



Petr Zasche

**Doubly Eclipsing Systems:
Divide Et Impera**

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The phrase "Divide et impera"

- Origin not clear
- Usually attributed to Julius Caesar
- Used before by Philip II of Macedon :
(lived 382-336BC, ruled 359-336BC)



**Philip II
Macedon**



**Silver
tetradrachm**

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Cut the problem into smaller pieces

What are the doubly eclipsing systems?

- **More eclipsing periods**
 - It can be either (2+2) quadruple, or 2+1, or (2+1)+1, (2+2)+1, etc.
- **One point source on the sky**
- **Range of periods, depths of eclipses, magnitudes, ...**
- **Selection effects - huge!**
- **Gravitationally bound system (?)**
- **Photometric surveys + Kepler + TESS +.. > 99 %**



An approach for DEBs analysis

- **We have in principle two different options:**
 - **Separately solve the individual LCs, RVs, ETV, ...**
 - **And then merging the solution together to get the self-consistent picture**
 - **I call this "Divide et impera" approach**
 - **Combined photodynamical analysis of all available means**
 - **Very complicated, time consuming, needed a lot of CPU time,....**
 - **Done by group around T.Borkovits and his fellows**

Why doubly eclipsing systems?

- Derive for both binaries: M , R , $\log g$, T_{eff} , ...
- Share the same: age, chemical compositions, distance, ...
- Study the 2+2 dynamics, perturbations, secular evolution, stability, ETV studies, model the future fate of the system, ...
- Statistical studies, modelling of stellar populations, ...

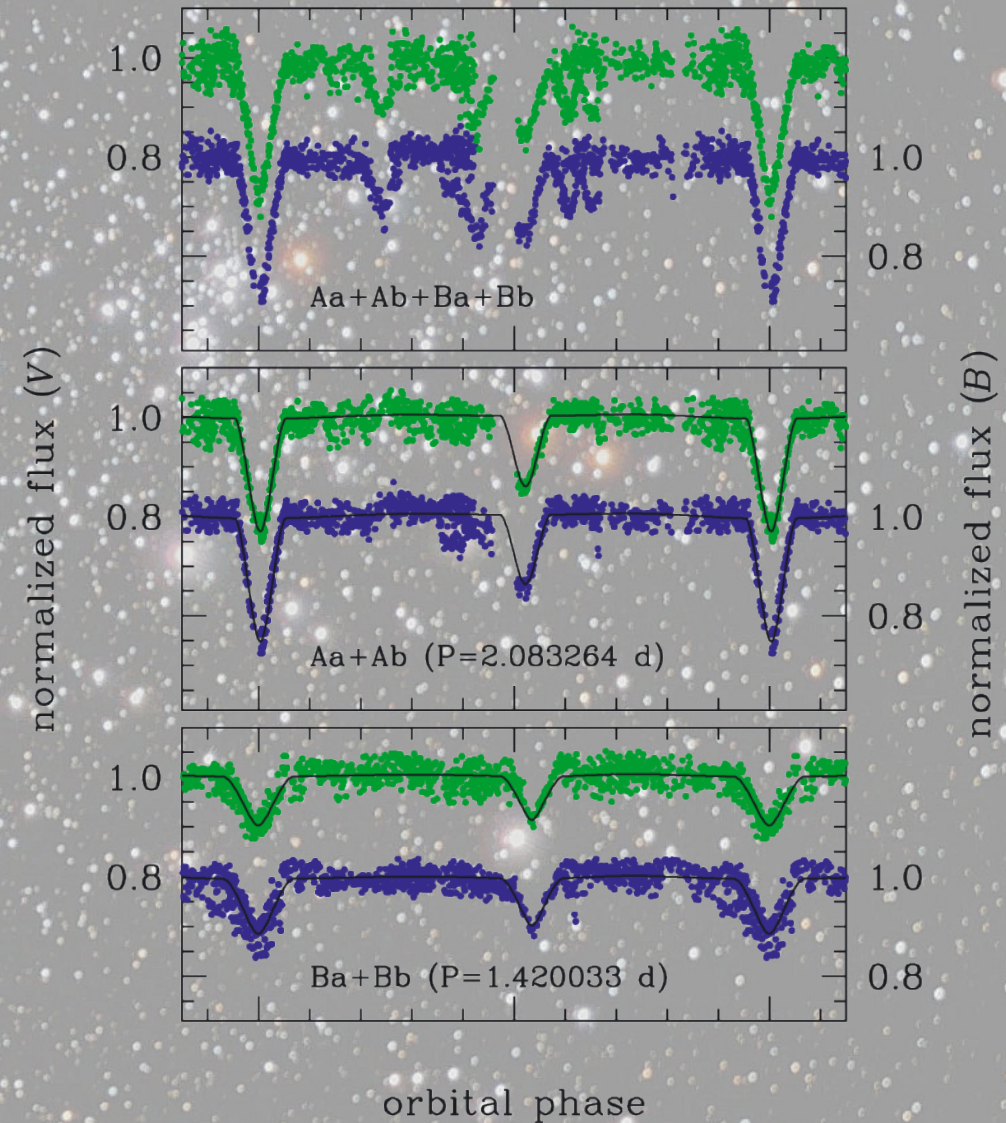
All these set rather strict constraints for the complete solution!

A little statistics

- Our goal is to detect both periods & prove grav.coupling
- Discovery statistics:
 - Year 2008: the first one (V994 Her)
 - Year 2018: in total 94 known
 - Year 2019: in total 146 known
 - Year 2020: in total 149 known
 - Year 2021: in total 159 known
 - Year 2022: in total 352 known
 - Year 2023: in total 771 known
 - Year 2024: in total 980 known
- But among these only 58 have confirmed mutual orbits (i.e. are definitely bound 2+2 quadruples)

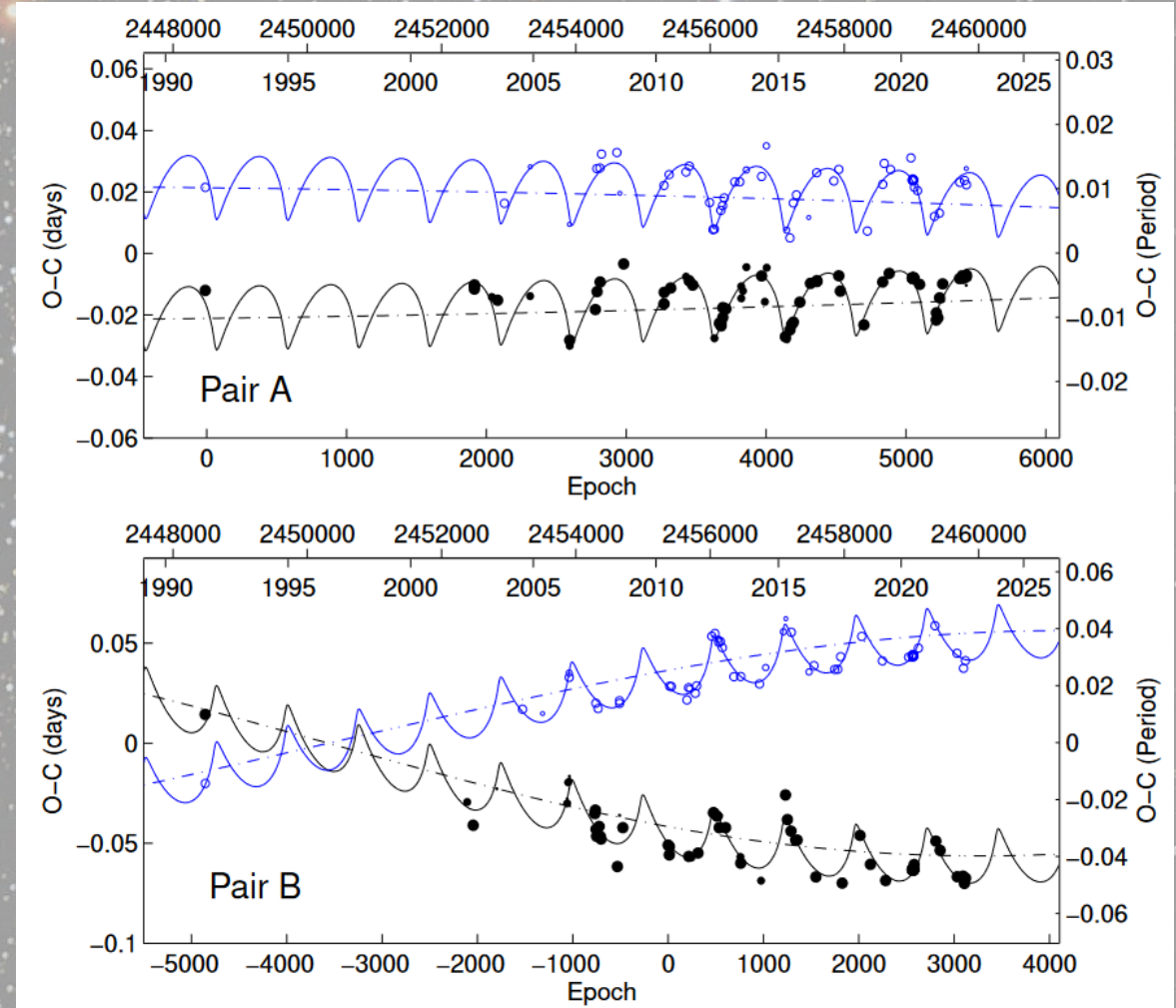
Where the story begun ...

