A THEORY OF MASS TRANSFER IN BINARY STARS

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Cehula & Pejcha (2023, MNRAS, 524, 471–490)

Cehula & Pejcha (in prep.)

Binary and Multiple Stars in the Era of Big Sky Surveys
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erc European Research Council

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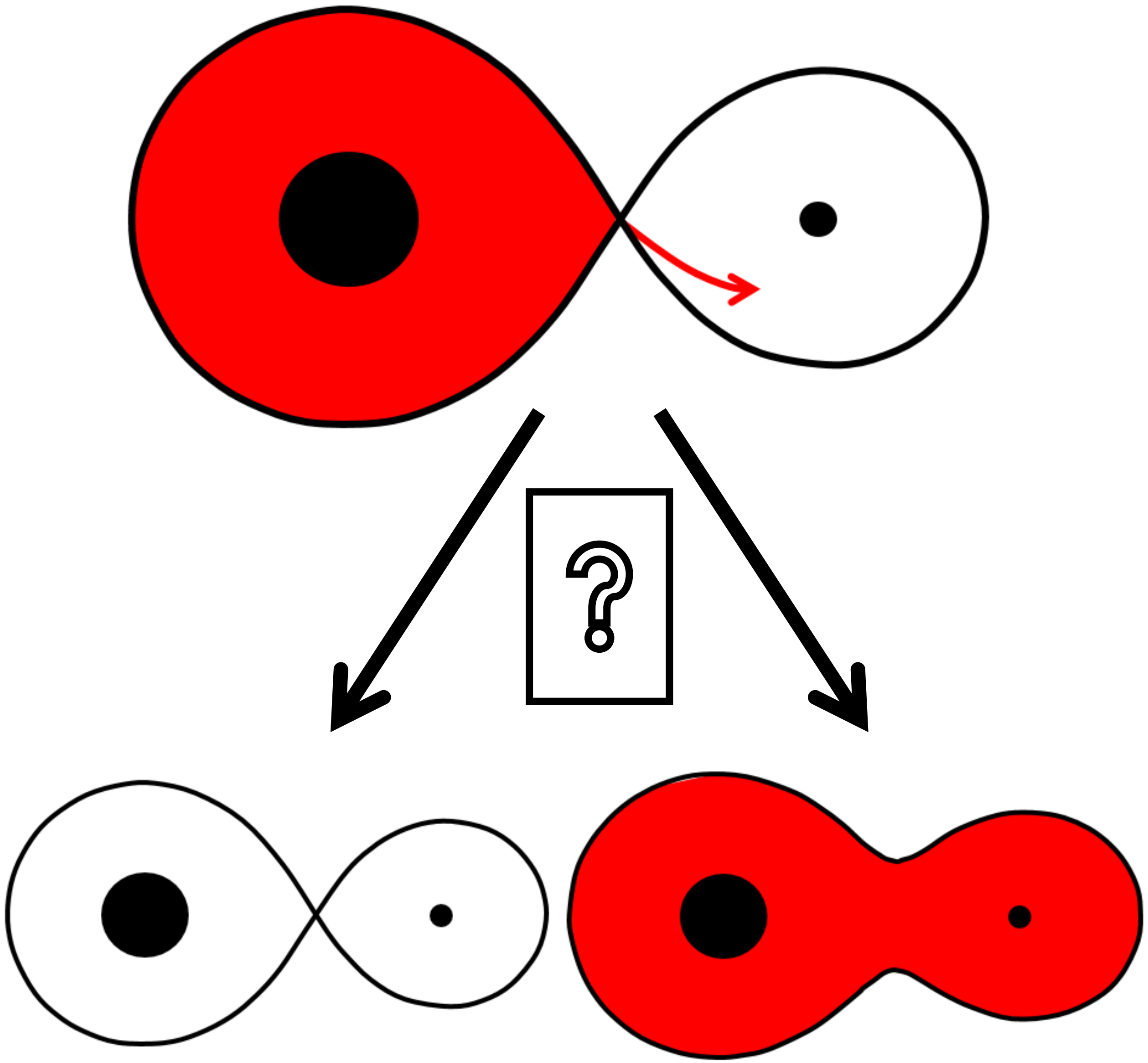
MOTIVATION

- mass transfer responsible for X-ray binaries, cataclysmic variables, type Ia supernovae, …
- understanding binary mass transfer => accurate differentiation between:
    1. stable mass transfer
    2. unstable mass transfer → common-envelope evolution
- standard mass-transfer models suffer from conceptual and practical difficulties => new model needed



MAIN GOAL

- donor's mass-loss rate:

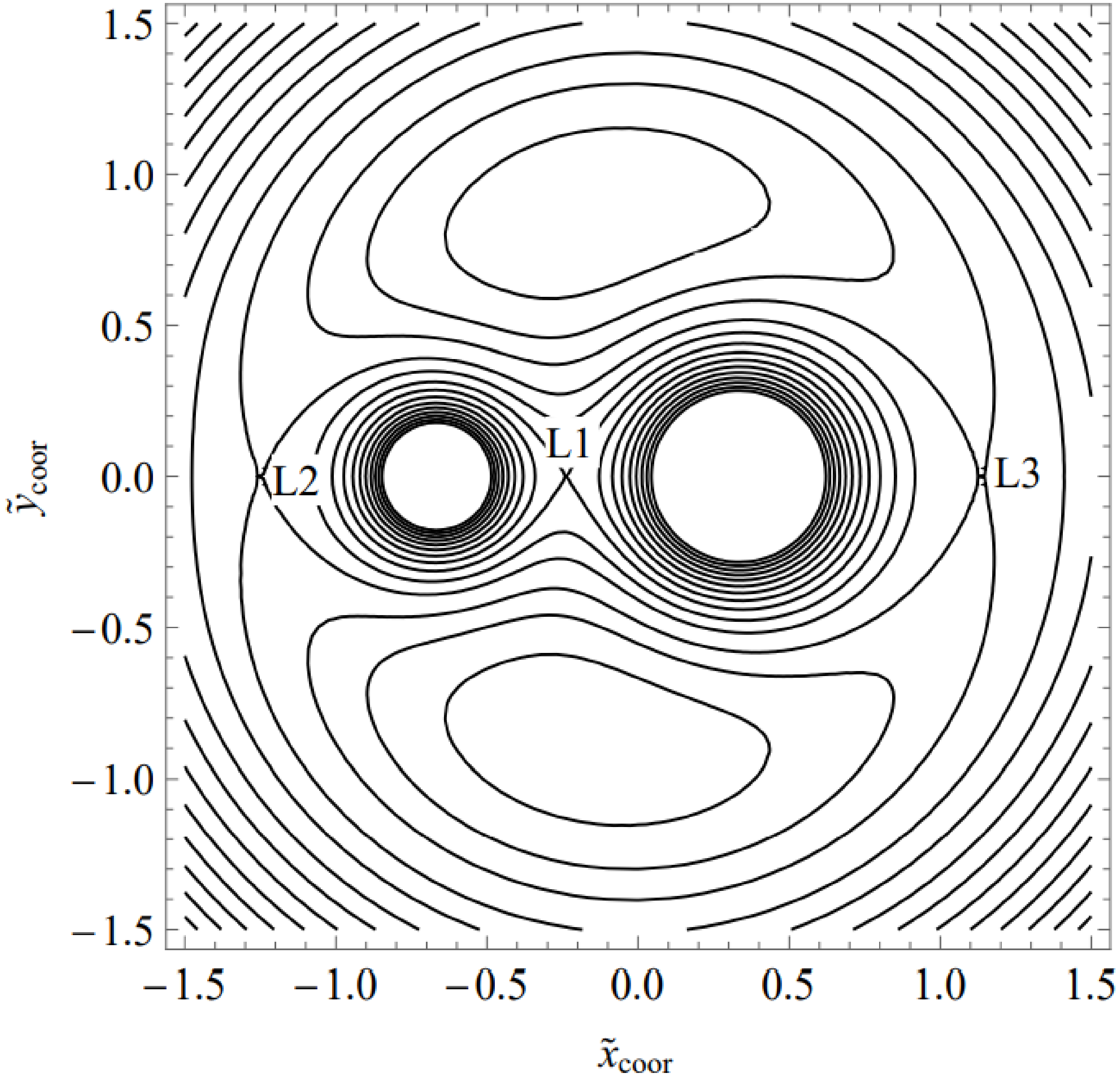
$$\dot{M}_{\mathrm{d}} = \dot{M}_{\mathrm{d}}(\delta R_{\mathrm{d}})$$

  - where δR_{d} is the relative radius excess:

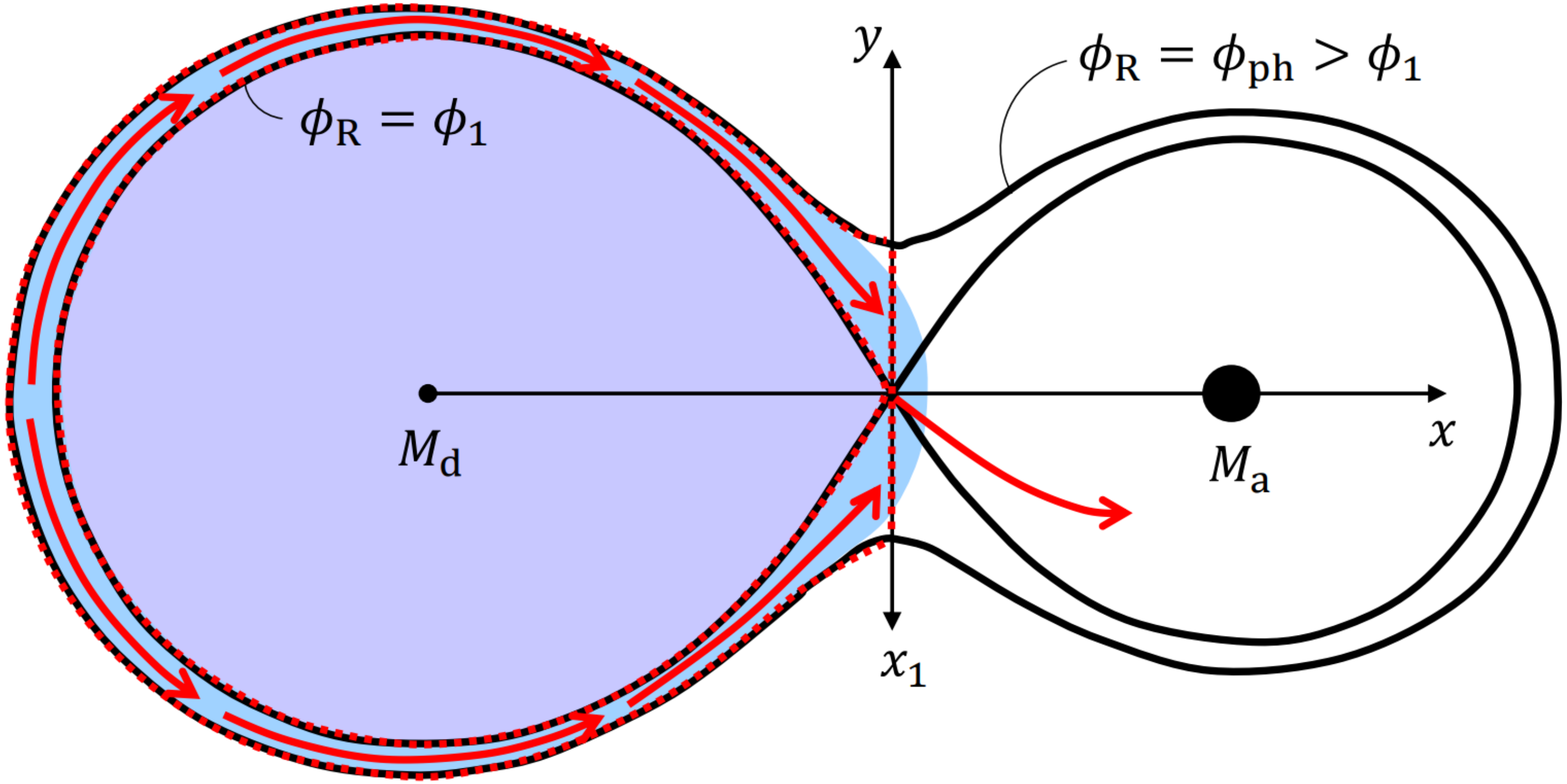
$$\delta R_{\mathrm{d}} \equiv \frac{\Delta R_{\mathrm{d}}}{R_{\mathrm{L}}} = \frac{R_{\mathrm{d}} - R_{\mathrm{L}}}{R_{\mathrm{L}}},$$

    R_{d} - donor's radius, R_{L} - Roche-lobe radius

  - serves as a **boundary condition** in a stellar evolution code (MESA)



