

THE INTERNATIONAL ASTROPHYSICS SERIES

VOLUME FIVE

Close Binary Systems

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NEW YORK
JOHN WILEY & SONS INC.

440 FOURTH AVENUE

1959

1

Heartbeat Binary Stars (HBs): Tidally Excited Oscillations (TEOs)

Brightness



Temperature: 7327 K
Period: 14.0 days

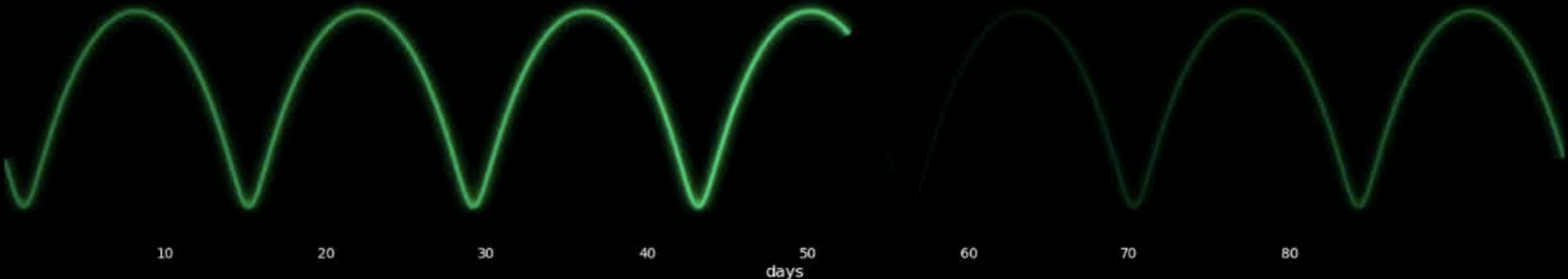


BPM

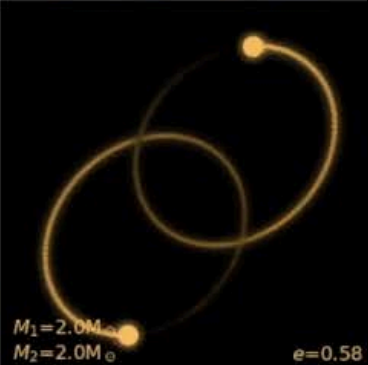
2.17

(beats/month)