- V994 Her: Lee et al. (2008)
- Two eclipsing systems, detached
- Bright star ($V = 7\text{mag}$)
- Distance about 240pc
- Pair A: $P=2.08\text{d}$, sp B8+A0, $e=0.03$
- Pair B: $P=1.42\text{d}$, sp A2+A4, $e=0.08$



V994 Her: really bound system

- **Zasche & Uhlar (2016)**
- **Two pairs A+B really orbit**
- **Mutual period 2.9yr**
- **Apsidal motion of both A&B**

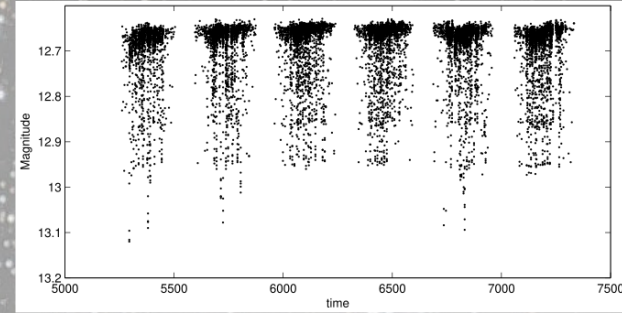


Selection effects

- **To detect the doubly eclipsing system :**
 - **Both inner periods A&B adequately short (too long - problem)**
 - **Not too short (contact systems - problem)**
 - **Selection effects due to data cadency of photometric surveys**
- **Mutual A-B outer period should be:**
 - **Short enough to be detected**
 - **Too short period – large dynamical effects**
 - **Long A-B period – large semimajor axis – interferometric detection**

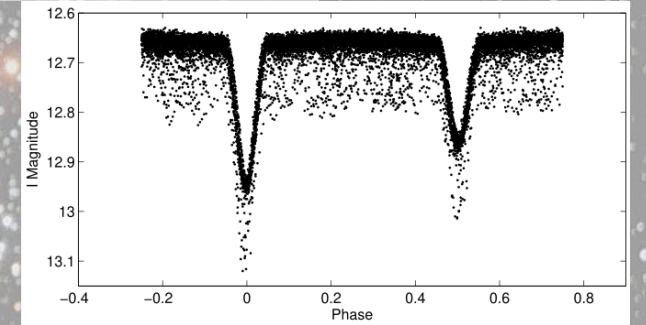
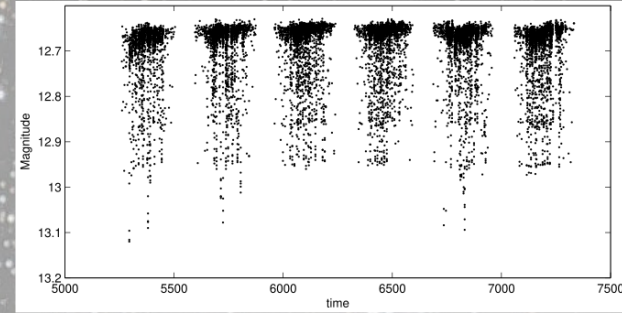
Divide et impera #1 :

- Complete combined photometry



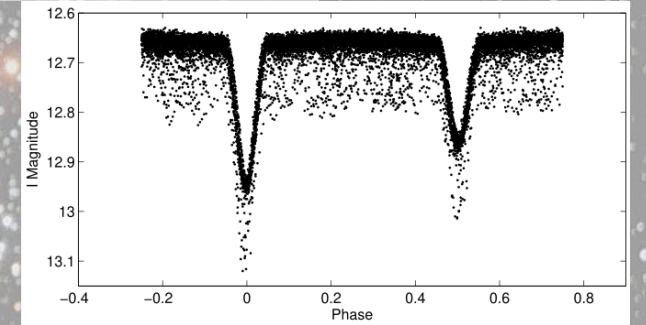
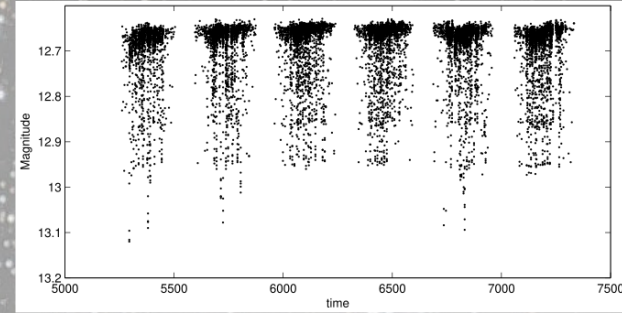
Divide et impera #1 :

- Complete combined photometry
- Detect the more pronounced period A



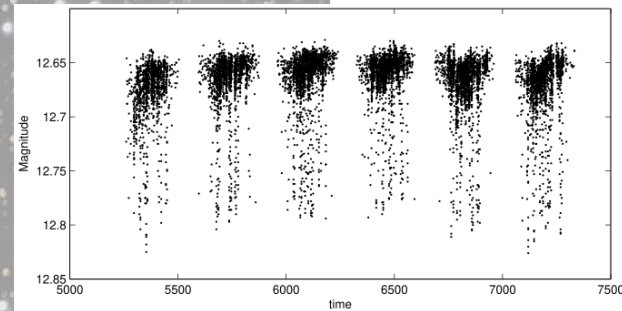
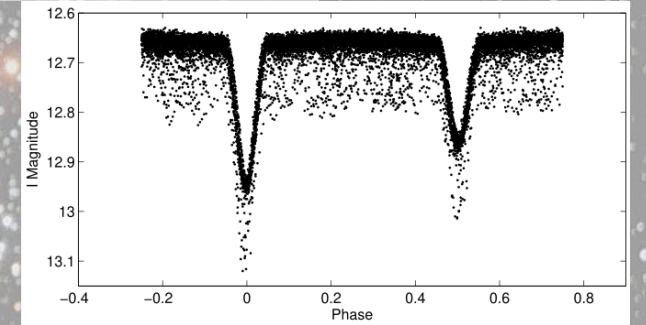
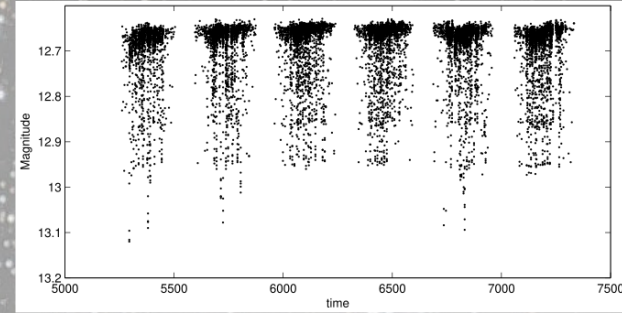
Divide et impera #1 :

- Complete combined photometry
- Detect the more pronounced period A
- Preliminary fit of pair A light curve



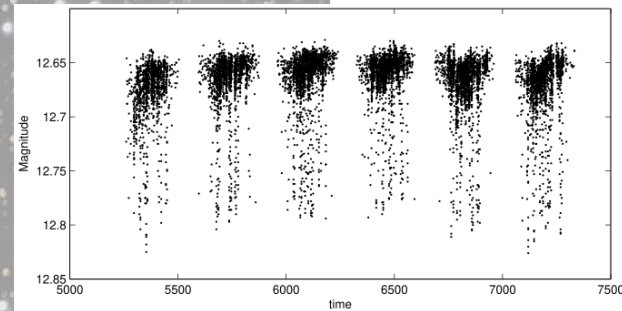
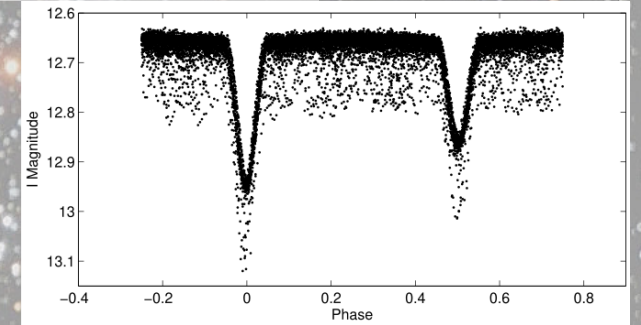
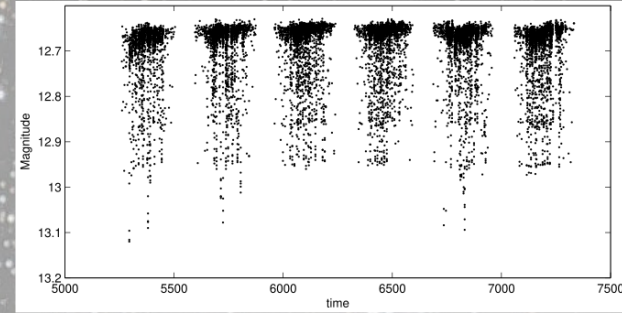
Divide et impera #1 :

- Complete combined photometry
- Detect the more pronounced period A
- Preliminary fit of pair A light curve
- Subtract of pair A \rightarrow residua
- Detect B and derive its period