STANDARD MODEL



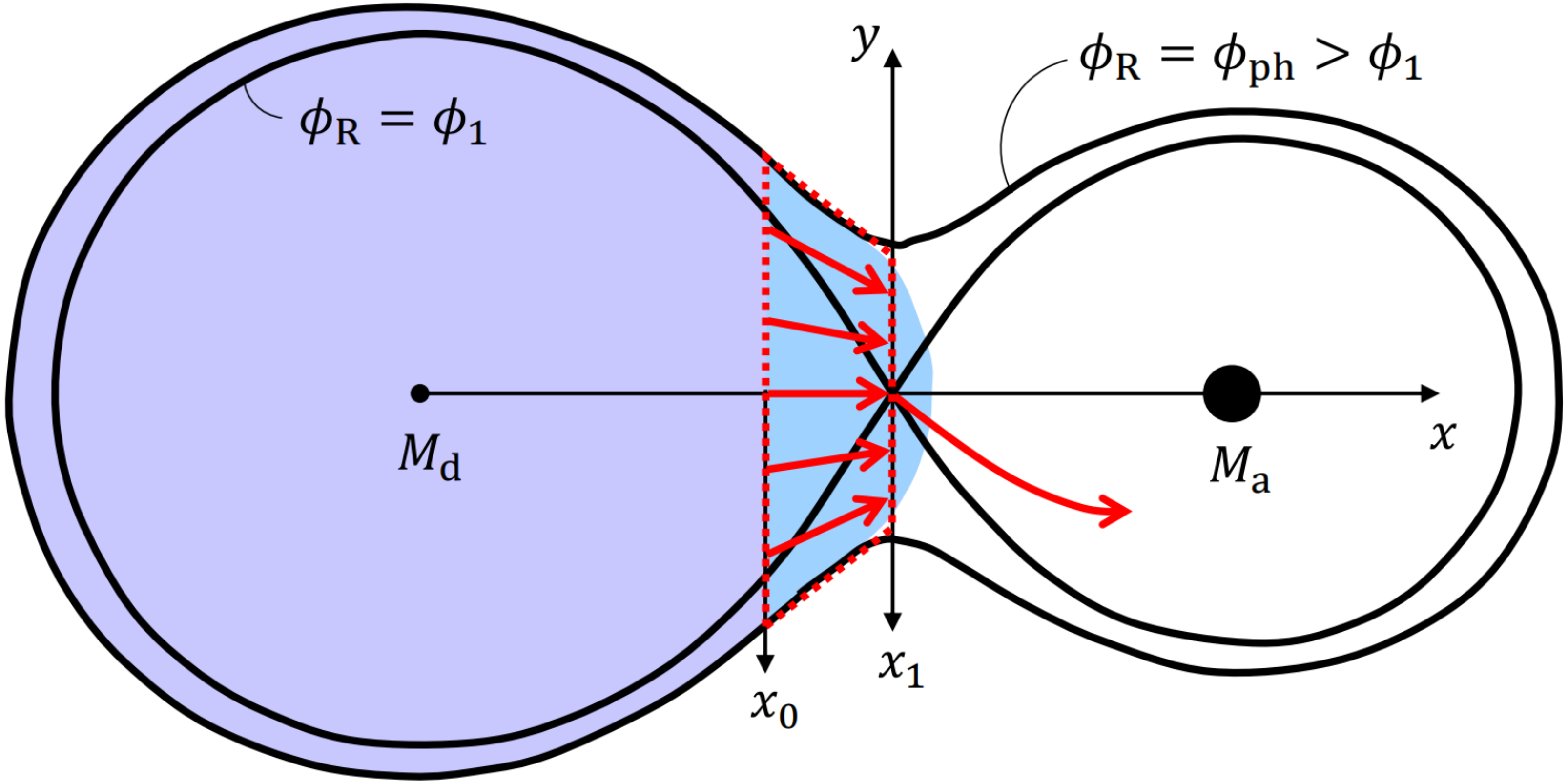
- possible systematic errors
- **instant** optically thin → thick transition
- stellar interior (sonically connected) does **not** influence mass loss
- **not** possible to include additional physics (radiation, mag. field, …)

$$\dot{M}_{\rm KR} = \frac{{\rm d}Q}{{\rm d}\phi}\bigg|_{\rm L1} \int_{\phi_1}^{\phi_{\rm ph}} F_3(\Gamma)\left(\frac{k\bar{T}}{\bar{m}}\right)^{\frac{1}{2}} \bar{\rho}\,{\rm d}\bar{\phi},$$

(Kolb & Ritter 1990)

(Lubow & Shu 1975, Ritter 1988, Kolb & Ritter 1990, Pavlovskii & Ivanova 2015, Jackson et al. 2017, Marchant et al. 2021)

NEW MODEL



(Cehula & Pejcha 2023)

ADVANTAGES

- testing for systematic errors
- stellar interior (sonically connected) **influences** mass loss
- **possible** to include additional physics (radiation, mag. field, …)
- clear analogy with stellar winds – de Laval nozzle

NEW MODEL IN EQUATIONS

START

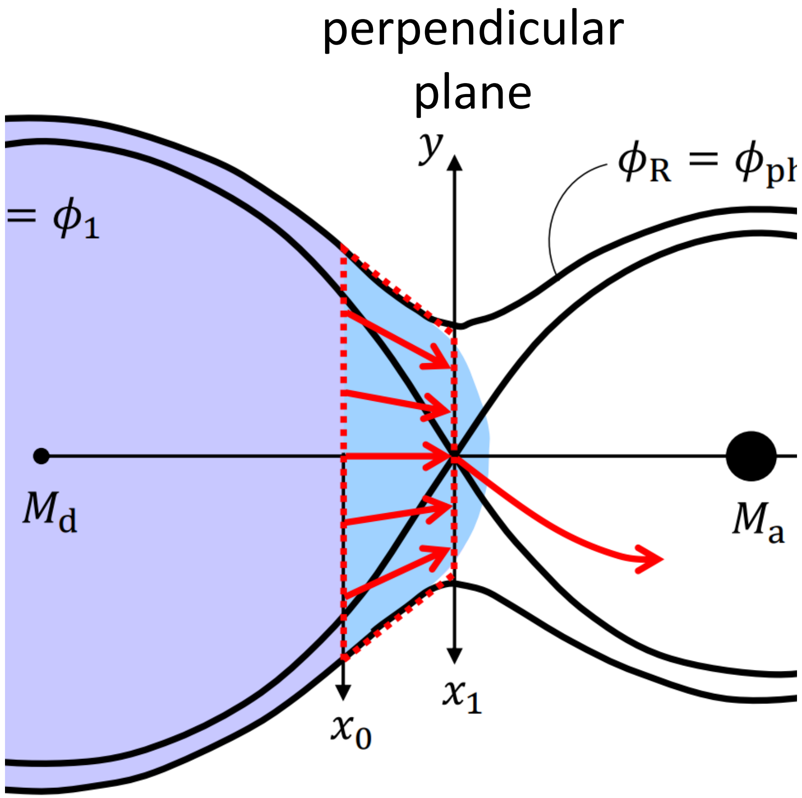
- ➢3D Euler equations with the Roche potential

ASSUMPTIONS

1. Stationarity: $\partial/\partial t \to 0$
2. Gas flow – effectively 1D ⇒ hydrostatic equilibrium in the perpendicular plane
3. Lowest order approximation of the Roche potential in the perpendicular plane
4. Polytropic approx. in the perpendicular plane

END

- ➢1D Euler equations with the Roche potential



$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho Q_\rho}\frac{\mathrm{d}}{\mathrm{d}x}(\rho Q_\rho) = 0,$$

$$v\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho Q_\rho}\frac{\mathrm{d}}{\mathrm{d}x}(P Q_P) = -\frac{\mathrm{d}\phi_{\rm R}}{\mathrm{d}x},$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\epsilon\frac{Q_P}{Q_\rho}\right) - \frac{P Q_P}{(\rho Q_\rho)^2}\frac{\mathrm{d}}{\mathrm{d}x}(\rho Q_\rho) = -\frac{\mathrm{d}}{\mathrm{d}x}\left(c_T^2\frac{Q_P}{Q_\rho}\right),$$

$$\dot{M}_{\rm new} = v\rho Q_\rho$$

SOLUTION OF NEW EQUATIONS

- 1D Euler equations with the Roche potential:

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho Q_\rho}\frac{\mathrm{d}}{\mathrm{d}x}(\rho Q_\rho) = 0,$$

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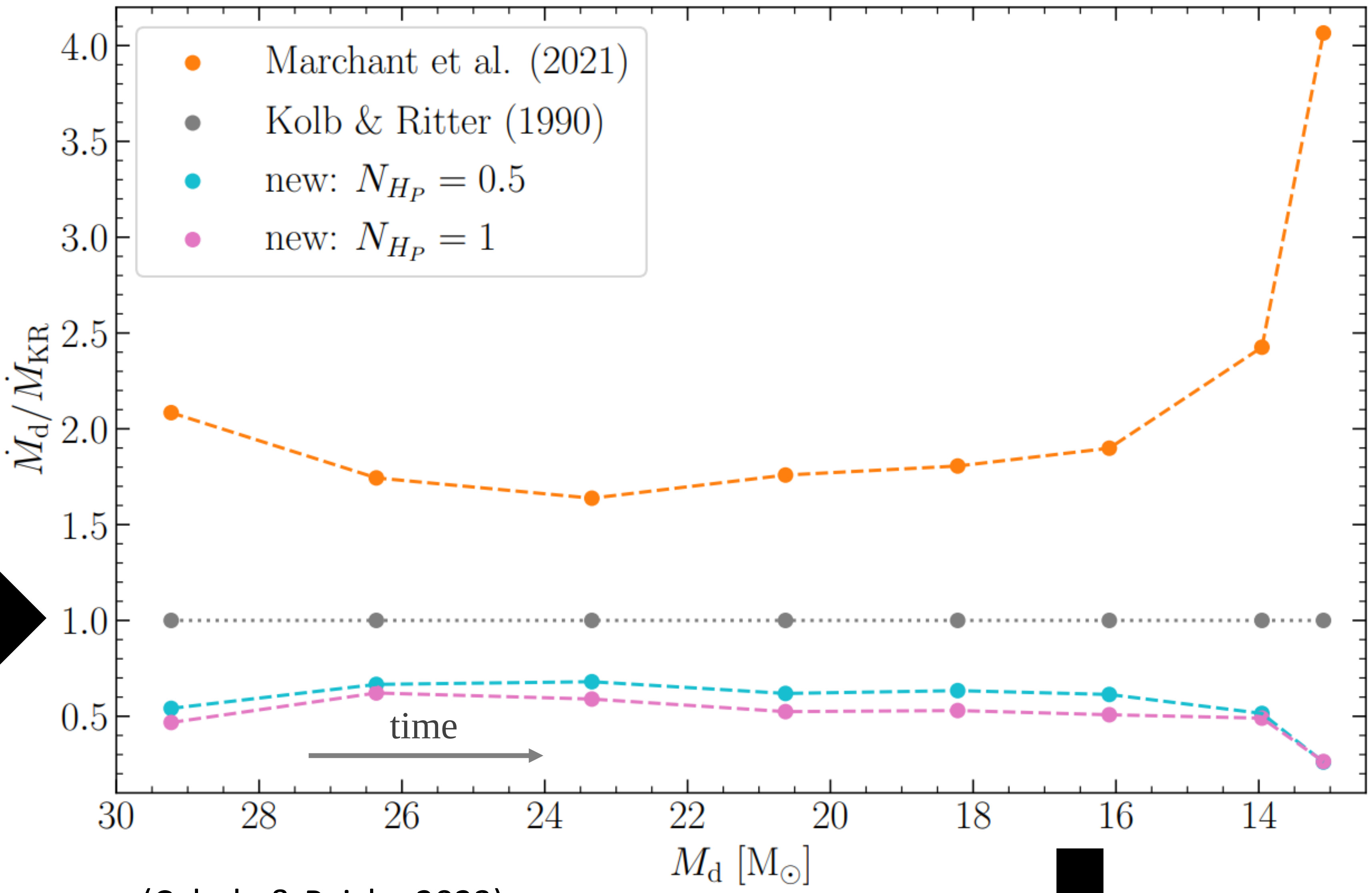
- 2-point BVP ⇒ numerical relaxation (Press et al. 2007)

- we still need the **EQUATION OF STATE**
  - isothermal: $P = K\rho$ } algebraic solution
  - polytropic: $P = K\rho^\Gamma$ } algebraic solution
  - realistic: MESA EOS module

(Saumon, Chabrier, & van Horn 1995; Irwin 2004; Timmes & Swesty 2000; Potekhin & Chabrier 2010; Jermyn et al. 2021)

RESULTS

- 30 $M_\odot$ star in a binary with 7.5 $M_\odot$ BH losing mass on thermal time scale evolved in Marchant et al. (2021) with MESA
- **a posteriori** $\dot{M}_\mathrm{d}$ comparison in different stages of star's evolution