Separation



Frequency Spectrum



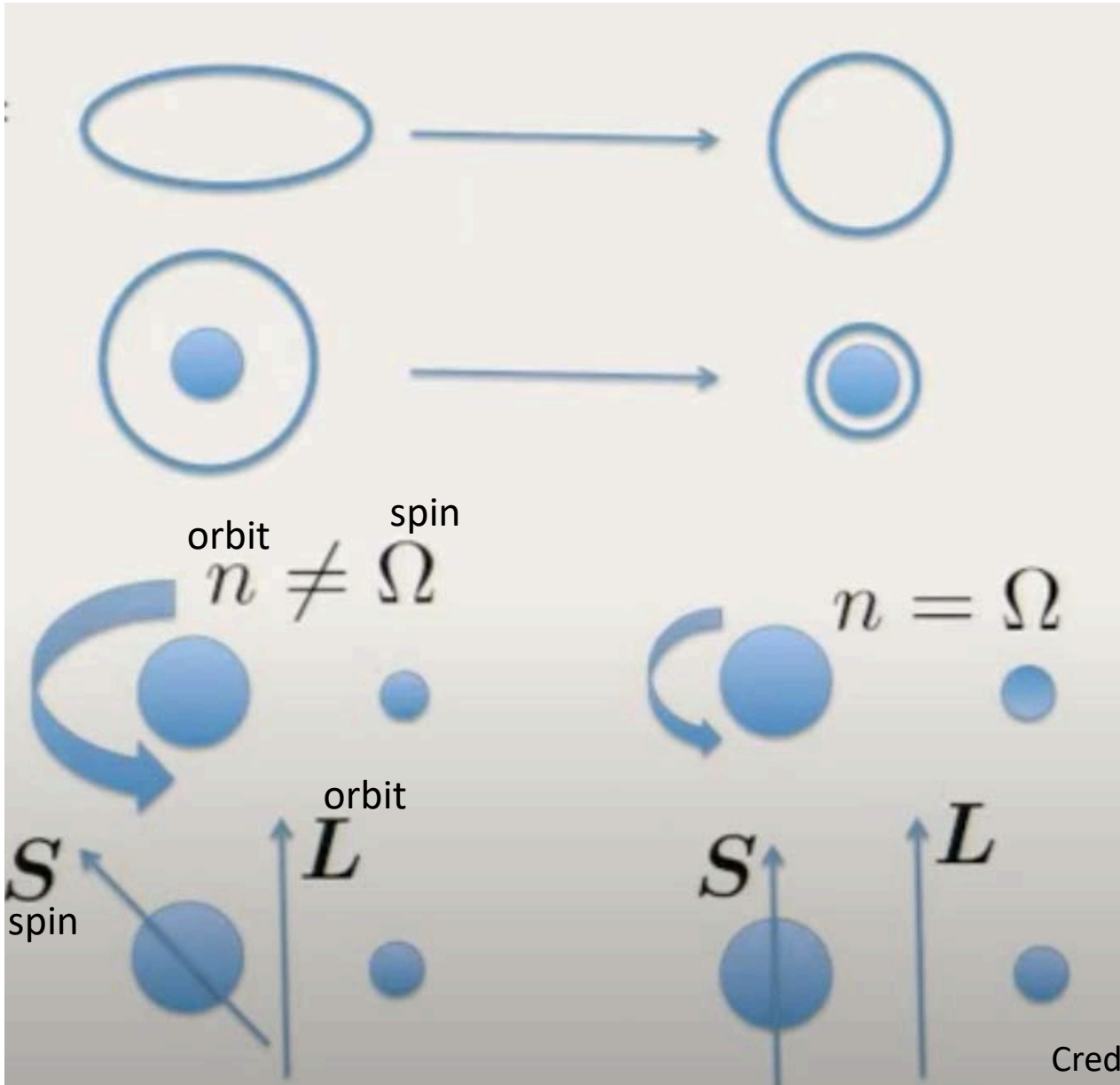
Tidal Evolution Effects

Circularization

Orbital Decay

Synchronization

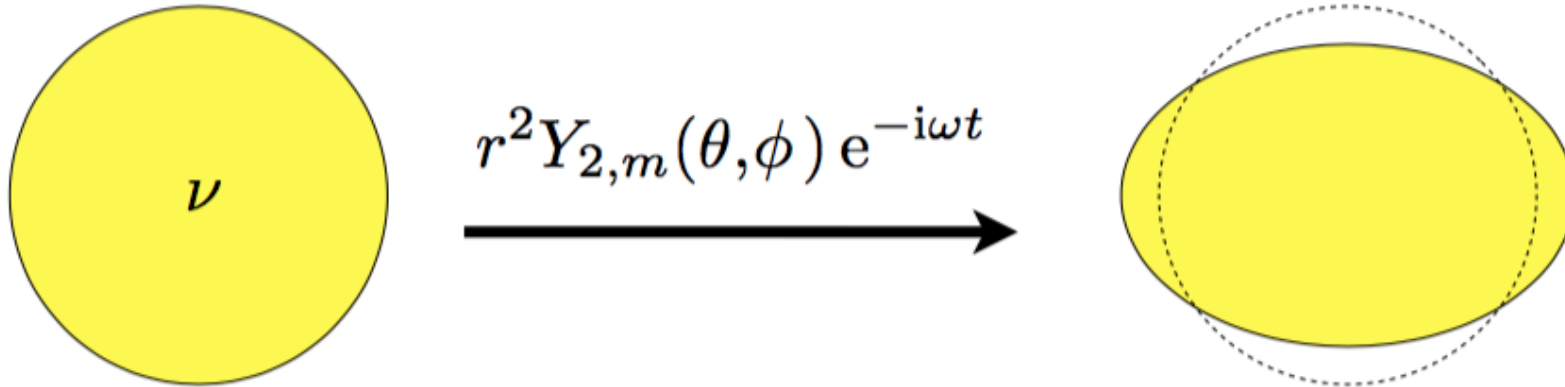
Spin-Orbit alignment



J-P Zahn's categorization :