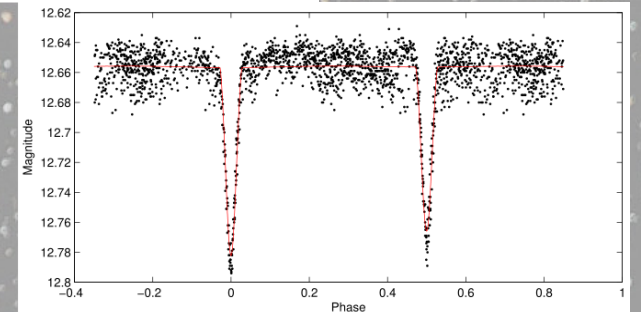
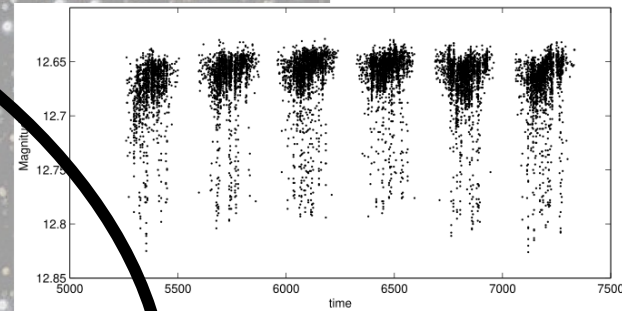
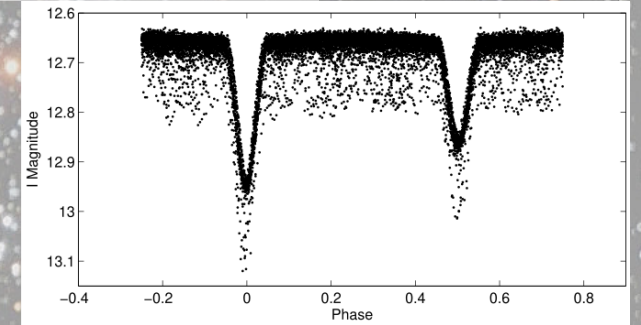
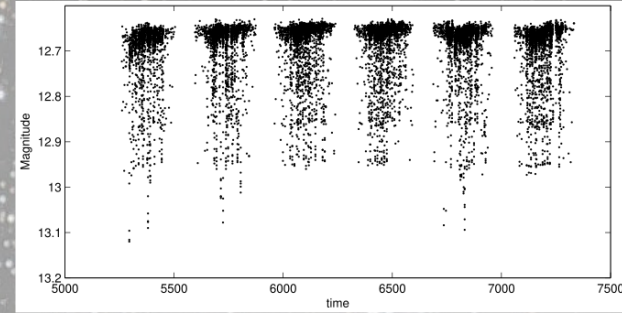
Divide et impera #1 :

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- Preliminary fit of pair B



Divide et impera #1 :

- Complete combined photometry
- Detect the more pronounced period A
- Preliminary fit of pair A light curve
- Subtract of pair A → residua
- Detect B and derive its period
- Preliminary fit of pair B
- Subtract pair B from combined photometry



Problematic detection

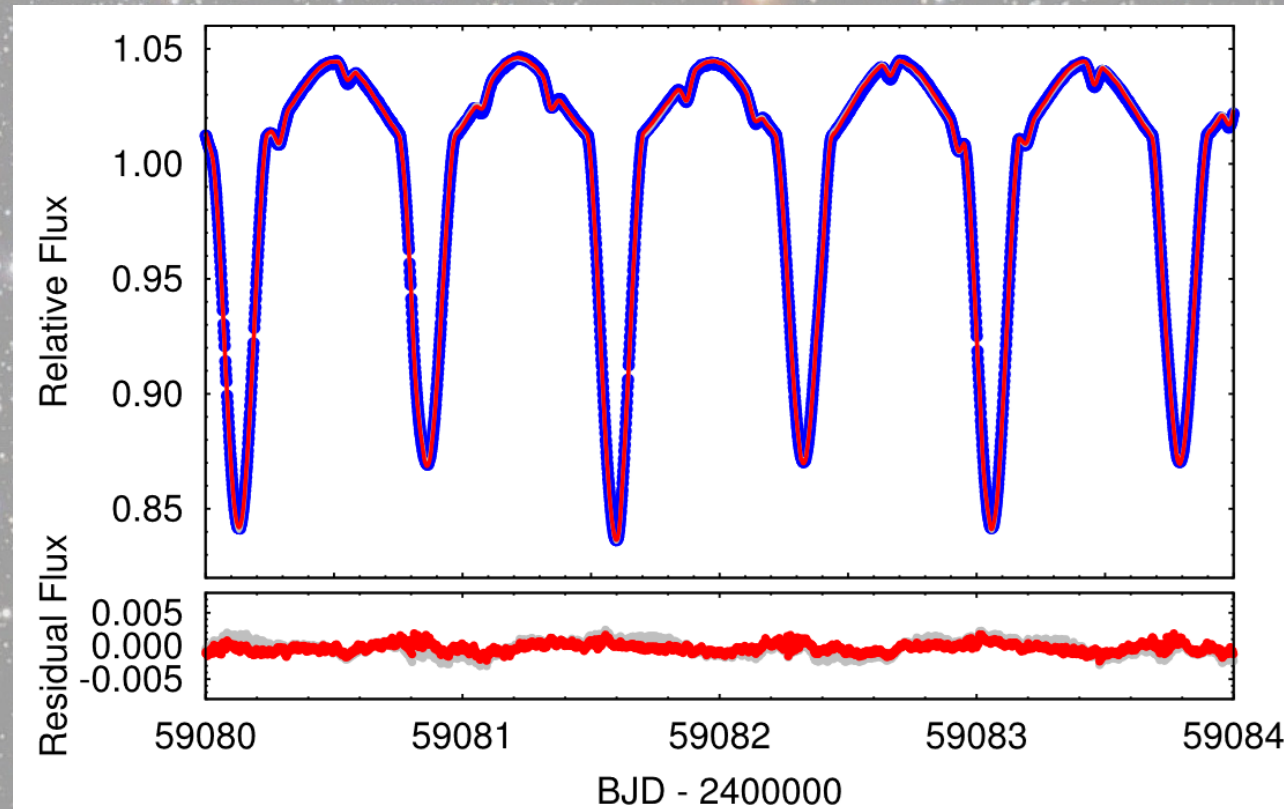
- From our sample: about each 500th EB is multiply eclipsing!!

Several examples here:

- V994 Her: known from 1999, but detected as DEB in 2008
- V482 Per: known from 1966, but detected as DEB in 2017
- AV CMi: known from 1968, but detected as DEB in 2010
- V839 Cep: known from 2006, but detected as DEB in 2021
- V2894 Cyg: known from 2004, but detected as DEB in 2021
- BG Ind: known from 1984, but detected as DEB in 2021
- BU CMi: known from 1999, but detected as DEB in 2021

Problematic detection

- BG Ind



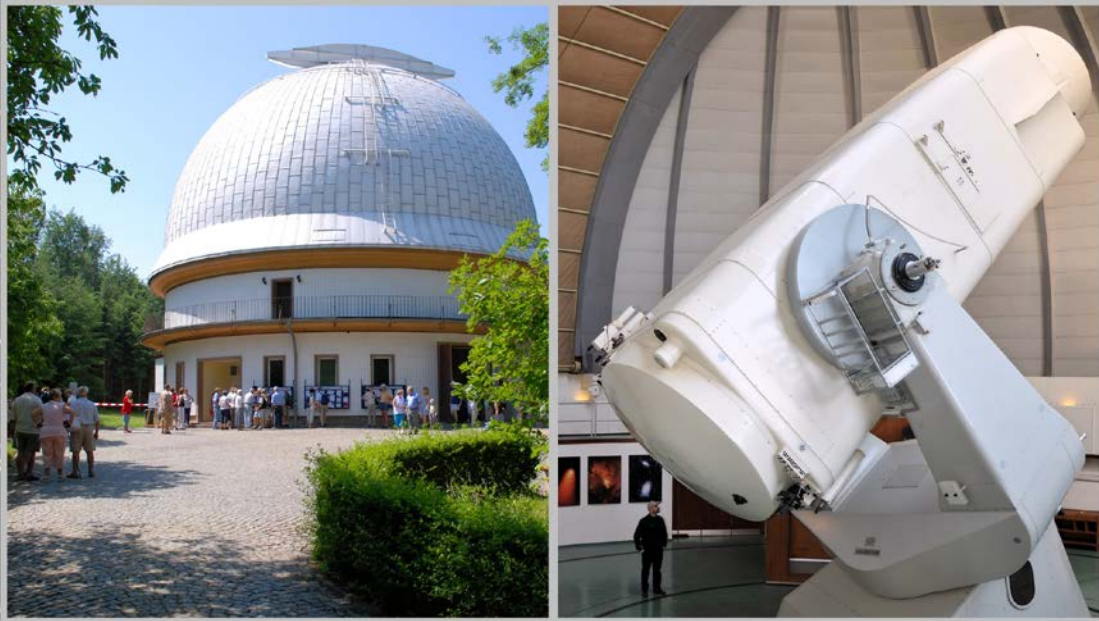
(Borkovits et al. 2021)

One typical example: CzeV1731

- Czech discovery! (Z. Henzl)
- Bright star (10.7mag), northern-hemisphere (DEC +57)
- Many photometric data (NSVS, ASAS-SN, SWASP, KELT)
- Two well-defined periods
- Both detached EA-type
- New data: D65, Henzl, FRAM, TESS
- Detailed study in: *Astronomy & Astrophysics* 642, A63 (2020)