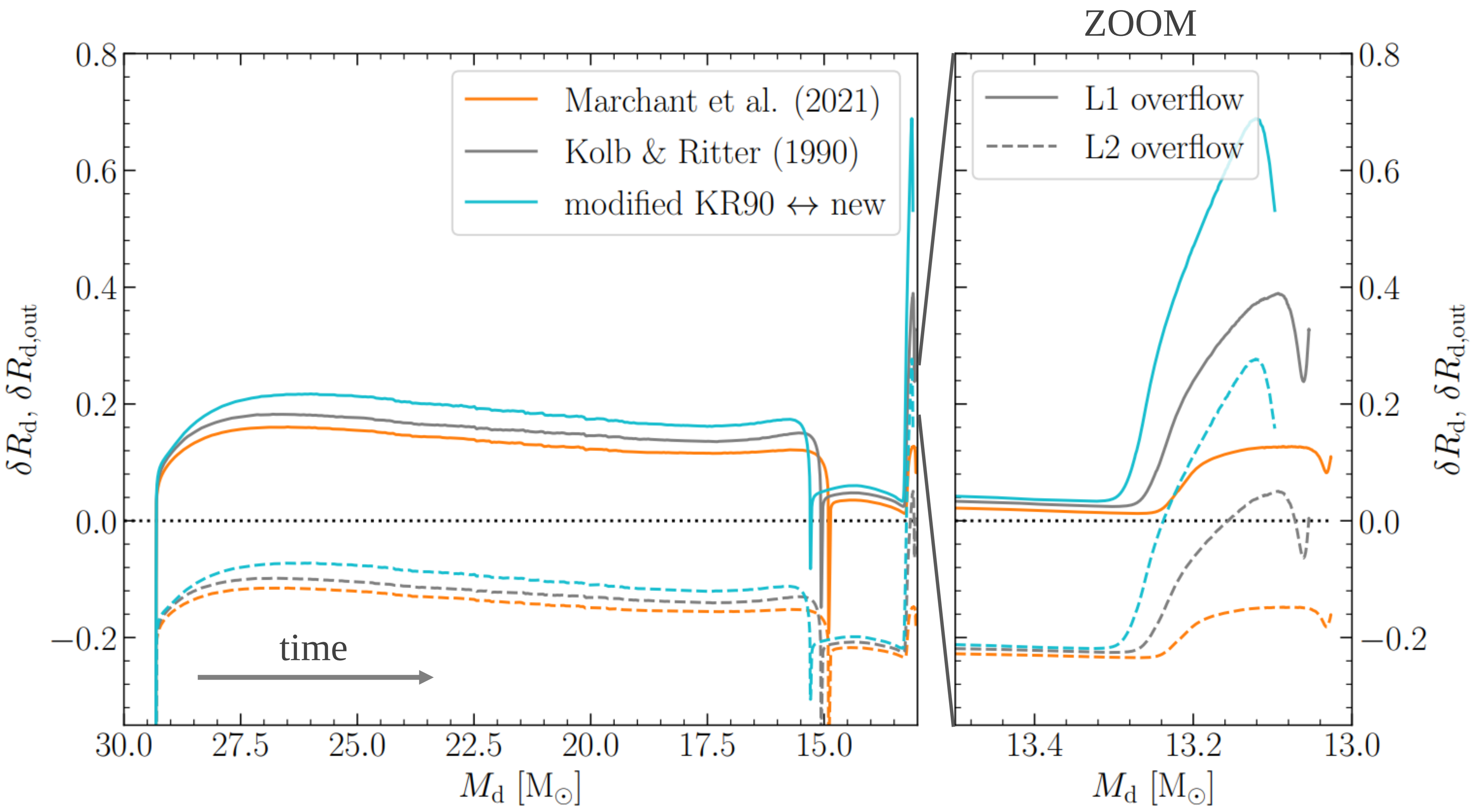


(Cehula & Pejcha 2023)

(MESA: Paxton et al. 2011, 2013, 2015, 2018, 2019)

RESULTS

- evolution **rerun** with ‘KR90’ mass-loss prescription decreased by a factor of 2 to simulate ‘new’ prescription ⇒ **less stable** mass transfer



(Cehula & Pejcha 2023)

CURRENT WORK

- implementation of radiative transfer

START

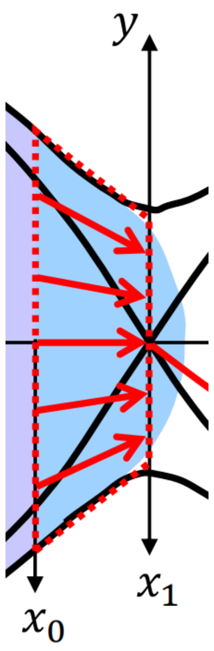
➢ 3D radiation hydrodynamics equations in the flux-limited diffusion approximation with the Roche potential

ASSUMPTIONS

1. Stationarity: $\partial/\partial t \rightarrow 0$
2. Gas flow – 1D ⇒ Q
3. LTE: $a_{\rm rad}T^4 - E_{\rm rad} = 0$
4. Optically thick limit: flux limiter $\lambda \rightarrow 1/3$
5. **von Zeipel theorem**

END

➢ 1D radiation hydrodynamics equations with the Roche potential and **radiative flux**



$$\dot{M}_{\rm new} = v\rho Q_\rho$$

$$\frac{1}{v}\frac{{\rm d}v}{{\rm d}x} + \frac{1}{\rho Q_\rho}\frac{\rm d}{{\rm d}x}\left(\rho Q_\rho\right) = 0,$$

$$v\frac{{\rm d}v}{{\rm d}x} + \frac{1}{\rho}\frac{{\rm d}P_{\rm gas}}{{\rm d}x} = -\left(1 - \frac{L_{\rm d}}{L_{\rm Edd}}\right)\frac{{\rm d}\phi_{\rm R}}{{\rm d}x},$$

$$\frac{1}{\rho}\frac{{\rm d}P_{\rm rad}}{{\rm d}x} = -\frac{\kappa_{\rm R}}{c}F_{\rm rad} = -\frac{L_{\rm d}}{L_{\rm Edd}}\frac{{\rm d}\phi_{\rm R}}{{\rm d}x},$$

$$P_{\rm rad} = \frac{1}{3}a_{\rm rad}T^4, \quad E_{\rm rad} = a_{\rm rad}T^4,$$

$$F_{\rm rad} = \frac{L_{\rm d}}{4\pi G M_{\rm d}}\frac{{\rm d}\phi_{\rm R}}{{\rm d}x}$$

COMPARISON TO OUR PREVIOUS WORK

$$\dot{M}_{\rm new} = v\rho Q_\rho$$

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$$P = P_{\rm gas} + \frac{1}{3}a_{\rm rad}T^4, \qquad Q_\rho = \frac{2\pi}{\sqrt{BC}}c_T^2$$

(Cehula & Pejcha 2023)

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$$F_{\rm rad} = \frac{L_\mathrm{d}}{4\pi G M_\mathrm{d}}\frac{\mathrm{d}\phi_\mathrm{R}}{\mathrm{d}x}, \qquad Q_\rho = \frac{2\pi}{\sqrt{BC}}c_T^2$$

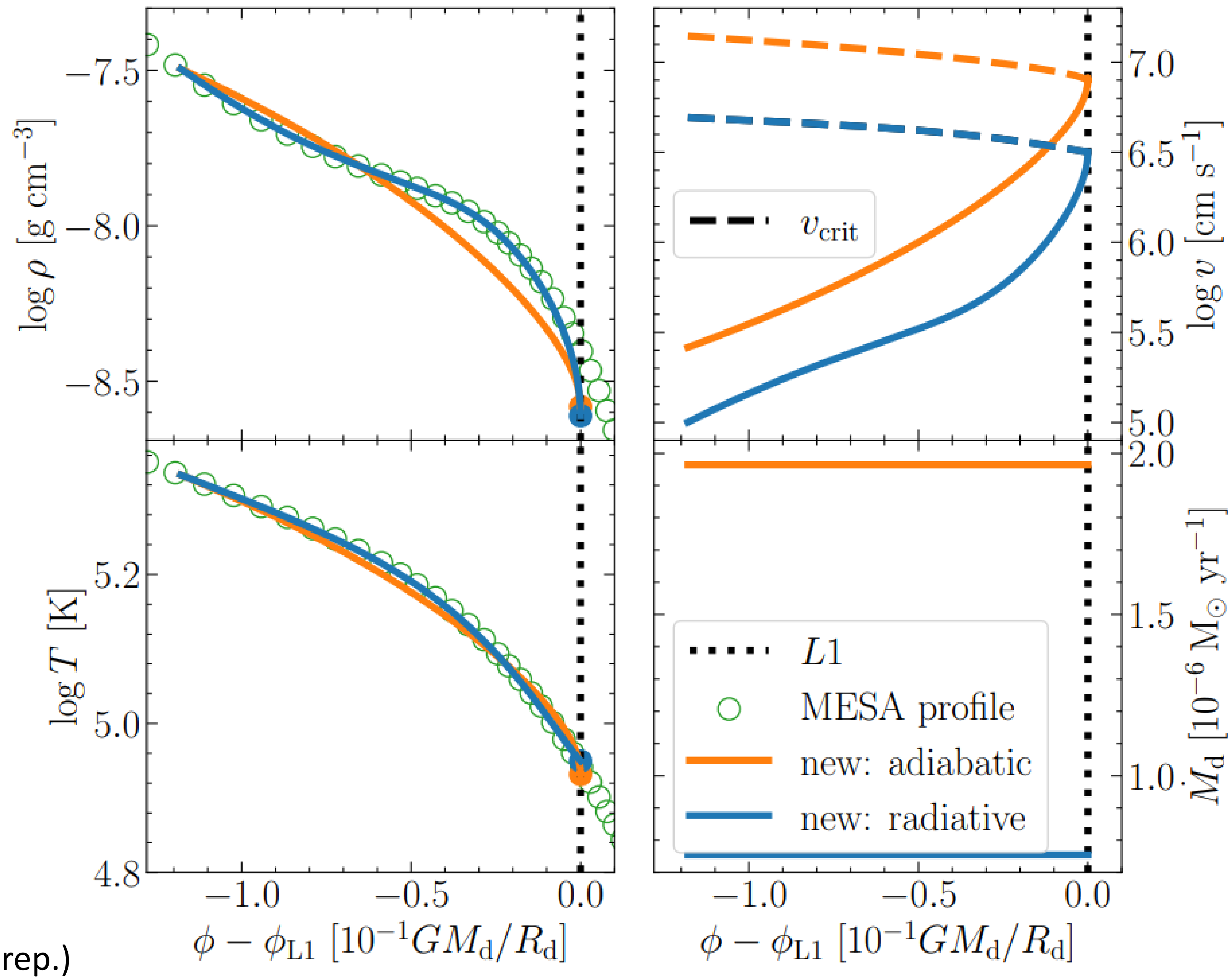
(Cehula & Pejcha in prep.)

- adiabatic *vs.* radiative
- critical point: $\frac{\mathrm{d}\phi_\mathrm{R}}{\mathrm{d}x} = 0$ *vs.* $\left(1 - \frac{L_\mathrm{d}}{L_{\rm Edd}}\right)\frac{\mathrm{d}\phi_\mathrm{R}}{\mathrm{d}x} = 0 \Rightarrow$ **super-Eddington boost** possible
- energy equation *vs.* Fick's law + von Zeipel theorem

RESULTS

RADIATIVE vs. **ADIABATIC**

- radiative model captures MESA profile better
- radiative model gives **lower** $\dot{M}_{\rm d}$



(Cehula & Pejcha in prep.)

RESULTS

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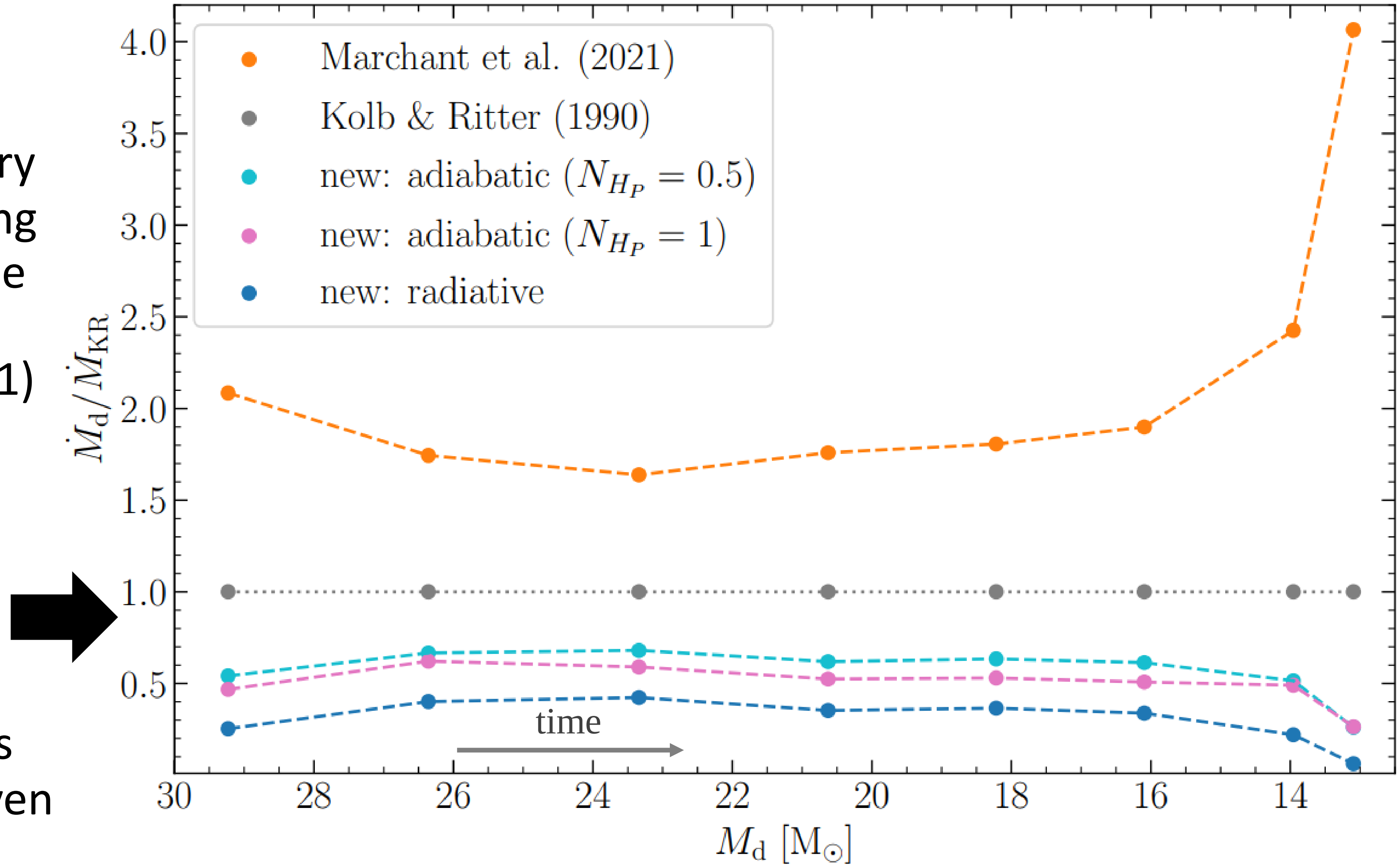
- **a posteriori** $\dot{M}_\mathrm{d}$ comparison in different stages of star's evolution
- radiative model gives even **lower** $\dot{M}_\mathrm{d}$ ⇒ even less stable mass transfer