- “Equilibrium tide” (ν: Turbulent viscosity in convection zone)

Dissipation associated with large-scale tidal bulge
(non-wave like)



Credit: Ogilvie

- “Dynamical tide”

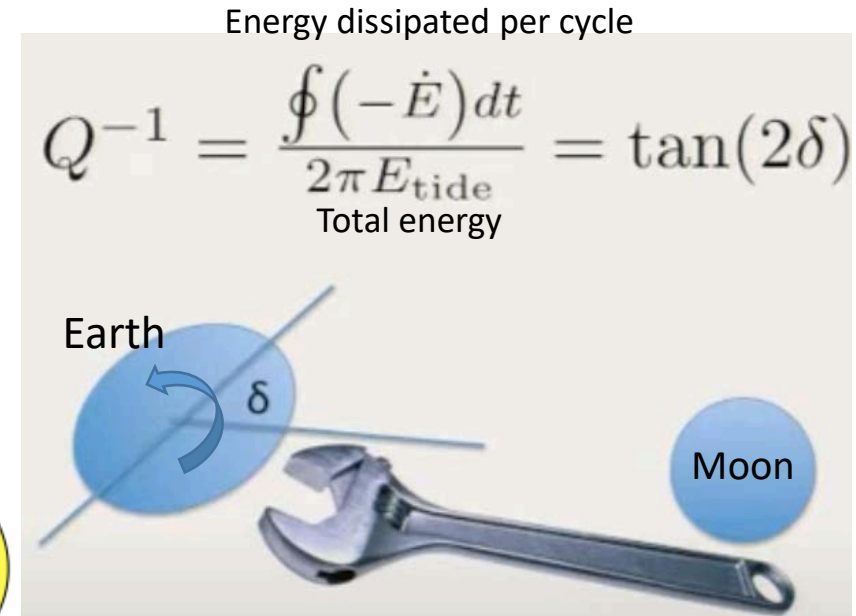
Dissipation associated with low-frequency waves

Convection zone: Inertial Wave

(N=0)

Radiative region: (Inertia-) Gravity Wave

(stably stratified)



credit: Arras

Tidal quality factor Q:

Earth ~ 10

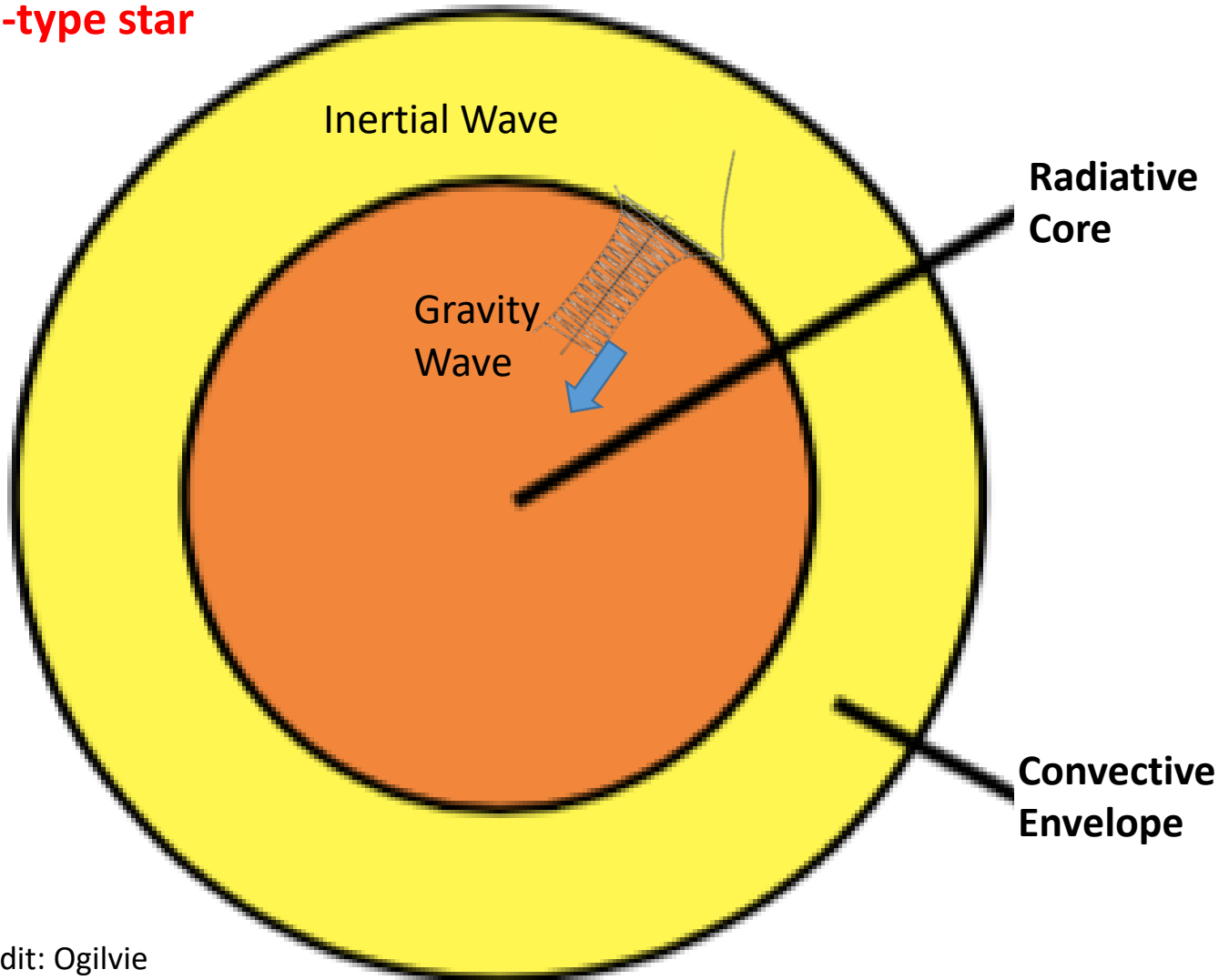
Sun ~ 10⁶

Jupiter ~ 10⁵ - 10⁶

● “Dynamical tide”