One typical example: CzeV1731

2m Tautenburg telescope

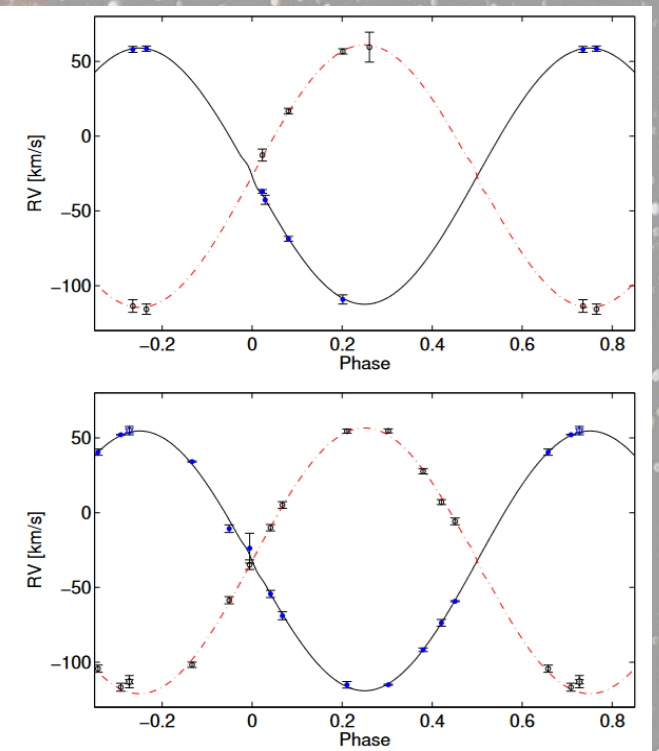
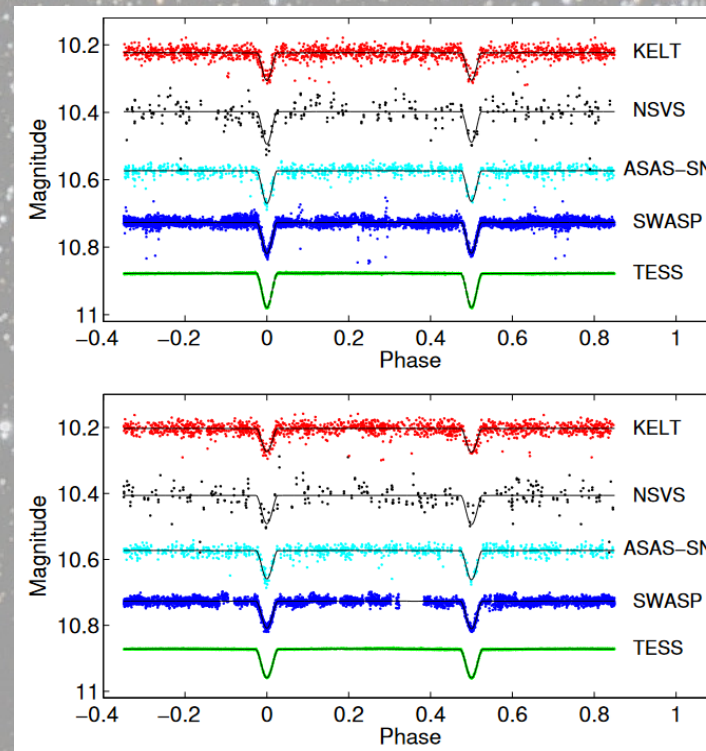
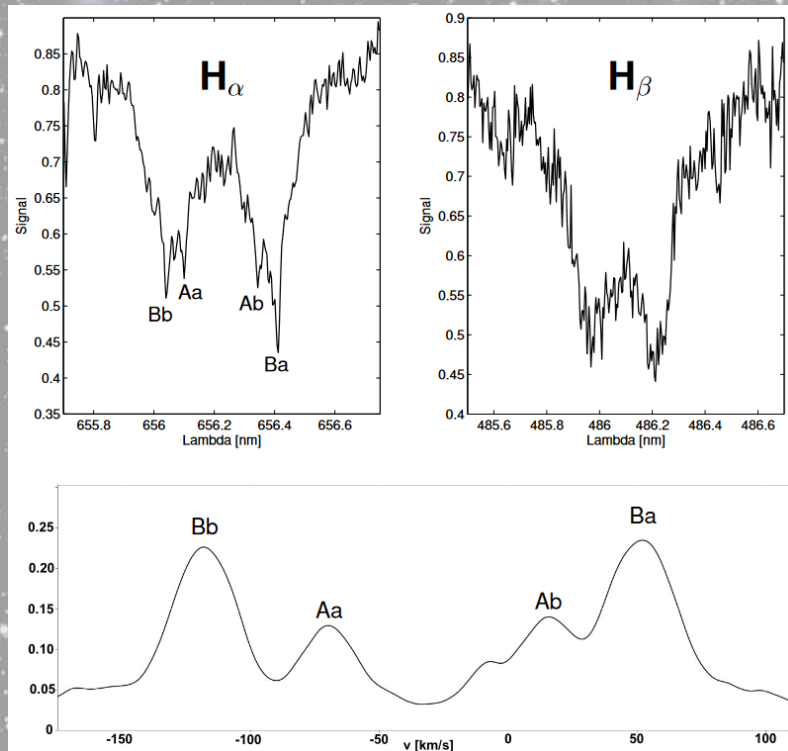


65cm Ondrejov telescope



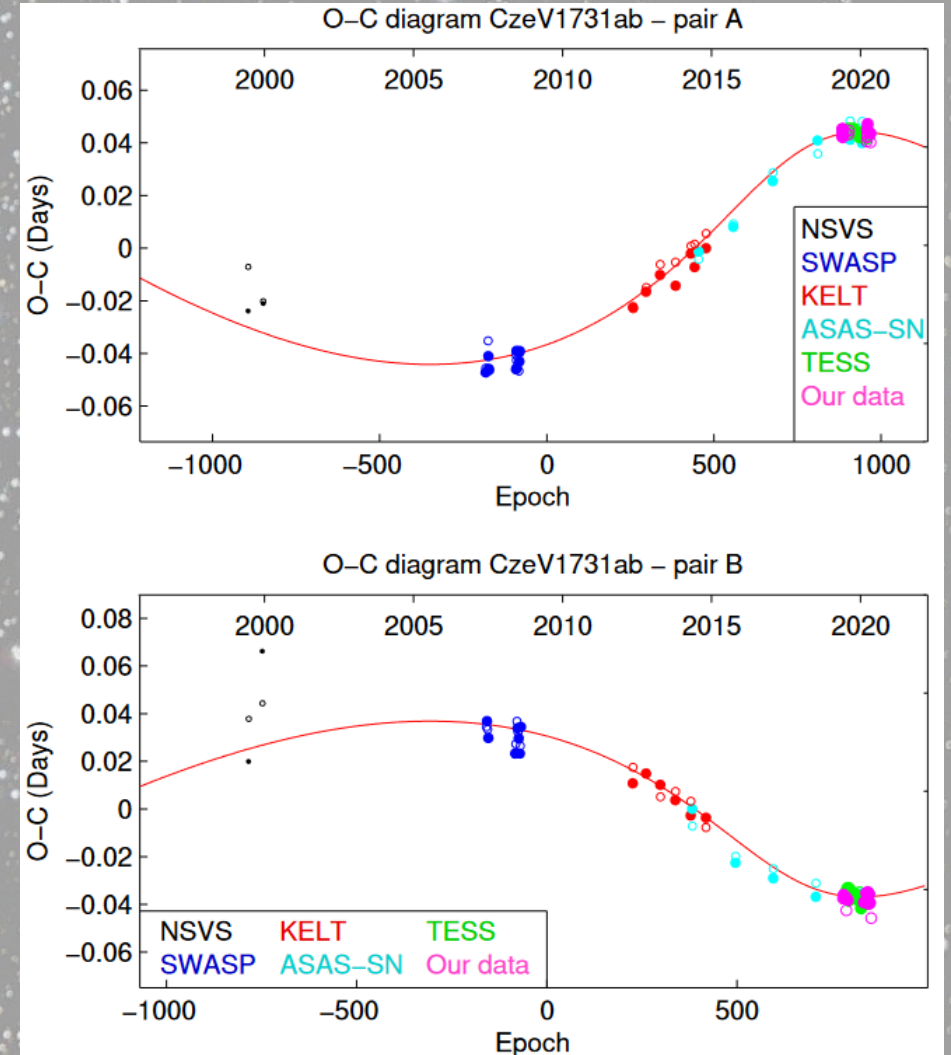
One typical example: CzeV1731

Spectroscopic data from 2m + photometric data --> LC+RV study



One typical example: CzeV1731

- Both pairs ETV period analysis
- Collecting available data
- Ground-based + satellite data
- Range > 20 yr
- Results:
 - $P_A = 4.10842$ d , $e=0.0$
 - $P_B = 4.67552$ d , $e=0.0$
 - Mutual orbit: $p_{AB} = 34$ yr, $e_{AB} = 0.38$
 - $M_B/M_A = 1.2$
 - Predicted: semiaxis ≈ 59 mas



Divide et impera #2 :