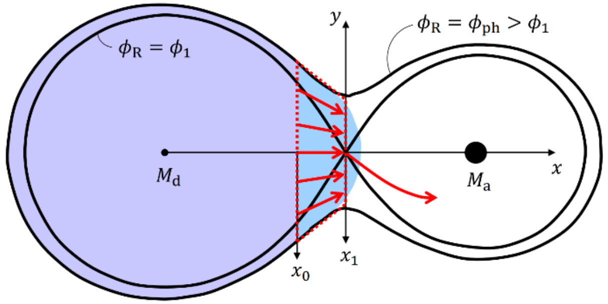


(Cehula & Pejcha in prep.)

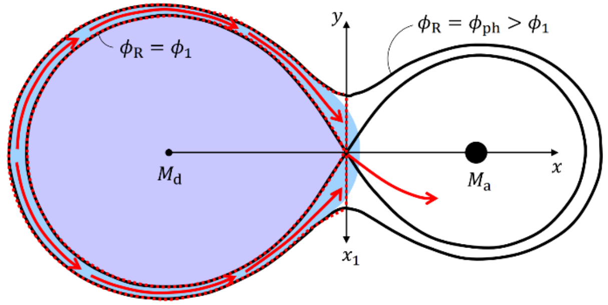
SUMMARY

- comparison with Marchant et al. (2021):
  - ➢factor of 4 lower $\dot{M}_{\mathrm{d}}$ ⇒ greater δR_{d} for given $\dot{M}_{\mathrm{d}}$ ⇒ less stable mass transfer ⇒ **favors CEE** over stable mass transfer
- comparison with Kolb & Ritter (1990):
  - ➢factor of 2 difference in $\dot{M}_{\mathrm{d}}$
- testing for **systematic differences** between models
- current work:
  - ➢including additional physics ≡ radiative transfer (**not** possible using the standard model)
  - ➢results: radiative model gives **lower** $\boldsymbol{\dot{M}_d}$ (than the adiabatic model) if radiation pressure is important but **super-Eddington boost** possible

***I am seeking a post-doc: jakub.cehula@mff.cuni.cz**



vs.

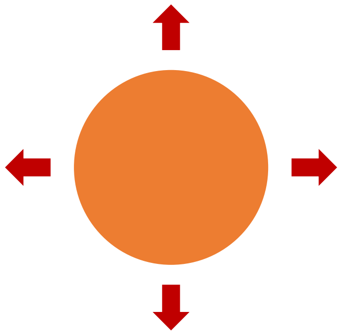


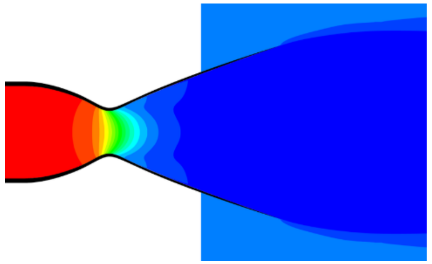
Cehula & Pejcha (2023, MNRAS, 524, 471–490)

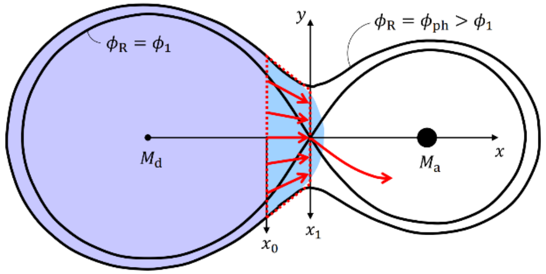
BACKUP SLIDES

STELLAR WINDS ≡ WAY TO NEW MT MODEL

- analogies between:
  - 1D isothermal stellar wind
  - flow through a rocket nozzle
  - **new model:** mass transfer through the nozzle created by the Roche potential around L1







- hydrodynamic equations governing 1D isothermal stellar wind:

$$-\dot{M}_* = 4\pi r^2 \rho(r)\, v(r) = \text{const},$$

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM_*}{r^2} = 0,$$

$$T(r) = T = \text{const}.$$

- assuming ideal gas EOS: $$\frac{1}{v}\frac{dv}{dr} = \frac{\frac{2c_T^2}{r} - \frac{GM_*}{r^2}}{v^2 - c_T^2}$$

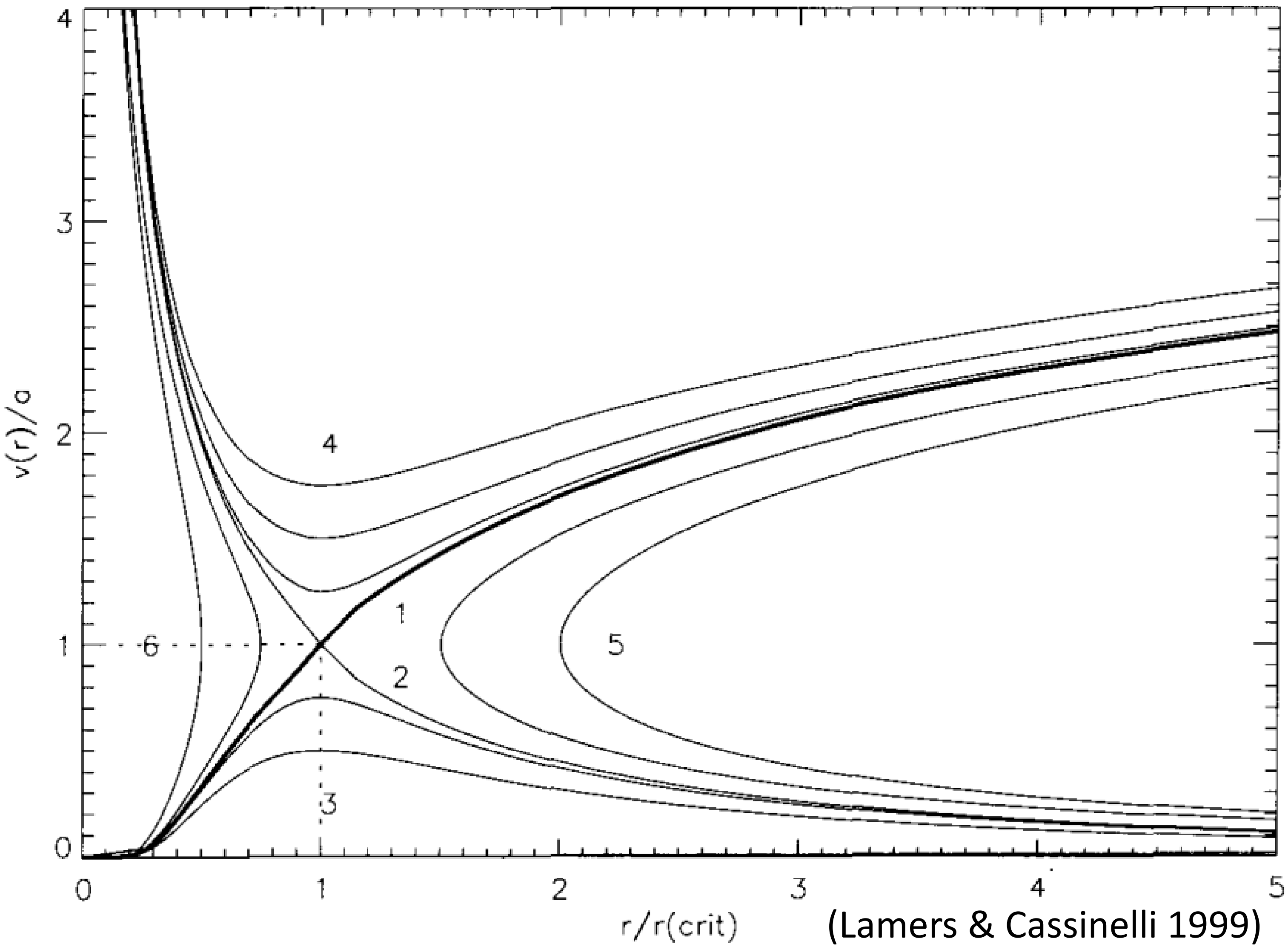
STELLAR WINDS ≡ WAY TO NEW MT MODEL

- solutions of:

$$\frac{1}{v}\frac{dv}{dr} = \frac{\frac{2c_T^2}{r} - \frac{GM_*}{r^2}}{v^2 - c_T^2}$$

- the critical point ($v = c_T$):

$$r_c = \frac{GM_*}{2c_T^2}$$



(Lamers & Cassinelli 1999)

ANALOGY TO ROCKET NOZZLES

- hydrodynamic equations governing isothermal gas flow through axially symmetric nozzle:

$$\dot{M}_N = \rho(l)v(l)A(l) = \rho_b v_b A_b = \text{const},$$

$$v\frac{dv}{dl} + \frac{1}{\rho}\frac{dP}{dl} = 0.$$

$$T(l) = T = \text{const}.$$

- assuming ideal gas EOS: $\frac{1}{v}\frac{dv}{dl} = \frac{\frac{c_T^2}{A}\frac{dA}{dl}}{v^2 - c_T^2},$

- the critical point ($v = c_T$): $dA/dl = 0$

ANALOGY TO ROCKET NOZZLES

- considering:

$$\frac{1}{v}\frac{dv}{dl} = \frac{\frac{c_T^2}{A}\frac{dA}{dl}}{v^2 - c_T^2},$$

- where ($A = \pi r_N^2$):

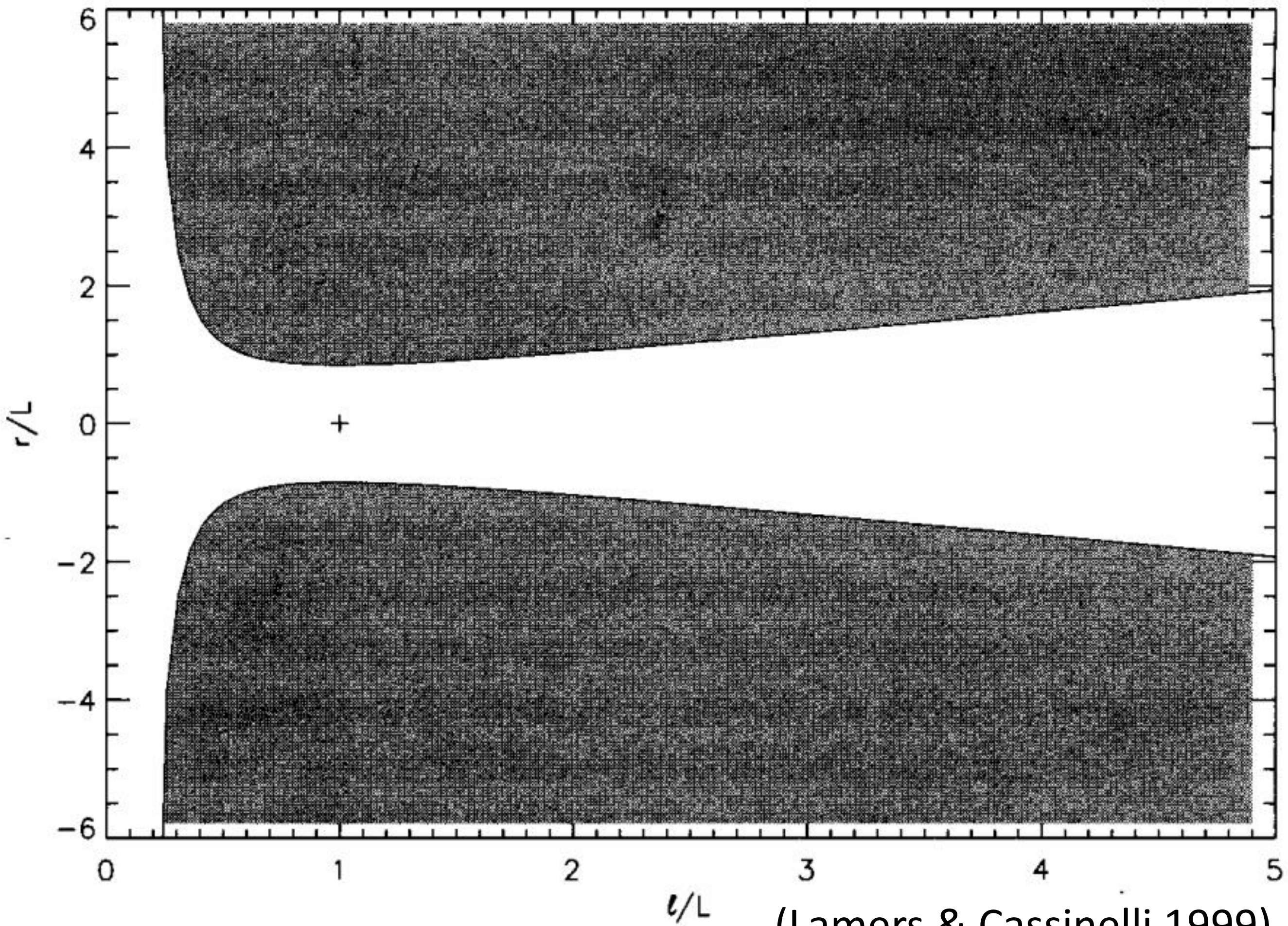
$$r_N(l) = \frac{l}{\pi}\exp\left(\frac{L}{l}\right), \quad \text{with} \quad L = \frac{GM_*}{2c_T^2},$$

