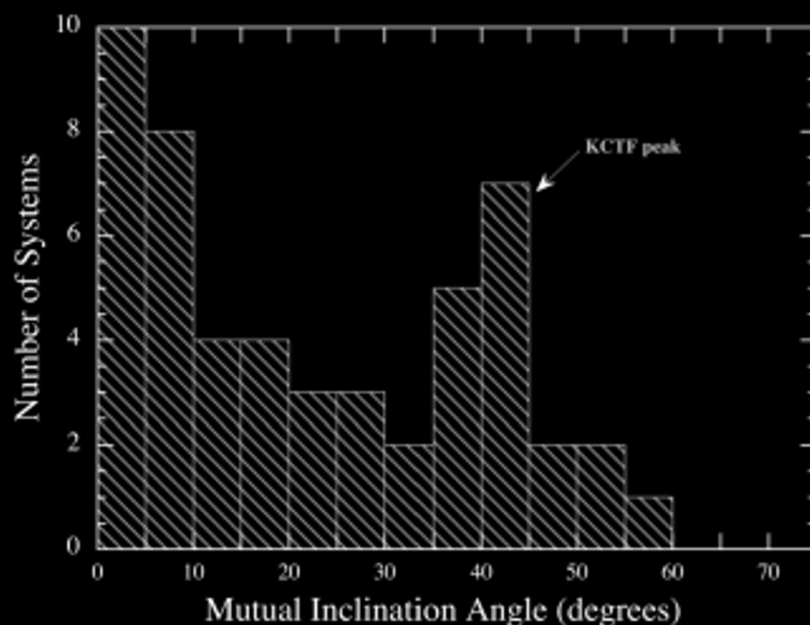


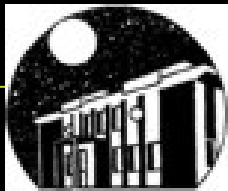


Results on hierarchical triples in the *Kepler* sample

Statistics

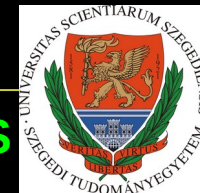
- **Mutual inclination:**  
(It can be calculated only for dynamical systems)
- The peak at  $i_m \sim 40^\circ$  may be a good evidence for KCTF mechanism



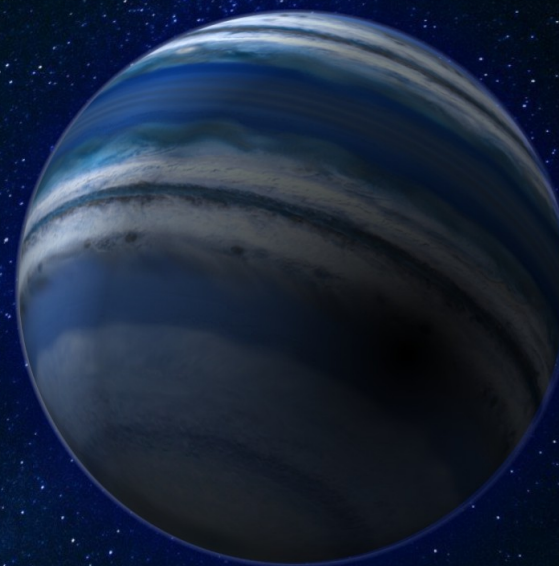


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**Thank you for the attention!**



by E. Forgács-Dajka