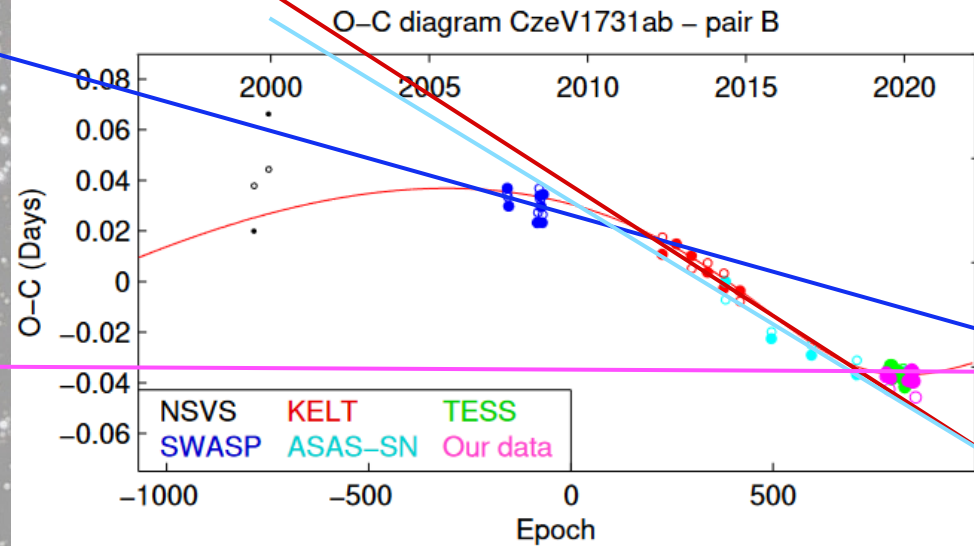
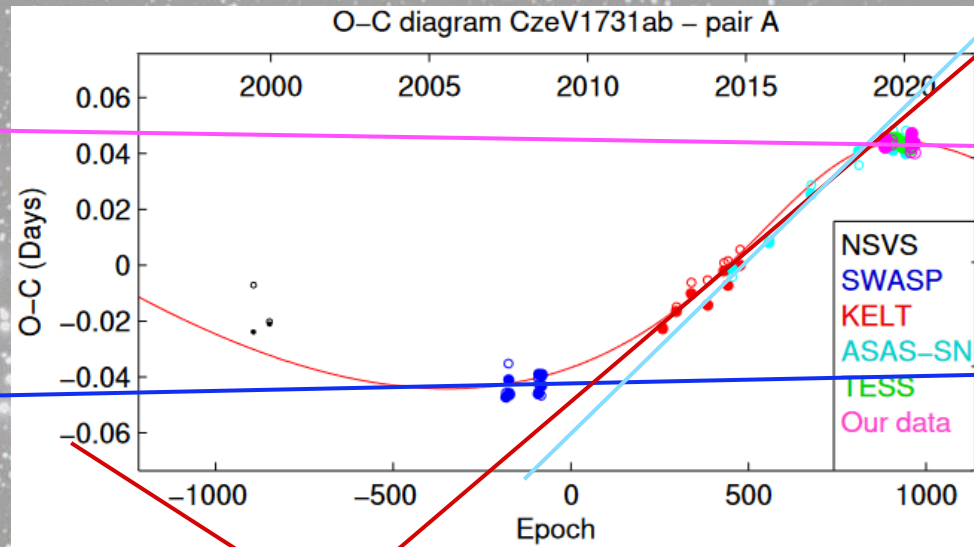
- **Cutting the entire data set into smaller parts**
 - **Assuming linear behavior on smaller time scales**
 - **Period constant during shorter time interval**
 - **All LC, RV, and ETV can be handled in this way**
- ➡ Can such an approach be used for all of the systems?**

Divide et impera #2 :

- **Cutting the entire data set into smaller parts**
 - **Assuming linear behavior on smaller time scales**
 - **Period constant during shorter time interval**
 - **All LC, RV, and ETV can be handled in this way**
- ➡ Can such an approach be used for all of the systems?**

DEFINITELY NOT! BUT SURPRISINGLY.....

Divide et impera #2 :

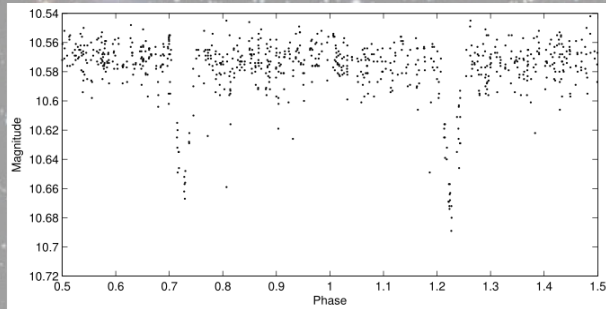


Divide et impera #2 :

- **Breaking the whole dataset to smaller parts is possible only when:**
 - **The outer period is long enough**
 - **The data set span is \ll than the outer period**
 - **The number of data points in the smaller subset is sufficient**

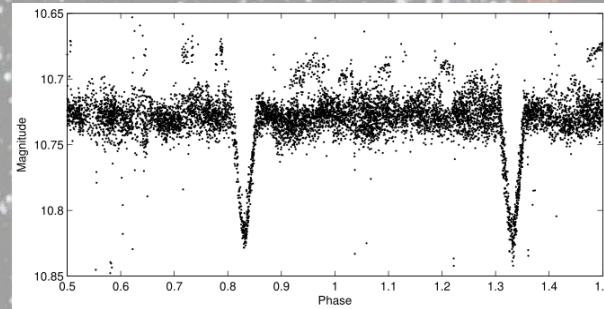
Divide et impera #2 :

- Some examples of LCs for czev1731 star from various datasets

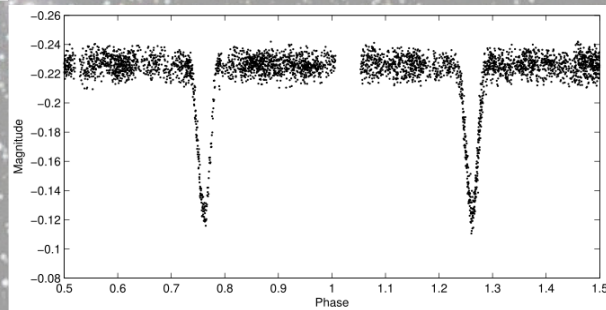


ASAS-SN

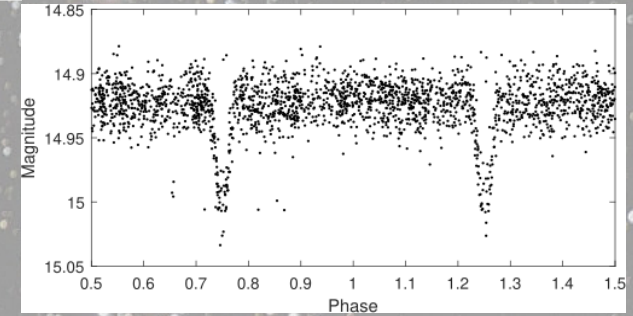
Our data



KELT



SuperWASP



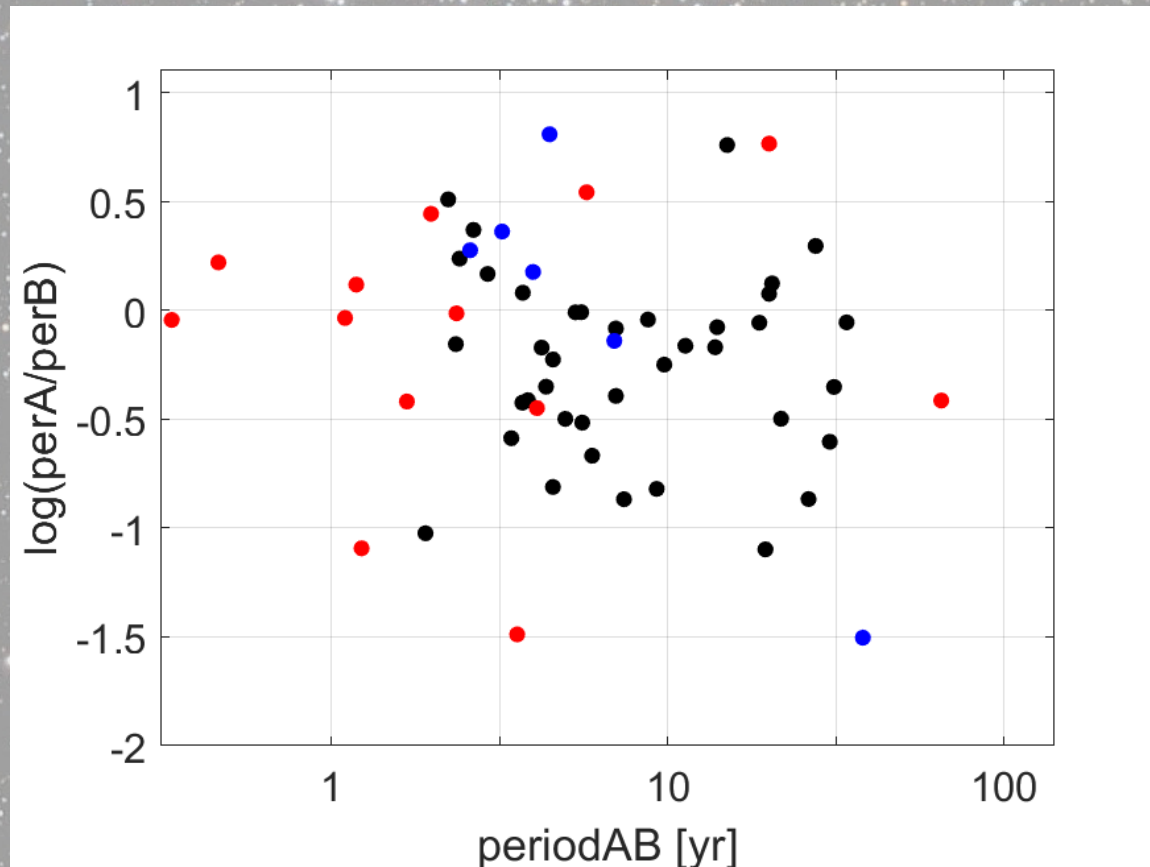
Our results

- Using our approach we have studied and discovered in total 38 proved 2+2 doubly eclipsing quadruples (from total number of 58 of proved ones yet!)
- The statistics: From total number of 58 systems:
 - Only 4 have $P_{AB}/P_A < 100$
 - Only 6 have $P_{AB}/P_B < 100$

Therefore, for the huge majority of systems our approach "divide et impera" is substantiated.