- yields:

$$\frac{c_T^2}{A}\frac{dA}{dl} \equiv \frac{2c_T^2}{r} - \frac{GM_*}{r^2}, \quad \text{for} \quad l = r,$$

- i.e. the same momentum equation and velocity distribution as **isothermal wind**:

$$\frac{1}{v}\frac{dv}{dr} = \frac{\frac{2c_T^2}{r} - \frac{GM_*}{r^2}}{v^2 - c_T^2}$$



(Lamers & Cassinelli 1999)

NEW MODEL IN EQUATIONS

START

- 3D Euler equations with the Roche potential:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + P\mathbf{I}) = -\rho \boldsymbol{\nabla} \phi_{\mathrm{R}},$$

$$\frac{\partial (\rho \epsilon_{\mathrm{tot}})}{\partial t} + \boldsymbol{\nabla} \cdot [(\rho \epsilon_{\mathrm{tot}} + P)\, \boldsymbol{v}] = 0,$$

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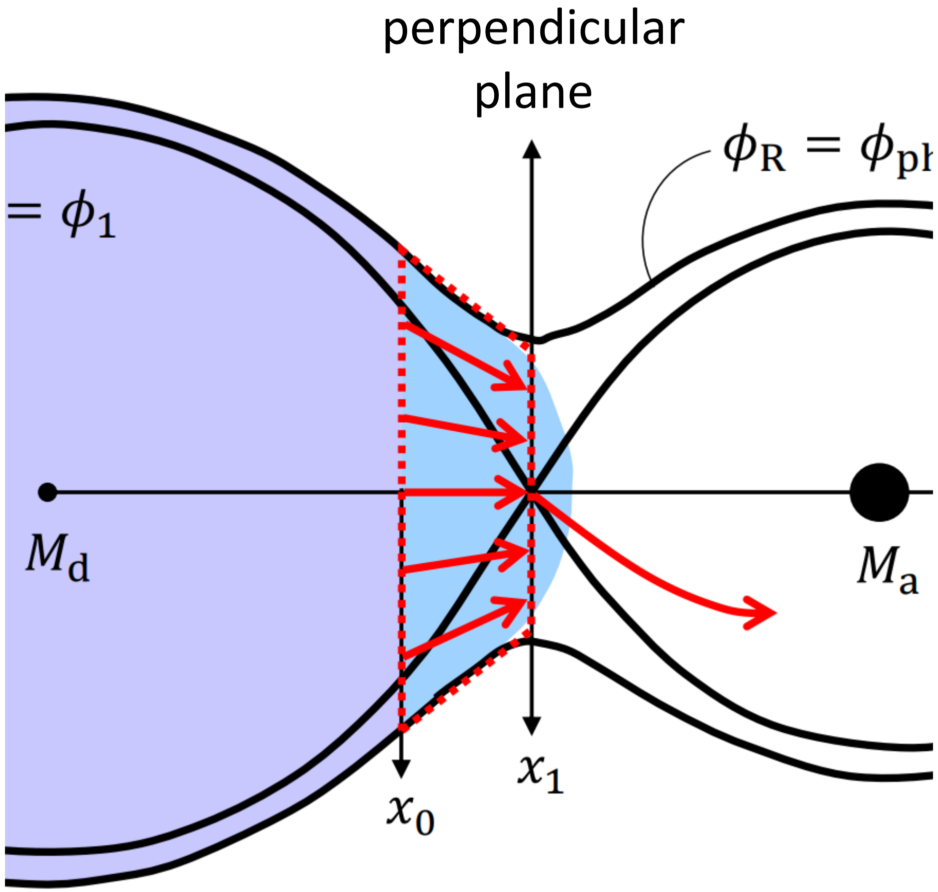

ASSUMPTIONS:

1. Stationarity
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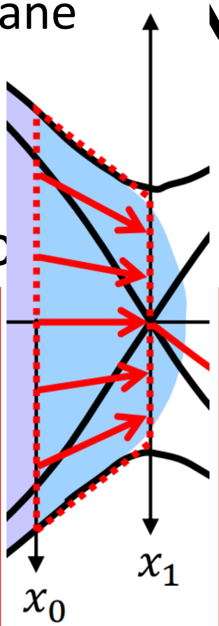
where we are averaging in the perpendicular plane: $\rho Q_\rho = \int_Q \rho' \mathrm{dQ}, \quad P Q_P = \int_Q P' \mathrm{dQ},$

$\frac{Q_P}{Q_\rho} = \frac{\Gamma}{2\Gamma - 1}$, and: $\dot{M}_{\mathrm{new}} = v \rho Q_\rho$

NEW MODEL IN EC NS



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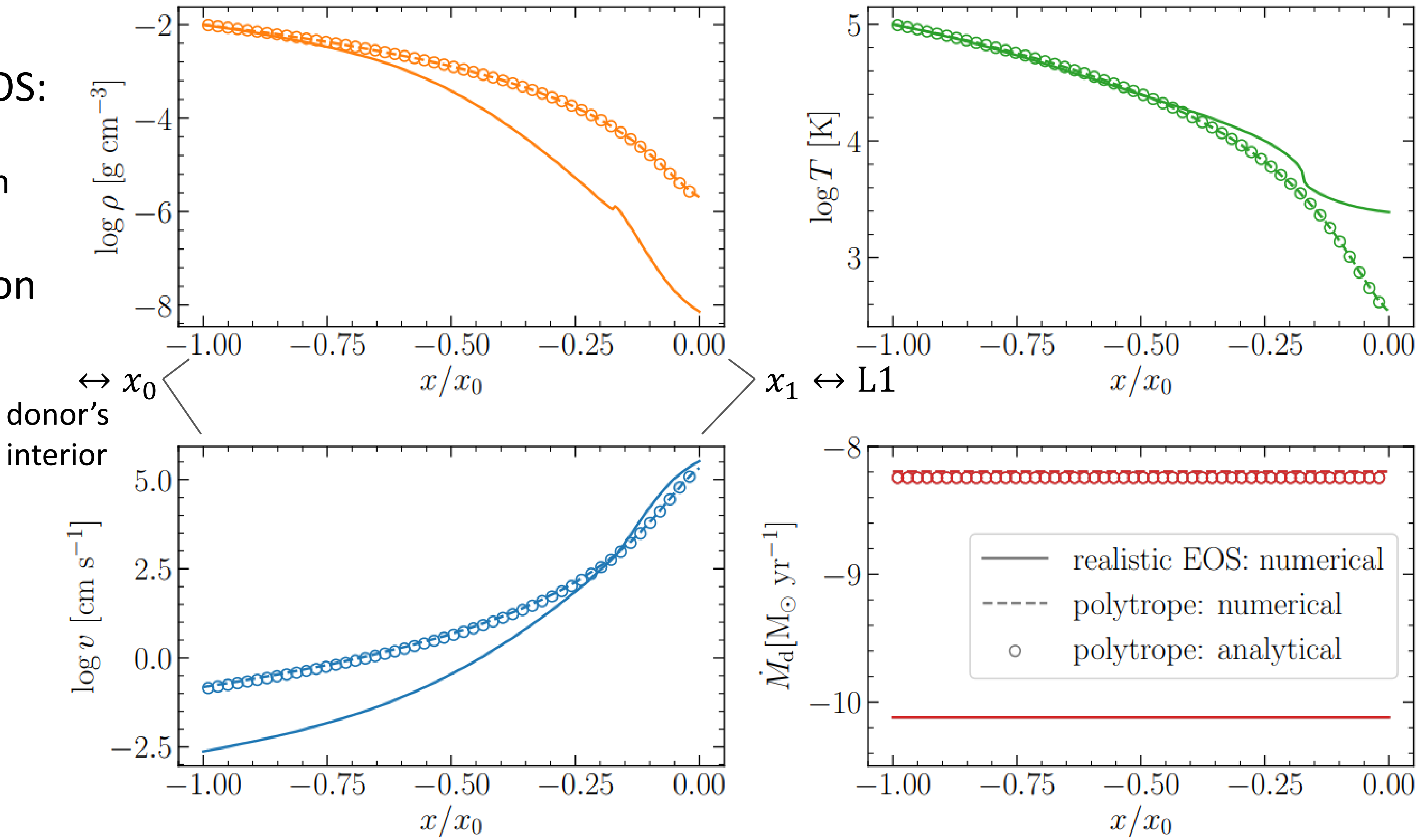
$\frac{Q_P}{Q_\rho} = \frac{\Gamma}{2\Gamma - 1}$, and: $\dot{M}_{\mathrm{new}} = v \rho Q_\rho$

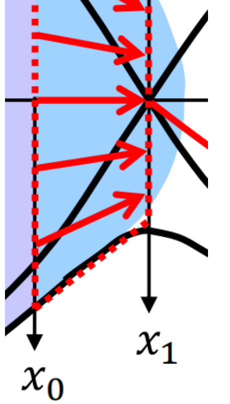
RESULTS

- polytropic vs. more realistic EOS:
  - factor of 10^2 difference in an extreme case!
- analytical solution agrees with the numerical for polytrope
- $\dot{M}_\mathrm{d}(x) = \mathrm{const.}$



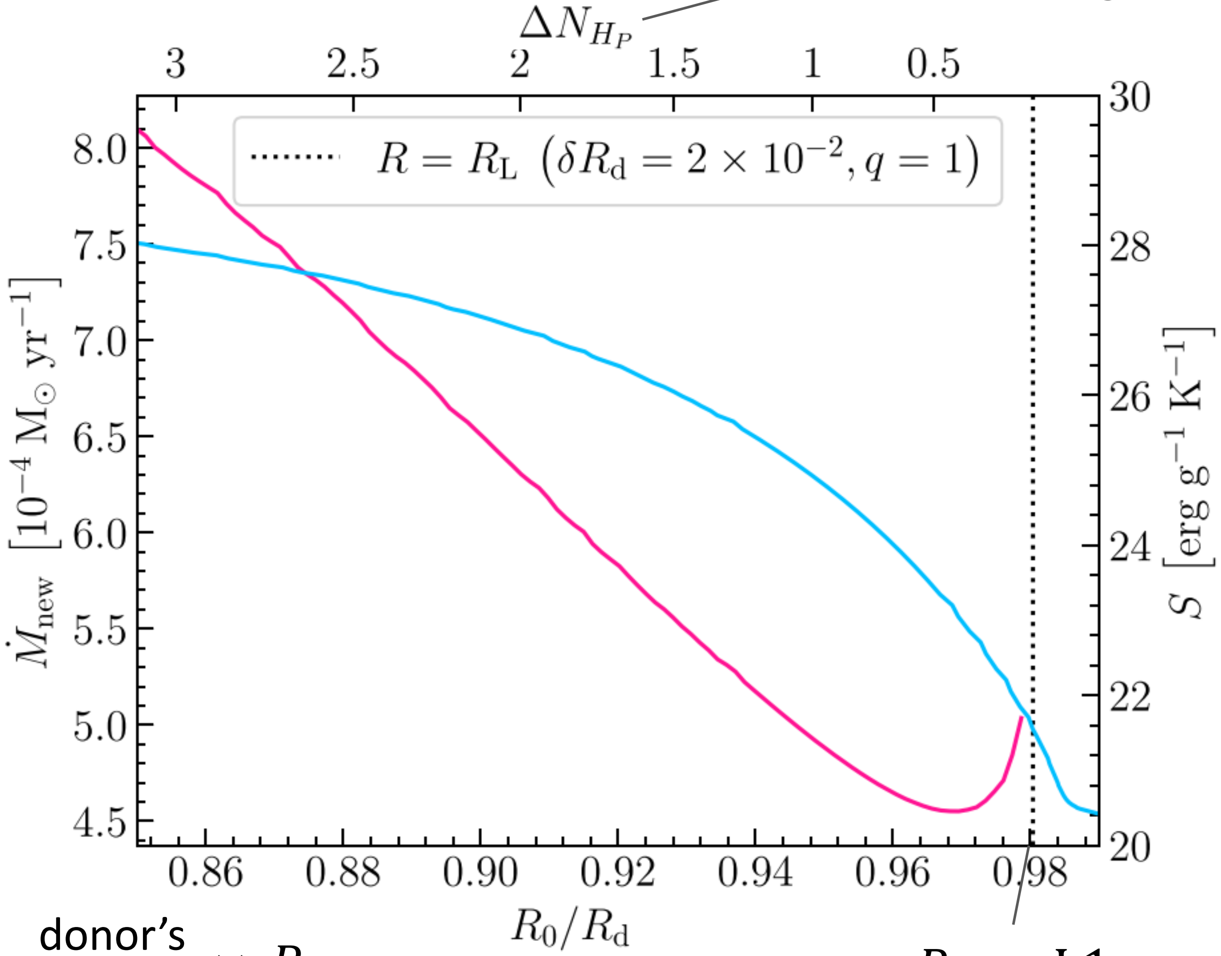
RESULTS

- $\dot{M}_{\rm new}(\Delta N_{H_P})$ <–> $\dot{M}_{\rm new}(x_0)$, $\delta R_{\rm d} = \text{const.}!$