Dissipation associated with low-frequency waves

Late-type star

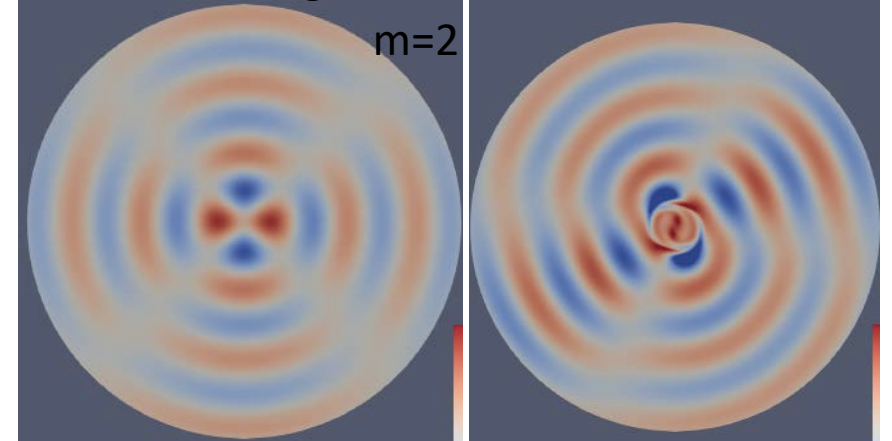


Credit: Ogilvie

Gravity waves may

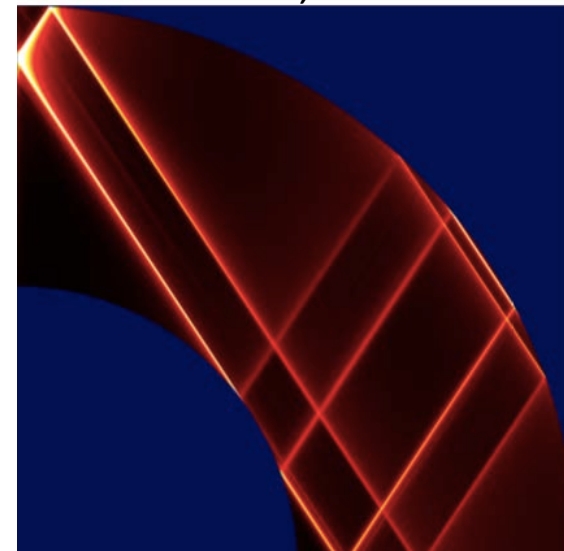
1 Form standing waves

2 Break near the center



Barker&Ogilvie2010
Guo+2023

Inertial wave in the convective shell
~ Narrow beams, not normal modes

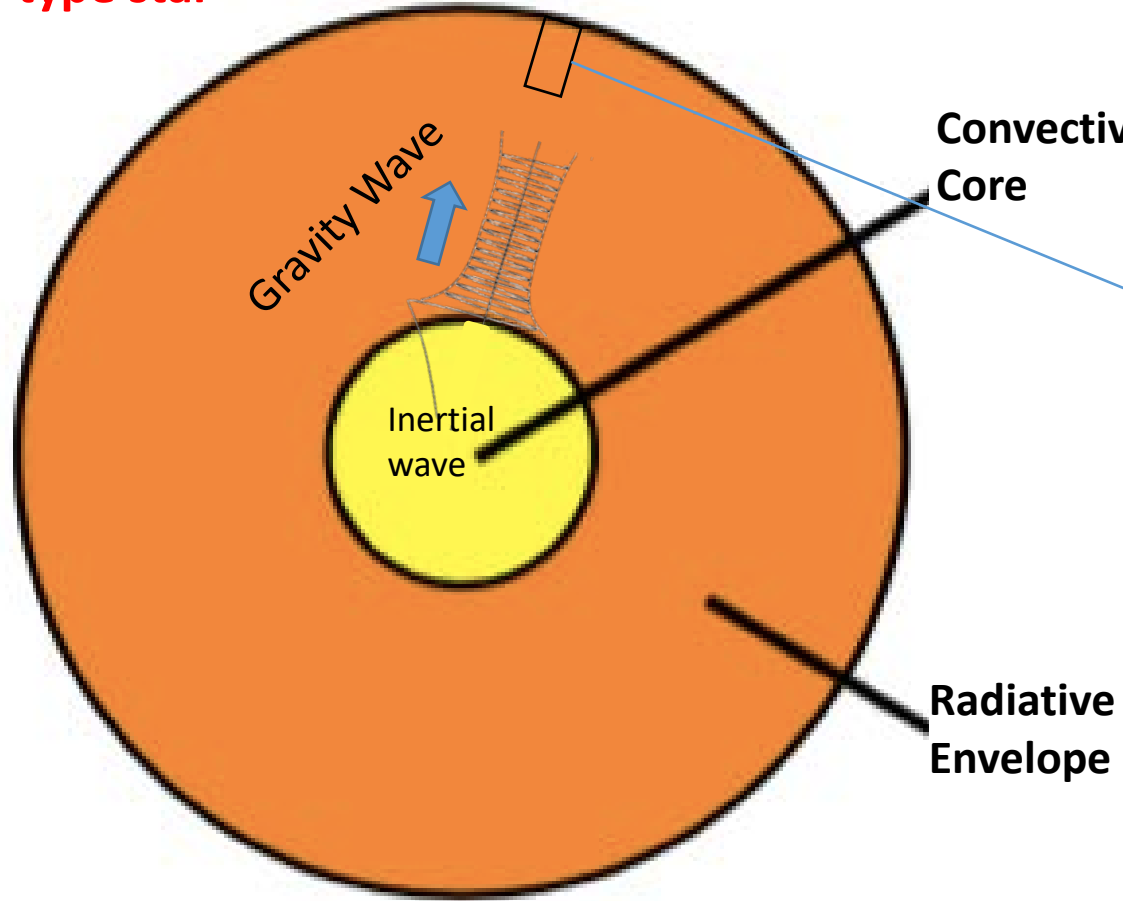


5
Ogilvie2004, 2007

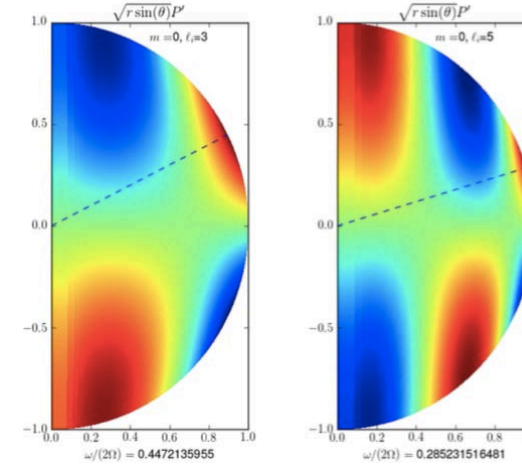
● “Dynamical tide”