Our results - comparison

- **Periods: inner versus mutual**



Black dots: our systems

Red dots: Borkovits et al.

Blue dots: others

Our approach and its assumptions

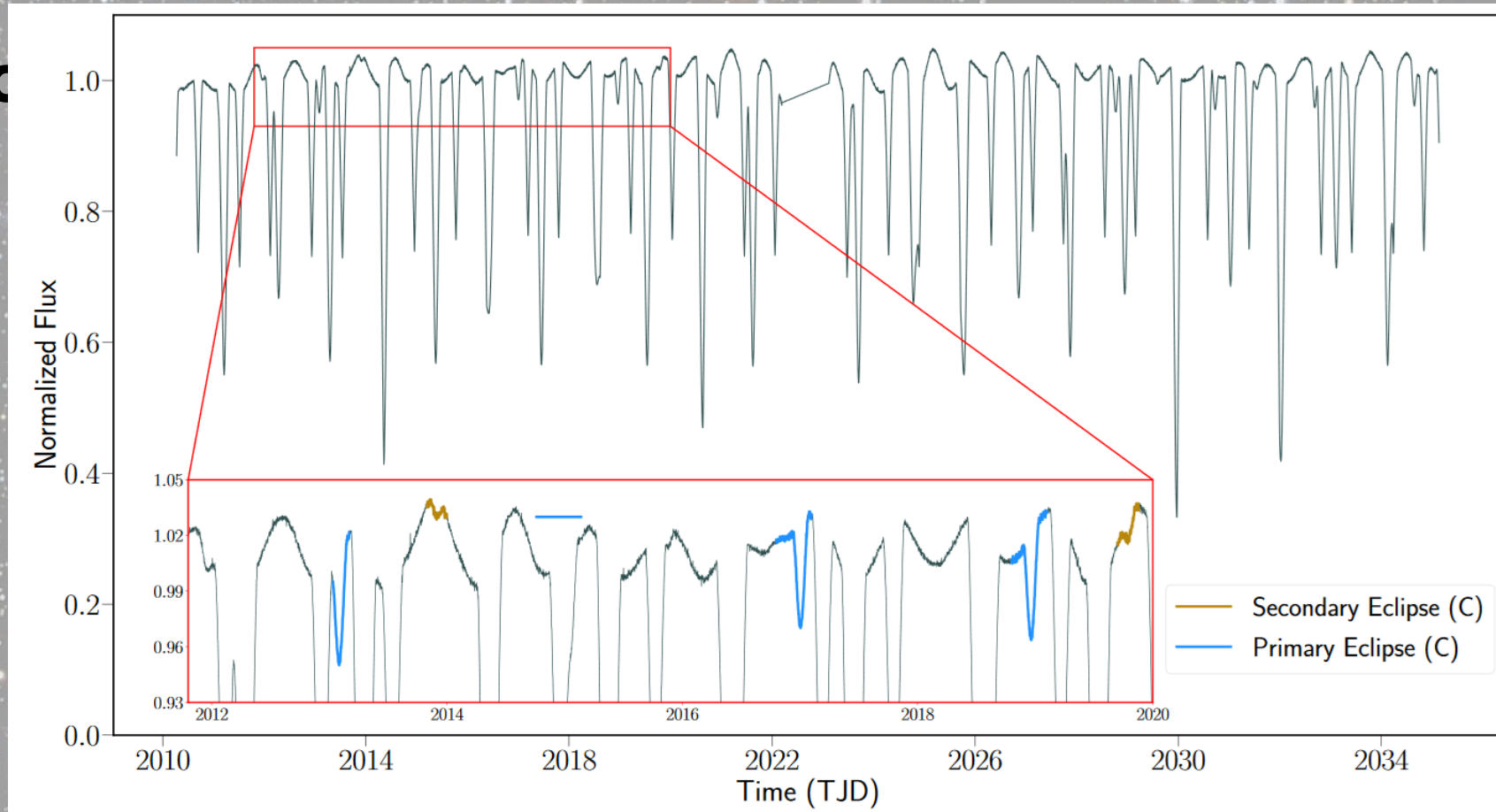
- **For whole our analysis we assume:**
 - **Constant luminosity fraction of both A&B pairs: $L_A/L_B = \text{const.}$**
 - **Constant value of the third light value (i.e. non-variable third light)**
 - **Inclination of both the orbits also constant**
- **On the other hand, what can even change and is computed:**
 - **Periods of both A&B, apsidal motion of both pairs, etc.**

Things are sometimes more complicated

- For some systems these assumptions do not work...
- V994 Her – more complicated system!