$1M_\odot$ donor on RGB

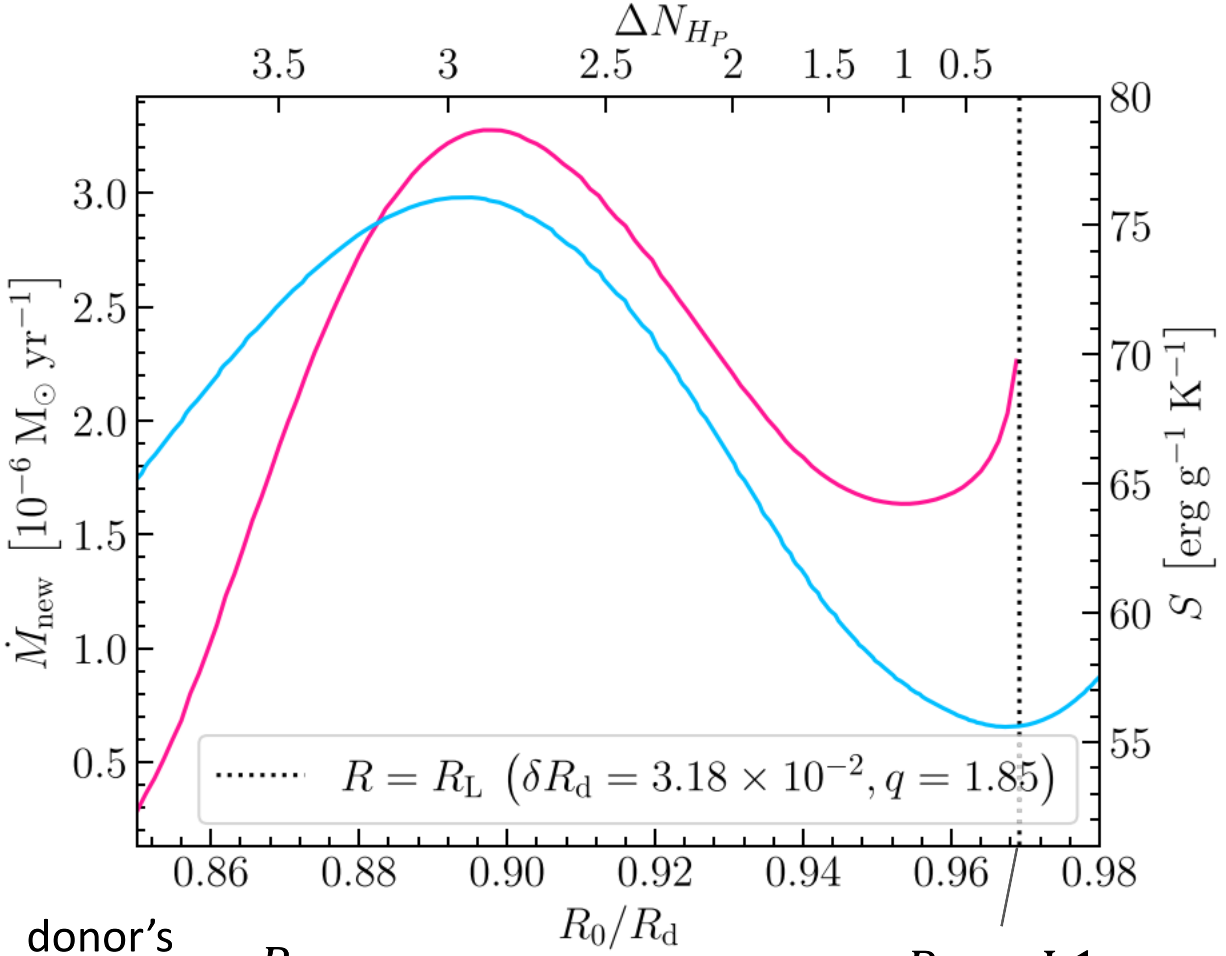
# of pressure scale heights



donor's interior ↔ R_0

$R_{\rm L}$ ↔ L1

$30M_\odot$ low-metallicity donor undergoing thermal MT



donor's interior ↔ R_0

$R_{\rm L}$ ↔ L1

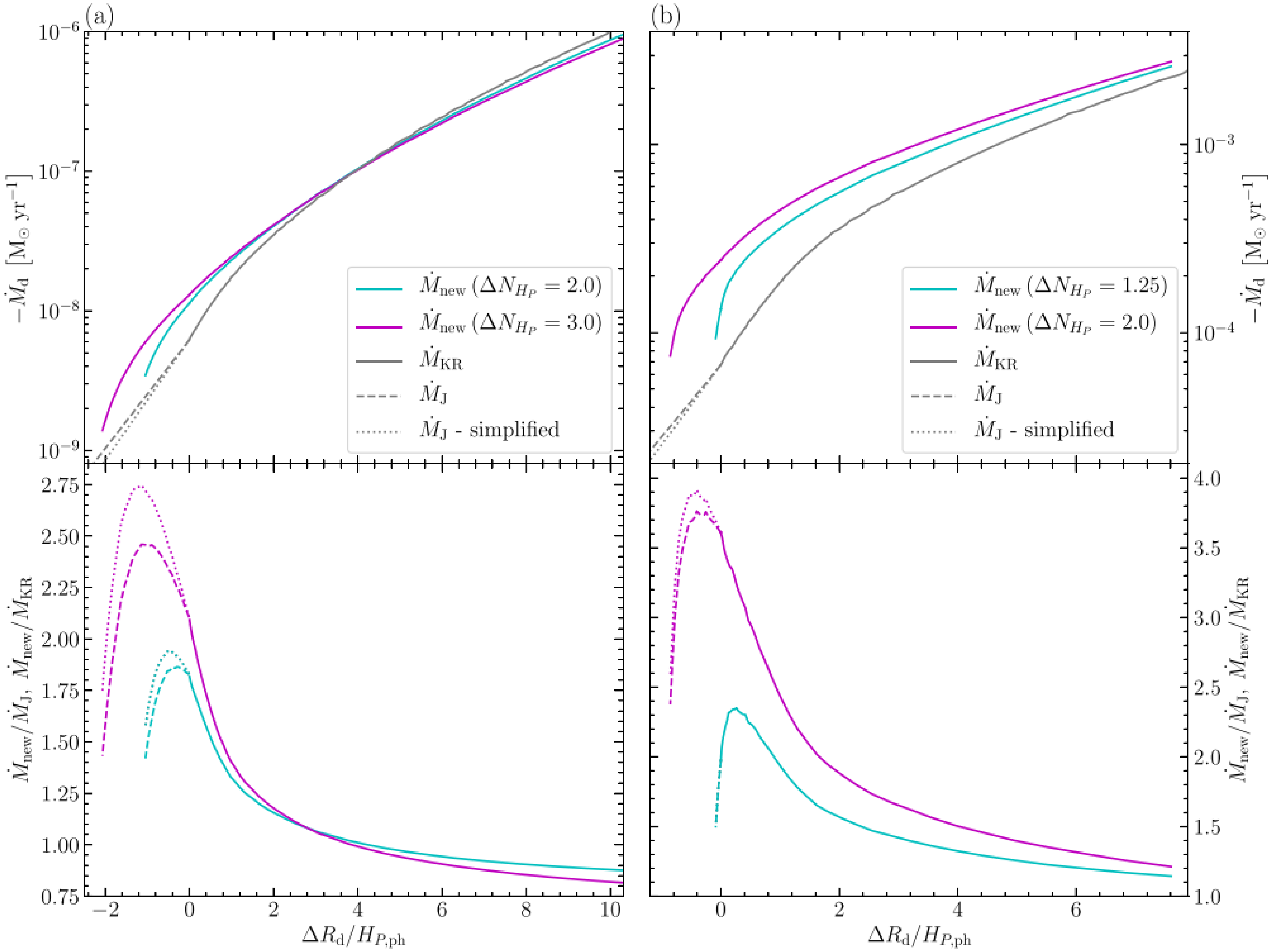
RESULTS

MT rate comparison

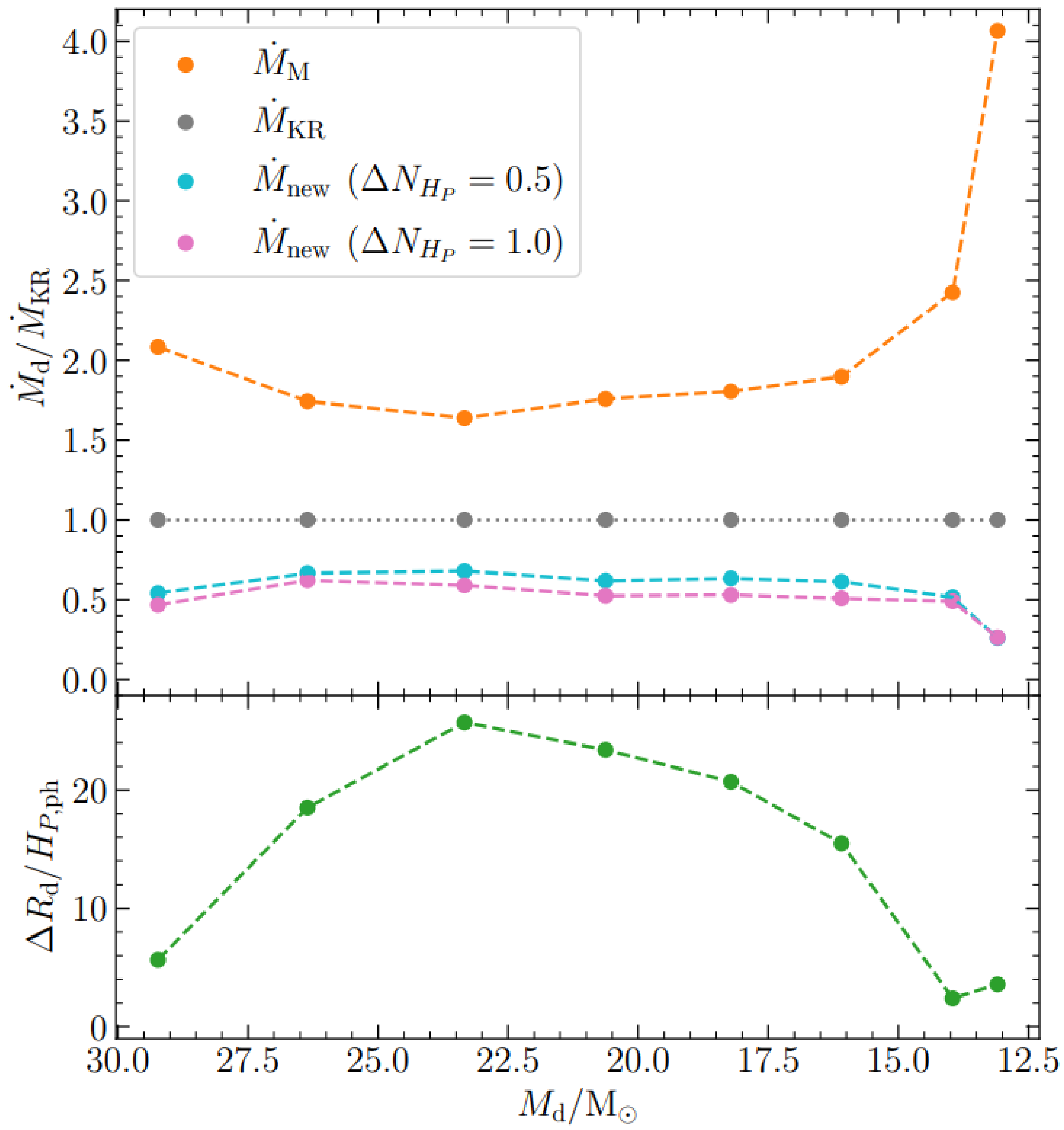
- $\dot{M}_{\mathrm{new}}(\Delta R_{\mathrm{d}})$, $\Delta N_{H_P} = \text{const.!}$
- vs. optically thin (Jackson et al. 2017)
- vs. optically thick (Kolb & Ritter 1990)

(a) $1\mathrm{M}_{\odot}$ donor on the main sequence

(b) $1\mathrm{M}_{\odot}$ donor on RGB

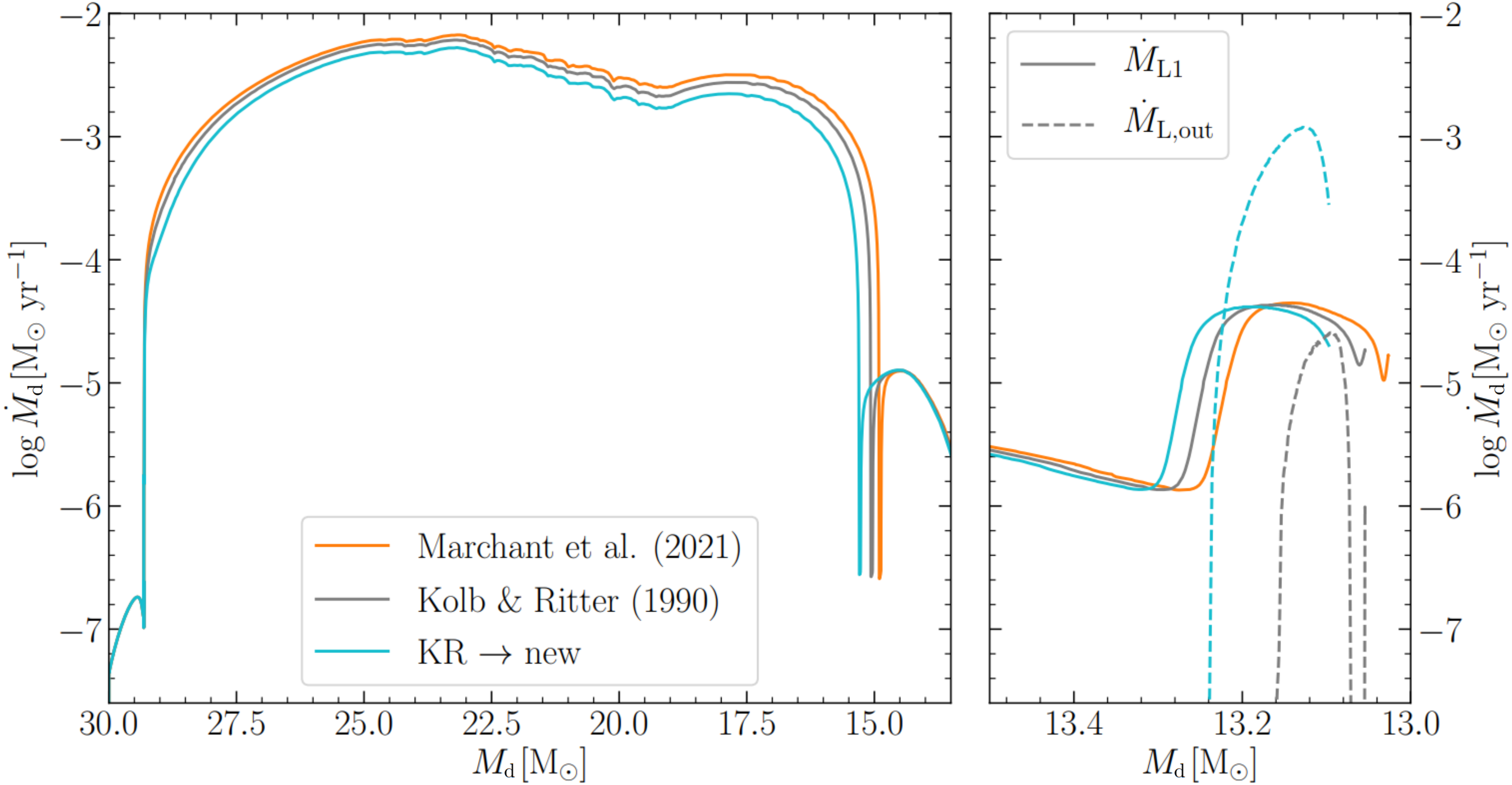


RESULTS



$\dot{M}_{\mathrm{M}}$
$\dot{M}_{\mathrm{KR}}$
$\dot{M}_{\mathrm{new}}$ $(\Delta N_{H_P} = 0.5)$
$\dot{M}_{\mathrm{new}}$ $(\Delta N_{H_P} = 1.0)$
$\dot{M}_{\mathrm{d}}/\dot{M}_{\mathrm{KR}}$
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\Delta R_{\mathrm{d}}/H_{P,\mathrm{ph}}$
20
10
0
30.0
27.5
25.0
22.5
20.0
17.5
15.0
12.5
$M_{\mathrm{d}}/\mathrm{M}_{\odot}$

RESULTS



-2
-3
-4
-5
-6
-7
$\log \dot{M}_{\rm d}\,[{\rm M}_\odot\,{\rm yr}^{-1}]$
Marchant et al. (2021)
Kolb & Ritter (1990)
KR → new
30.0
27.5
25.0
22.5
20.0
17.5
15.0
$M_{\rm d}\,[{\rm M}_\odot]$
$\dot{M}_{\rm L1}$
$\dot{M}_{\rm L,out}$
13.4
13.2
13.0

CURRENT WORK IN EQUATIONS

START

- radiation hydrodynamics equations in the flux-limited diffusion approximation in the mixed-frame formulation (e.g. Calderón et al. 2021):

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v}) + \boldsymbol{\nabla} P_{\mathrm{gas}} + \lambda \boldsymbol{\nabla} E_{\mathrm{rad}} = -\rho \boldsymbol{\nabla} \phi_{\mathrm{R}},$$

$$\frac{\partial (\rho \epsilon^{*})}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \epsilon^{*} \boldsymbol{v} + P_{\mathrm{gas}} \boldsymbol{v}) + \lambda \boldsymbol{v} \cdot \boldsymbol{\nabla} E_{\mathrm{rad}} = -c \rho \kappa_{\mathrm{P}} \left(a T^{4} - E_{\mathrm{rad}}^{(0)} \right),$$

$$\frac{\partial E_{\mathrm{rad}}}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{3 - f}{2} E_{\mathrm{rad}} \boldsymbol{v} \right) - \lambda \boldsymbol{v} \cdot \boldsymbol{\nabla} E_{\mathrm{rad}} = c \rho \kappa_{\mathrm{P}} \left(a T^{4} - E_{\mathrm{rad}}^{(0)} \right) + \boldsymbol{\nabla} \cdot \left(\frac{c \lambda}{\rho \kappa_{\mathrm{R}}} \boldsymbol{\nabla} E_{\mathrm{rad}} \right),$$

$$\boldsymbol{F}_{\mathrm{rad}}^{(0)} = -\frac{c \lambda}{\rho \kappa_{\mathrm{R}}} \boldsymbol{\nabla} E_{\mathrm{rad}}^{(0)},$$

$$P_{\mathrm{rad}}^{(0)} = f^{(0)} E_{\mathrm{rad}}^{(0)},$$

CURRENT WORK IN EQUATIONS

START

- radiation hydrodynamics equations in the flux-limited diffusion approximation in the mixed-frame formulation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

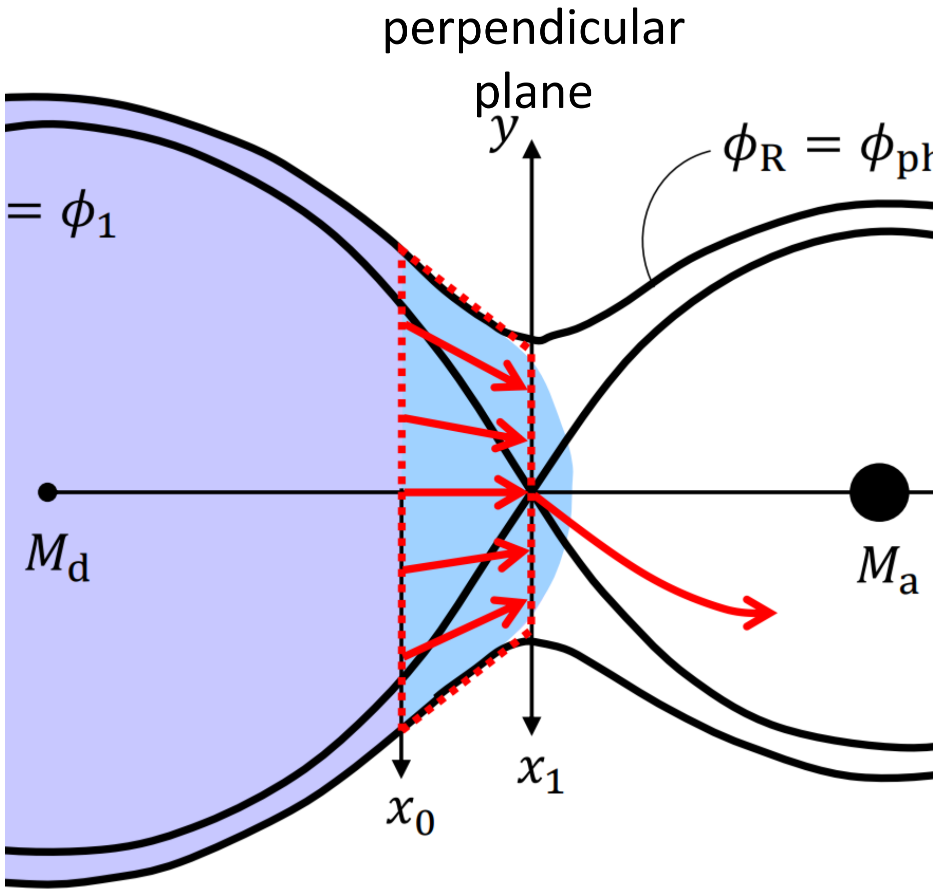
$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v}) + \nabla P_{\rm gas} + \lambda \nabla E_{\rm rad} = -\rho \nabla \phi_{\rm R},$$

$$\frac{\partial (\rho \epsilon^*)}{\partial t} + \nabla \cdot (\rho \epsilon^* \boldsymbol{v} + P_{\rm gas} \boldsymbol{v}) + \lambda \boldsymbol{v} \cdot \nabla E_{\rm rad} = -c\rho\kappa_{\rm P} \left(aT^4 - E^{(0)}_{\rm rad} \right),$$

$$\frac{\partial E_{\rm rad}}{\partial t} + \nabla \cdot \left(\frac{3-f}{2} E_{\rm rad} \boldsymbol{v} \right) - \lambda \boldsymbol{v} \cdot \nabla E_{\rm rad} = c\rho\kappa_{\rm P} \left(aT^4 - E^{(0)}_{\rm rad} \right) + \nabla \cdot \left(\frac{c\lambda}{\rho\kappa_{\rm R}} \nabla E_{\rm rad} \right),$$

$$\boldsymbol{F}^{(0)}_{\rm rad} = -\frac{c\lambda}{\rho\kappa_{\rm R}} \nabla E^{(0)}_{\rm rad},$$

$$P^{(0)}_{\rm rad} = f^{(0)} E^{(0)}_{\rm rad},$$



ASSUMPTIONS:

1. Stationarity
2. Gas flow – 1D
3. LTE: $aT^4 - E_{\rm rad} = 0$
4. Optically thick limit: $\lambda \to 1/3$

CURRENT WORK IN EQUATIONS

START

- radiation hydrodynamics equations in the flux-limited diffusion approximation in the mixed-frame formulation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

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$$F^{(0)}_{\rm rad} = -\frac{c\lambda}{\rho \kappa_{\rm R}} \nabla E^{(0)}_{\rm rad},$$

$$P^{(0)}_{\rm rad} = f^{(0)} E^{(0)}_{\rm rad},$$

ASSUMPTIONS:

1. Stationarity
2. Gas flow – 1D
3. LTE: $aT^4 - E_{\rm rad} = 0$
4. Optically thick limit: $\lambda \to 1/3$

END

- 1D radiation hydrodynamics equations with the Roche potential and radiative flux:

$$\frac{1}{v}\frac{{\rm d}v}{{\rm d}x} + \frac{1}{\rho}\frac{{\rm d}\rho}{{\rm d}x} = 0,$$

$$v\frac{{\rm d}v}{{\rm d}x} + \frac{1}{\rho}\frac{{\rm d}P_{\rm gas}}{{\rm d}x} = \frac{\kappa}{c} F_{\rm rad} - \frac{{\rm d}\phi_{\rm R}}{{\rm d}x},$$

$$\frac{\rm d}{{\rm d}x}\left[(\epsilon_{\rm tot}\rho + P) v + F_{\rm rad} \right] = 0,$$

$$F_{\rm rad} = -\frac{c}{\kappa}\frac{1}{\rho}\frac{{\rm d}P_{\rm rad}}{{\rm d}x},$$

where: $P_{\rm rad} = \frac{1}{3} a T^4, \quad E_{\rm rad} = a T^4.$

CURRENT WORK IN EQUATIONS

START

- radiation hydrodynamics equations in th flux-limited diffusion approximation in th mixed-frame formulation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