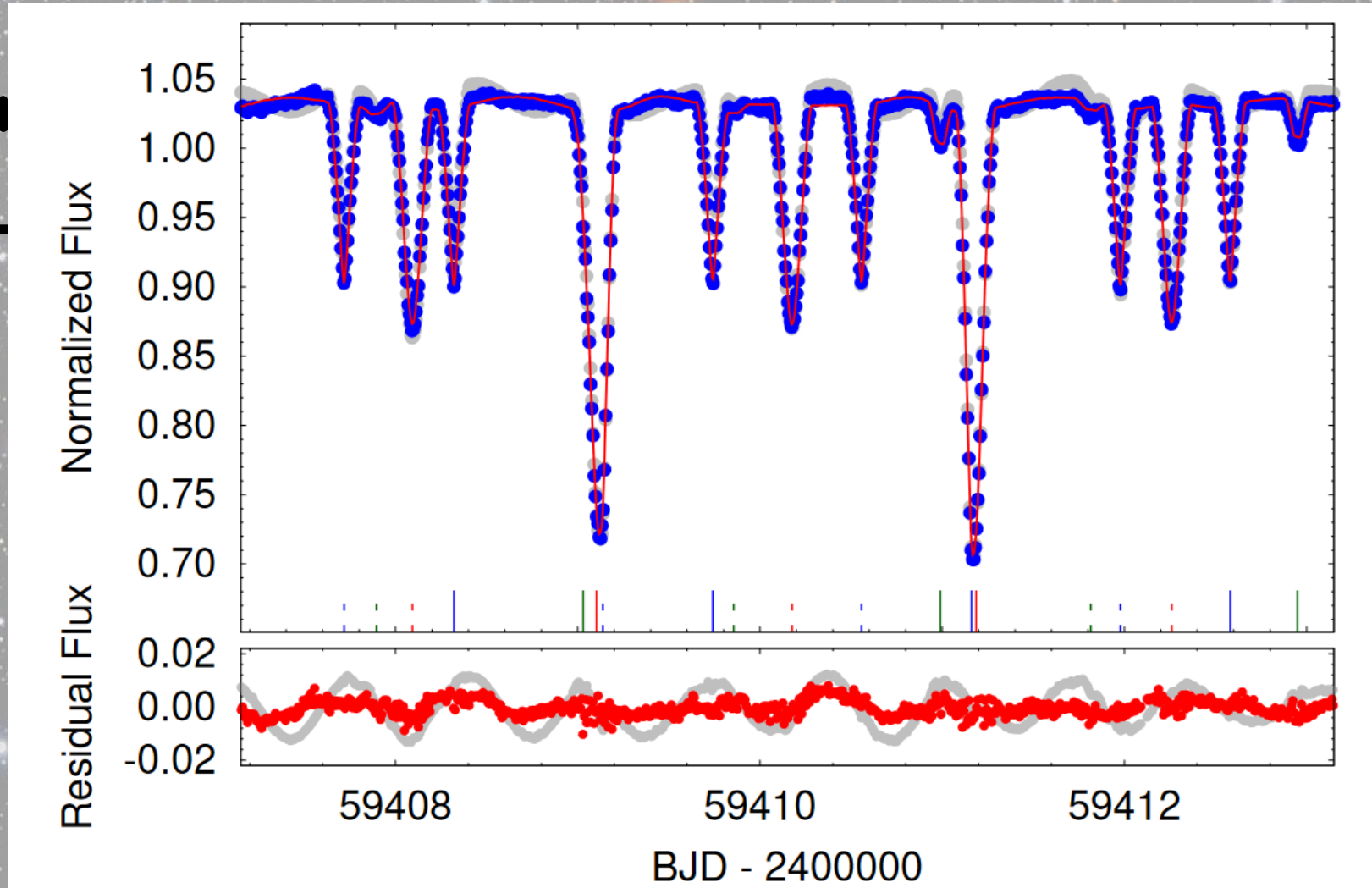
Things are sometimes more complicated

- For so
- V994



Things are sometimes more complicated

- For some
- V994 H



Things are sometimes more complicated

- For
- V99

Parameter	Binary A		Binary B		Binary C		A-B orbit
P_a [days]	2.0832039 ^{+0.0000042} _{-0.0000039}		1.4200981 ^{+0.0000033} _{-0.0000040}		1.9601064 ^{+0.0000018} _{-0.0000018}		1062.3 ^{+2.2} _{-1.6}
a [R_\odot]	11.90 ^{+0.18} _{-0.21}		8.27 ^{+0.08} _{-0.07}		9.00 ^{+0.08} _{-0.07}		911 ⁺¹¹ ₋₁₂
i^a [deg]	84.26 ^{+0.40} _{-0.36}		89.37 ^{+0.37} _{-0.35}		85.90 ^{+1.17} _{-1.02}		85.0 ^{+3.2} _{-3.7}
e	0.0271 ^{+0.0012} _{-0.0013}		0.1187 ^{+0.0007} _{-0.0007}		0.1701 ^{+0.0064} _{-0.0087}		0.694 ^{+0.027} _{-0.027}
ω [deg]	206.2 ^{+4.3} _{-6.1}		173.6 ^{+2.2} _{-2.8}		314.0 ^{+3.4} _{-2.3}		59.1 ^{+3.0} _{-3.0}
$\dot{\omega}$ [deg/yr]	1.85 ^{+0.70} _{-0.63}		3.60 ^{+0.14} _{-0.18}		2.04 ^{+0.30} _{-0.35}		—
τ [BJD - 2 400 000]	59 010.842 ^{+0.025} _{-0.035}		59 010.389 ^{+0.009} _{-0.011}		59 009.338 ^{+0.018} _{-0.012}		58 167.6 ^{+9.7} _{-6.9}
$t_{\text{prim eclipse}}$ [BJD - 2 400 000]	59 011.1822 ^{+0.0008} _{-0.0010}		59 010.7330 ^{+0.0017} _{-0.0018}		59 011.1252 ^{+0.0003} _{-0.0003}		—
q ($= m_2/m_1$)	0.760 ^{+0.008} _{-0.008}		1.007 ^{+0.008} _{-0.007}		0.534 ^{+0.031} _{-0.038}		0.724 ^{+0.021} _{-0.018}
K_{pri} [km s ⁻¹]	124 ⁺² ₋₃		149 ⁺² ₋₁		82 ⁺³ ₋₄		25 ⁺¹ ₋₁
K_{sec} [km s ⁻¹]	163 ⁺² ₋₂		148 ⁺¹ ₋₁		154 ⁺⁴ ₋₄		35 ⁺² ₋₂
γ [km/s]	—		—		—		-39.0 ^{+0.5} _{-0.4}
individual stars	Aa	Ab	Ba	Bb	Ca	Cb	
Relative Quantities:							
fractional radius ^b [R/a]	0.1773 ^{+0.0027} _{-0.0022}	0.1461 ^{+0.0017} _{-0.0021}	0.1907 ^{+0.0021} _{-0.0027}	0.1915 ^{+0.0020} _{-0.0028}	0.1673 ^{+0.0019} _{-0.0021}	0.0874 ^{+0.0084} _{-0.0044}	
fractional luminosity in <i>TESS</i> -band	0.4198 ^{+0.0086} _{-0.0095}	0.2077 ^{+0.0113} _{-0.0121}	0.1290 ^{+0.0023} _{-0.0019}	0.1315 ^{+0.0024} _{-0.0021}	0.0930 ^{+0.0112} _{-0.0126}	0.0067 ^{+0.0012} _{-0.0012}	
fractional luminosity in <i>V</i> -band	0.4406 ^{+0.0124} _{-0.0176}	0.2084 ^{+0.0130} _{-0.0153}	0.1200 ^{+0.0033} _{-0.0031}	0.1232 ^{+0.0035} _{-0.0034}	0.0763 ^{+0.0130} _{-0.0148}	0.0032 ^{+0.0008} _{-0.0007}	
Physical Quantities:							
T_{eff}^c [K]	11879 ⁺³⁶² ₋₃₂₅	9915 ⁺²⁸⁹ ₋₂₆₄	8643 ⁺²⁴¹ ₋₁₇₉	8695 ⁺²⁴² ₋₁₉₀	7785 ⁺³⁵¹ ₋₂₈₅	5181 ⁺¹⁵⁶ ₋₂₂₀	
mass [M_\odot]	2.957 ^{+0.133} _{-0.146}	2.251 ^{+0.107} _{-0.125}	1.876 ^{+0.050} _{-0.048}	1.889 ^{+0.055} _{-0.049}	1.662 ^{+0.052} _{-0.055}	0.888 ^{+0.039} _{-0.046}	
radius ^c [R_\odot]	2.110 ^{+0.024} _{-0.023}	1.741 ^{+0.039} _{-0.057}	1.580 ^{+0.024} _{-0.032}	1.586 ^{+0.024} _{-0.034}	1.507 ^{+0.020} _{-0.023}	0.789 ^{+0.073} _{-0.044}	
luminosity ^c [L_\odot]	79.5 ^{+11.4} _{-9.4}	26.3 ^{+4.4} _{-4.1}	12.5 ^{+1.4} _{-1.1}	12.9 ^{+1.5} _{-1.2}	7.5 ^{+1.4} _{-1.1}	0.40 ^{+0.08} _{-0.07}	
[M_{bol}]	0.02 ^{+0.14} _{-0.15}	1.22 ^{+0.18} _{-0.17}	2.03 ^{+0.10} _{-0.12}	2.00 ^{+0.10} _{-0.12}	2.59 ^{+0.17} _{-0.18}	5.76 ^{+0.22} _{-0.20}	
$\log g^c$ [cgs]	4.260 ^{+0.014} _{-0.019}	4.309 ^{+0.007} _{-0.006}	4.314 ^{+0.011} _{-0.009}	4.314 ^{+0.011} _{-0.009}	4.302	4.597 ^{+0.026} _{-0.075}	
$\log(\text{age})$ [dex]			7.79 ^{+0.26} _{-0.21}				
$(M_V)_{\text{tot}}^c$			-0.24 ^{+0.07} _{-0.08}				
distance [pc]			271 ⁺⁷ ₋₆				

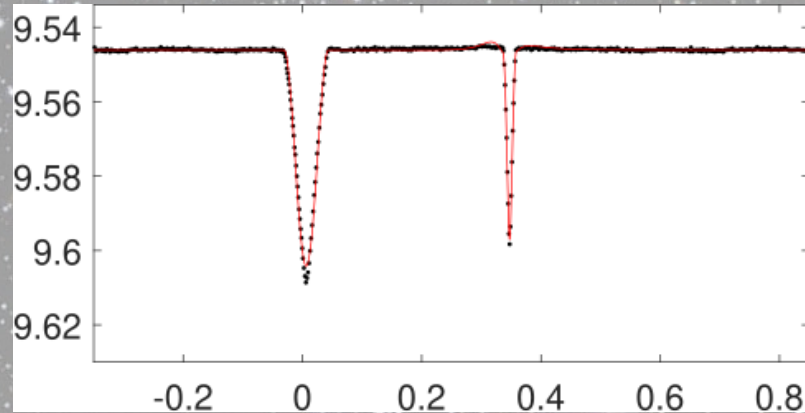
Things are sometimes more complicated

- For some systems these assumptions do not work...
- The star named CzeV4315:
 - Discovered by Z.Henzl
 - Bright star, $V = 9.6\text{mag}$
 - In nebula, dense area, near galactic plane
 - Visual double star ($2''$, $\Delta M \approx 2\text{ mag}$), or CPM pair
 - Several spectral type estimates: A0, B8, B9

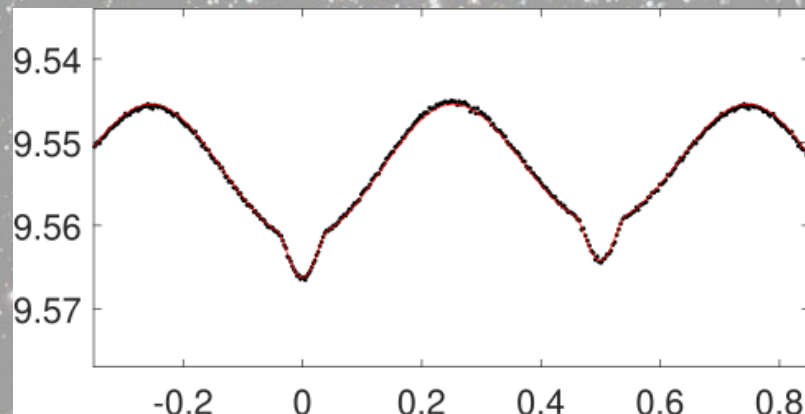
CzeV4315

- TESS data shows two periodicities:
- 1:
 - Detached LC, period 6.74d
 - Very eccentric orbit
 - Prim eclipse: 12h, sec eclipse 3.5h
- 2:
 - Close but still detached pair, period 0.919d
 - Eclipses on TESS data deeper and deeper
- Unfortunately, no other usable photometry (ZTF, SWASP, Atlas, ASASsn)