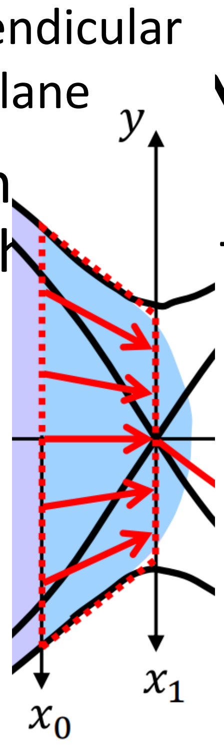
$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v}) + \nabla P_{\mathrm{gas}} + \lambda \nabla E_{\mathrm{rad}} = -\rho \nabla \phi_{\mathrm{R}},$$

$$\frac{\partial (\rho \epsilon^*)}{\partial t} + \nabla \cdot (\rho \epsilon^* \boldsymbol{v} + P_{\mathrm{gas}} \boldsymbol{v}) + \lambda \boldsymbol{v} \cdot \nabla E_{\mathrm{rad}} = -c \rho \kappa_{\mathrm{P}} \left(a T^4 - E_{\mathrm{rad}}^{(0)} \right),$$

$$\frac{\partial E_{\mathrm{rad}}}{\partial t} + \nabla \cdot \left(\frac{3 - f}{2} E_{\mathrm{rad}} \boldsymbol{v} \right) - \lambda \boldsymbol{v} \cdot \nabla E_{\mathrm{rad}} = c \rho \kappa_{\mathrm{P}} \left(a T^4 - E_{\mathrm{rad}}^{(0)} \right) + \nabla \cdot \left(\frac{c \lambda}{\rho \kappa_{\mathrm{R}}} \nabla E_{\mathrm{rad}} \right),$$

$$F_{\mathrm{rad}}^{(0)} = -\frac{c \lambda}{\rho \kappa_{\mathrm{R}}} \nabla E_{\mathrm{rad}}^{(0)},$$

$$P_{\mathrm{rad}}^{(0)} = f^{(0)} E_{\mathrm{rad}}^{(0)},$$



ASSUMPTIONS:

1. Stationarity
2. Gas flow – 1D
3. LTE: $aT^4 - E_{\mathrm{rad}} = 0$
4. Optically thick limit: $\lambda \to 1/3$

ND

- 1D radiation hydrodynamics equations with the Roche potential and radiative flux:

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x} = 0,$$

$$v\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}P_{\mathrm{gas}}}{\mathrm{d}x} = \frac{\kappa}{c}F_{\mathrm{rad}} - \frac{\mathrm{d}\phi_{\mathrm{R}}}{\mathrm{d}x},$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[(\epsilon_{\mathrm{tot}}\rho + P)\, v + F_{\mathrm{rad}}\right] = 0,$$

$$F_{\mathrm{rad}} = -\frac{c}{\kappa}\frac{1}{\rho}\frac{\mathrm{d}P_{\mathrm{rad}}}{\mathrm{d}x},$$

where: $$P_{\mathrm{rad}} = \frac{1}{3}aT^4, \quad E_{\mathrm{rad}} = aT^4.$$

CURRENT WORK

- implementation of radiative transfer

START

➢ 3D radiation hydrodynamics equations in the flux-limited diffusion approximation with the Roche potential

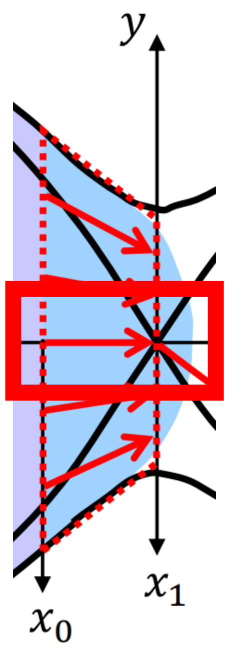
ASSUMPTIONS

1. Stationarity: $\partial/\partial t \to 0$
2. Gas flow – 1D: $\partial/\partial y \to 0$, $\partial/\partial z \to 0$
3. LTE: $aT^4 - E_{\mathrm{rad}} = 0$
4. Optically thick limit: $\lambda \to 1/3$

END

➢ 1D radiation hydrodynamics equations with the Roche potential and **radiative flux**

perpendicular plane



$$\dot{m} = v\rho$$

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x} = 0,$$

$$v\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}P_{\mathrm{gas}}}{\mathrm{d}x} = \frac{\kappa}{c}F_{\mathrm{rad}} - \frac{\mathrm{d}\phi_{\mathrm{R}}}{\mathrm{d}x},$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[(\epsilon_{\mathrm{tot}}\rho + P)\,v + F_{\mathrm{rad}}\right] = 0,$$

$$F_{\mathrm{rad}} = -\frac{c}{\kappa}\frac{1}{\rho}\frac{\mathrm{d}P_{\mathrm{rad}}}{\mathrm{d}x},$$

$$P_{\mathrm{rad}} = \frac{1}{3}aT^4, \quad E_{\mathrm{rad}} = aT^4.$$

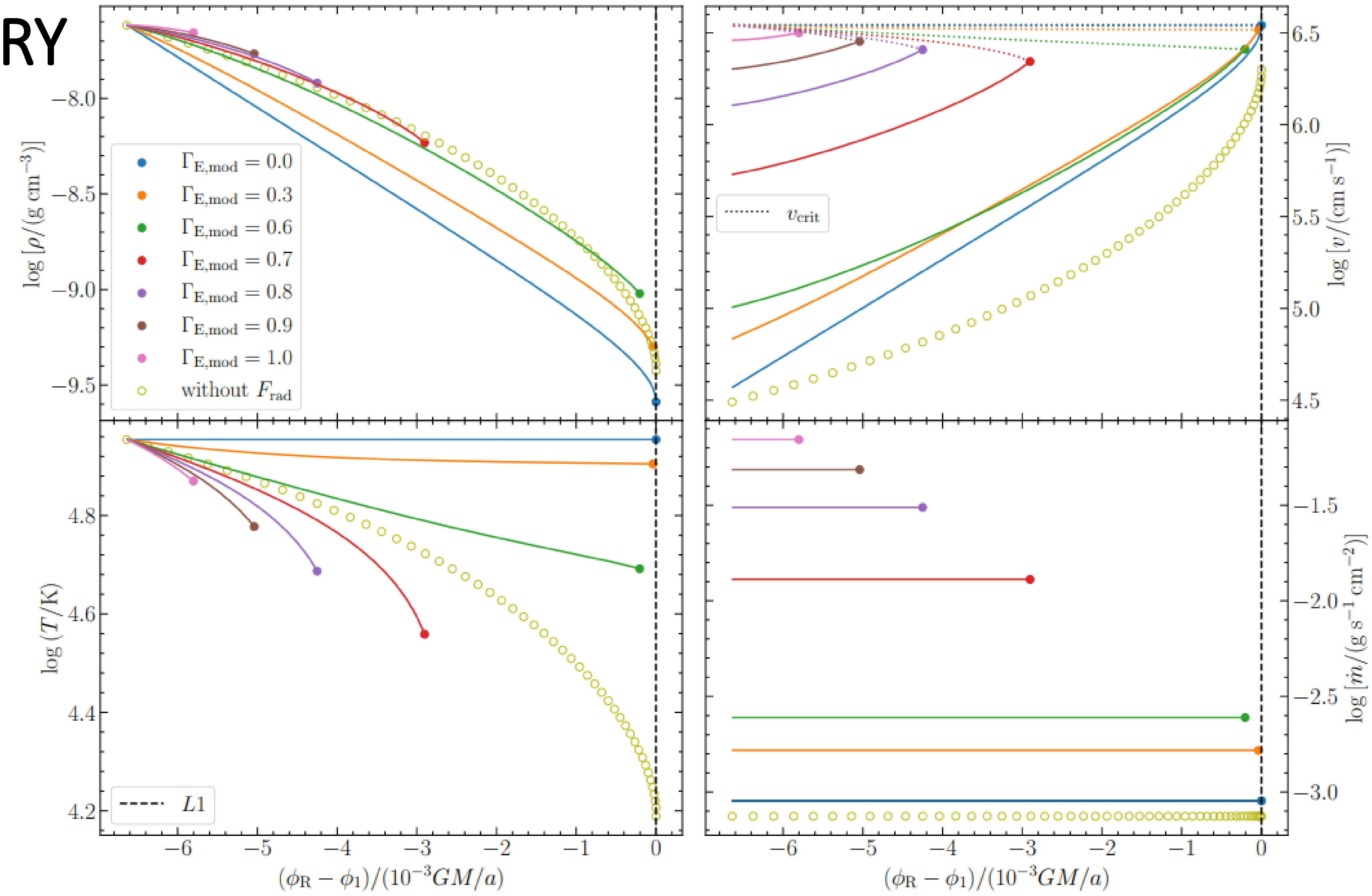
PRELIMINARY RESULTS

SETUP

- $M_{\mathrm{d}} = 30\mathrm{M}_{\odot}$
- $q = 1$
- $\delta R_{\mathrm{d}} = 0$
- $\kappa = 1.2\ \mathrm{cm}^2\ \mathrm{g}^{-1}$
- $P_{\mathrm{gas}} = \frac{k}{\mu m_u} \rho T$
- $\phi_{\mathrm{R}} = \zeta(x) \frac{G M_{\mathrm{d}}}{R_{\mathrm{L}} + x - x_1}$
- $\Gamma_{\mathrm{E,mod}}$ – modified Eddington factor

RESULTS

- shift of the critical point



(Cehula & Pejcha 2024, in prep.)

PRELIMINARY RESULTS

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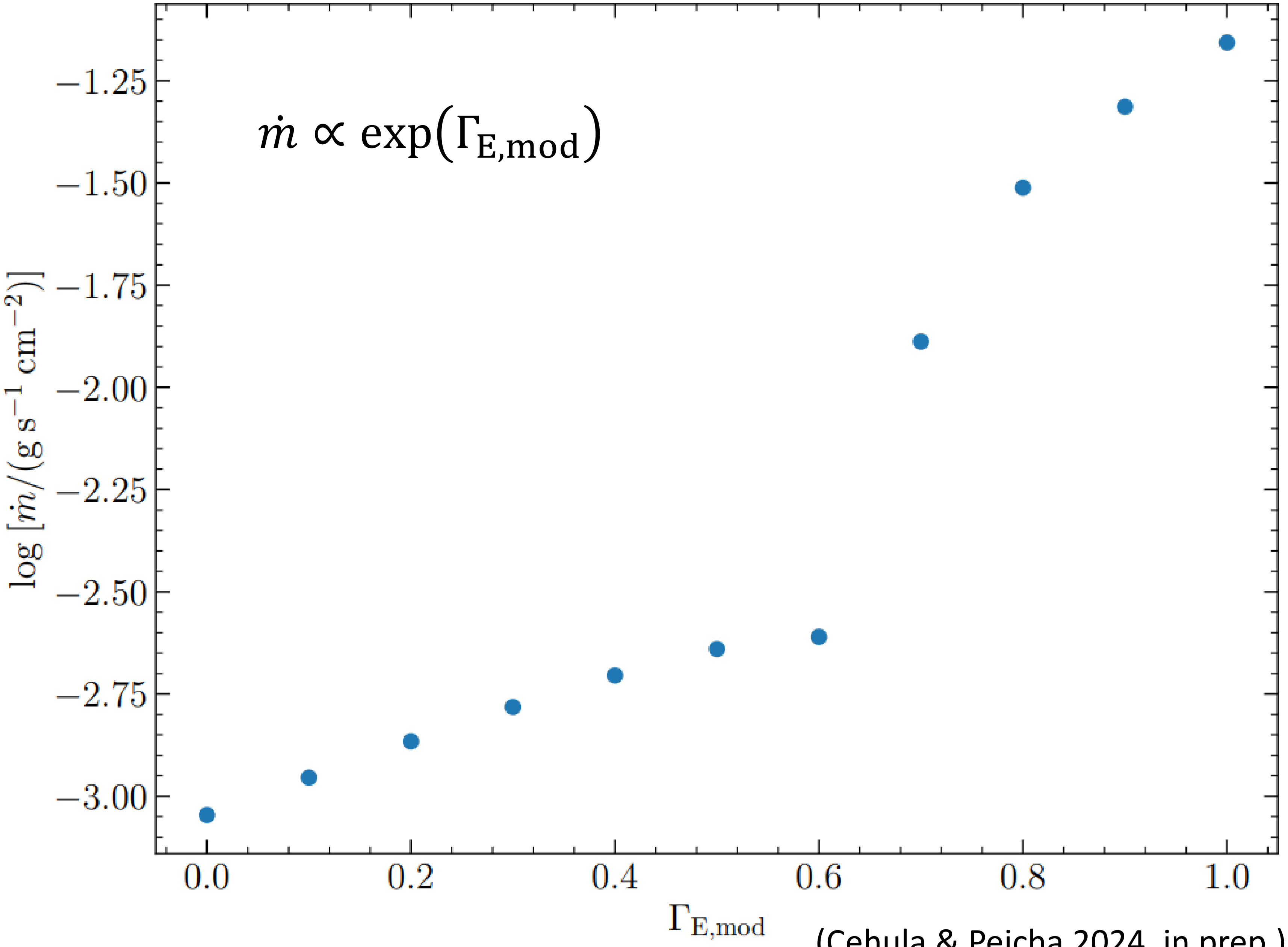
- $\Gamma_\mathrm{E,mod}$ – modified Eddington factor

RESULTS

- $\dot{m} \propto \exp(\Gamma_\mathrm{E,mod})$



(Cehula & Pejcha 2024, in prep.)