CzeV4315

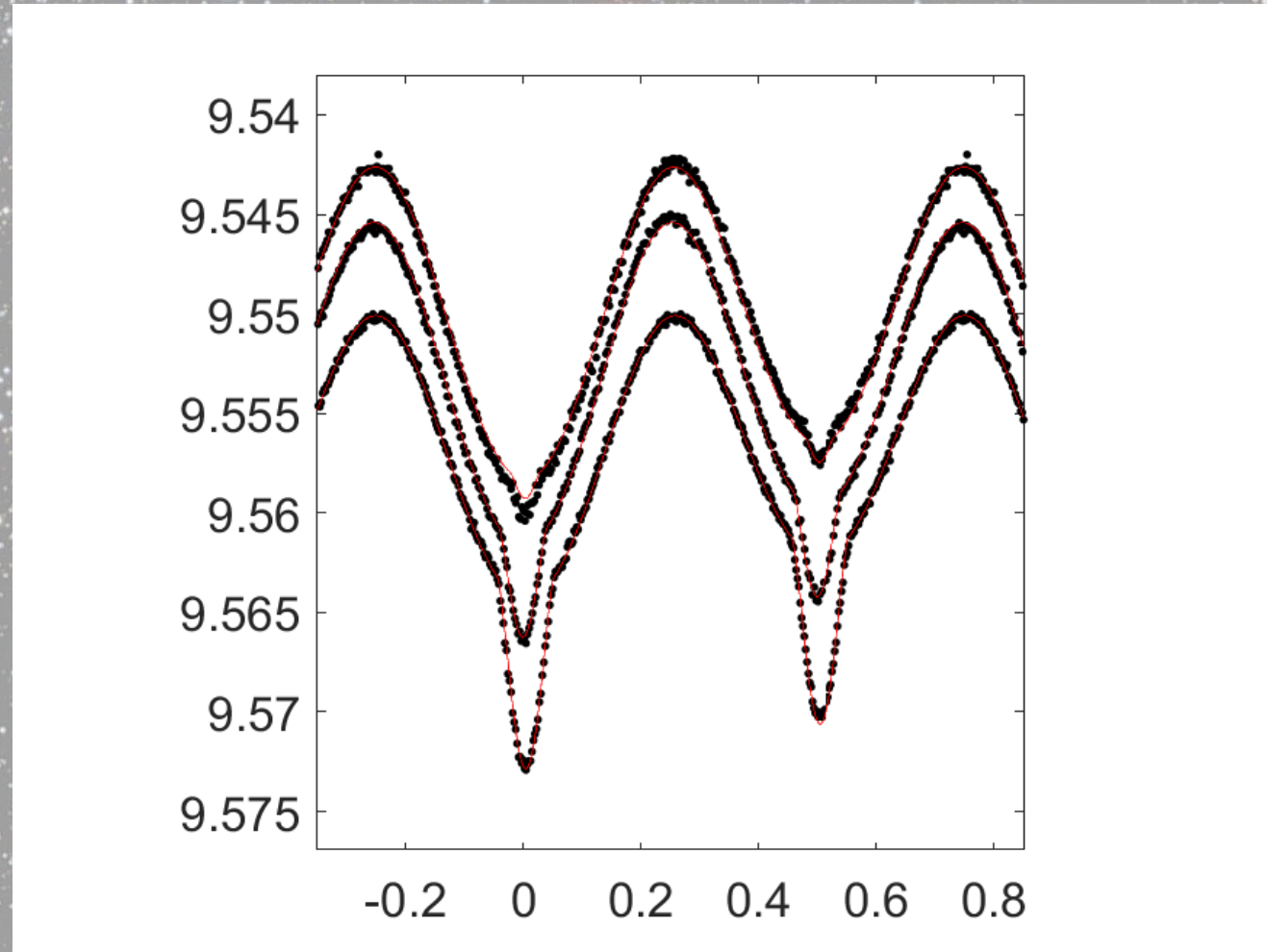


- **Preliminary pair A:**
 - **P = 6.739 d**
 - **e = 0.69**
 - **Slow apsidal motion**
 - **Constant inclination**

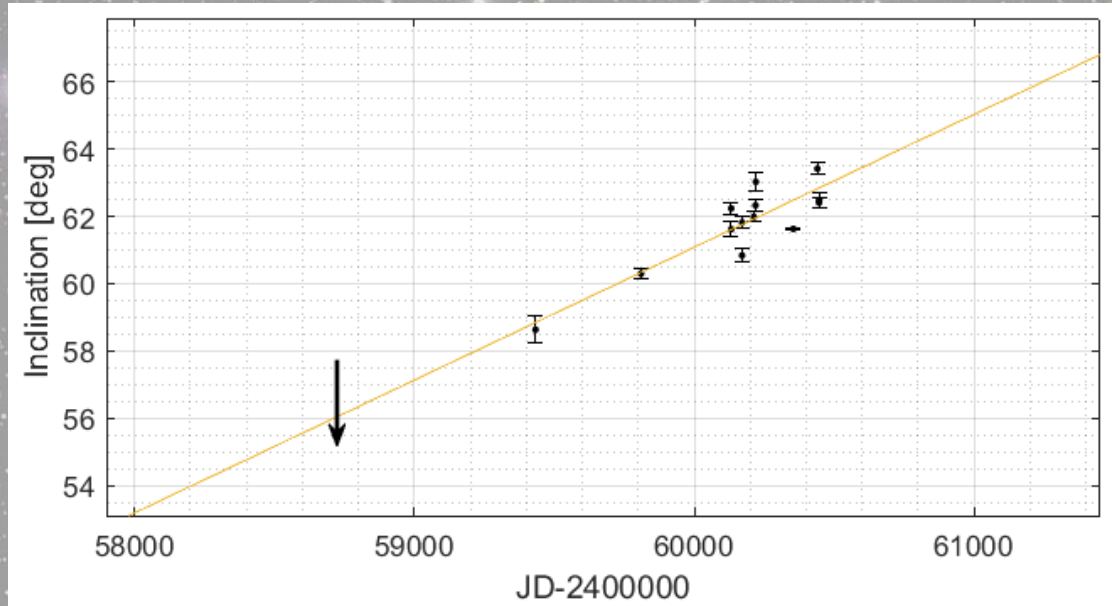


- **Preliminary pair B**
 - **P = 0.9193 d, circular**
 - **TESS sect 14+15: only ELL variations**
 - **TESS sect 41: start of eclipses**
 - **TESS sect 55: clearly visible eclipses**
 - **TESS sect 75: deep enough for ground based observations**

CzeV4315 – evolution during TESS sectors

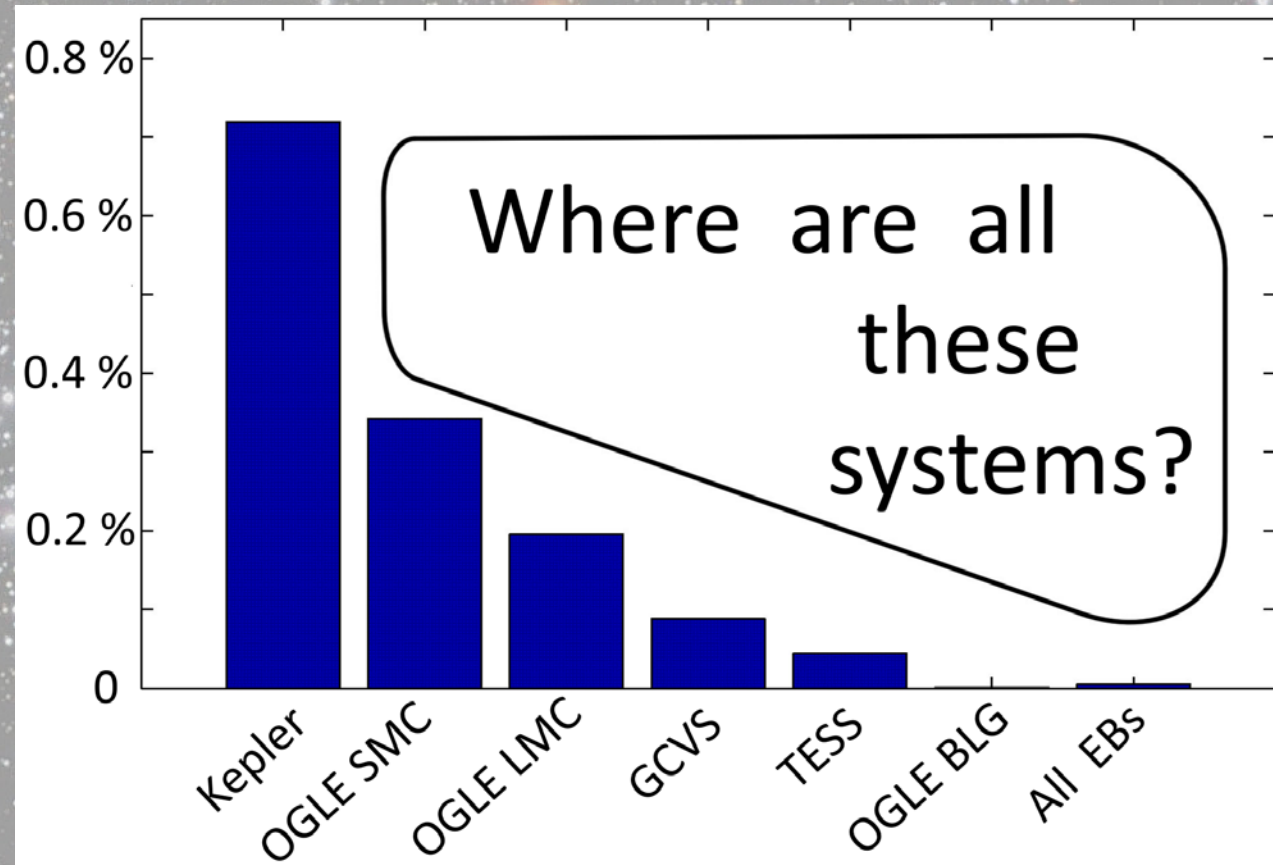


CzeV4315 – inclination change



- **Rapid inclination change**
- **Now visible on ground-based data**
- **Rough estimation of nodal precession: second fastest!**
- **Rough estimation of mutual period**
A-B \approx 200-300 days

Inclination changing stars – very rare!



Conclusion

- Doubly eclipsing systems as unique celestial mechanics laboratory
- Complicated for modelling – taking all constraints into account
- Still very limited number of proved 2+2 quadruples
- For most of them the "divide et impera" method sufficient
- In general, the dynamical interactions should always be tested



This is the end...
my only friend, the end