Dissipation associated with low-frequency waves

Early-type star



Inertial wave in the core
 \sim can form normal modes



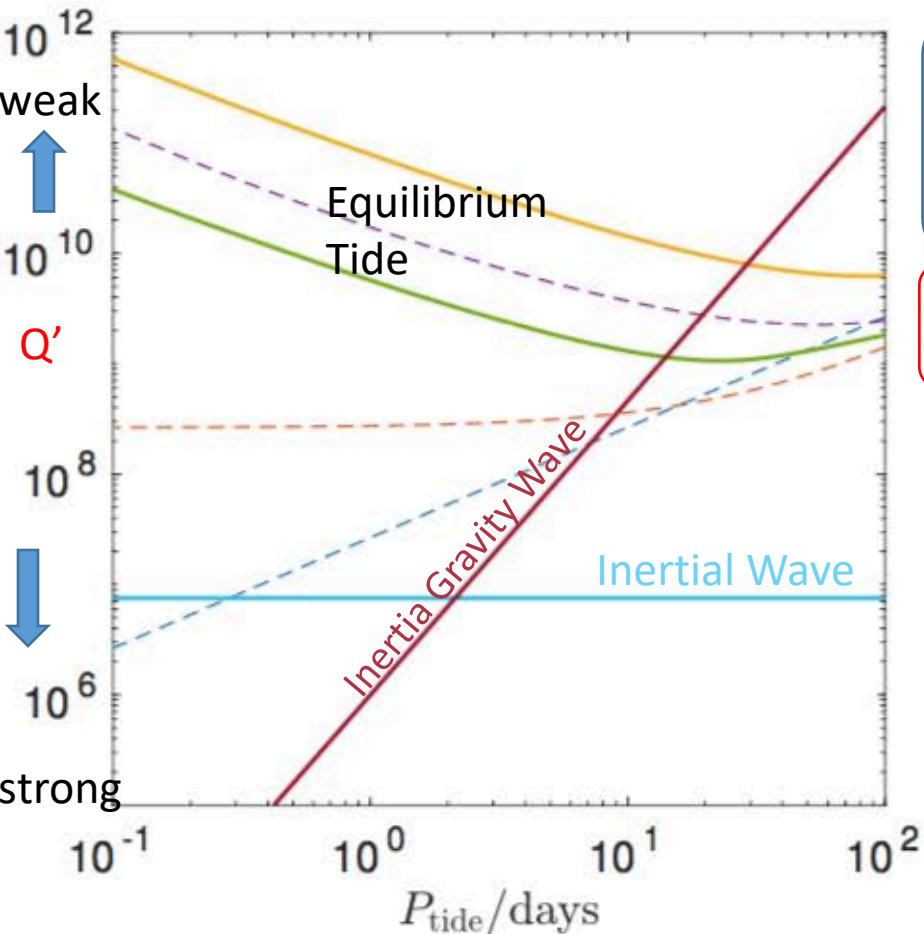
Dips in ΔP -P
 Saio, Ouazzani, Aerts

Gravity waves may
 1 Form standing waves
 -> g-mode seismology

2 Break near the surface
 -> form critical layer

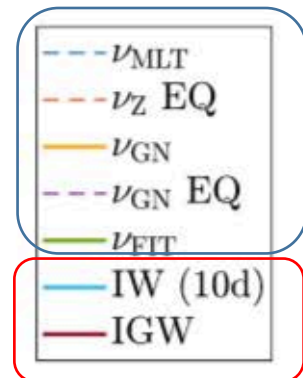


Tidal Dissipation



(c) $M/M_{\odot} = 1$, age/yr = 4.70×10^9

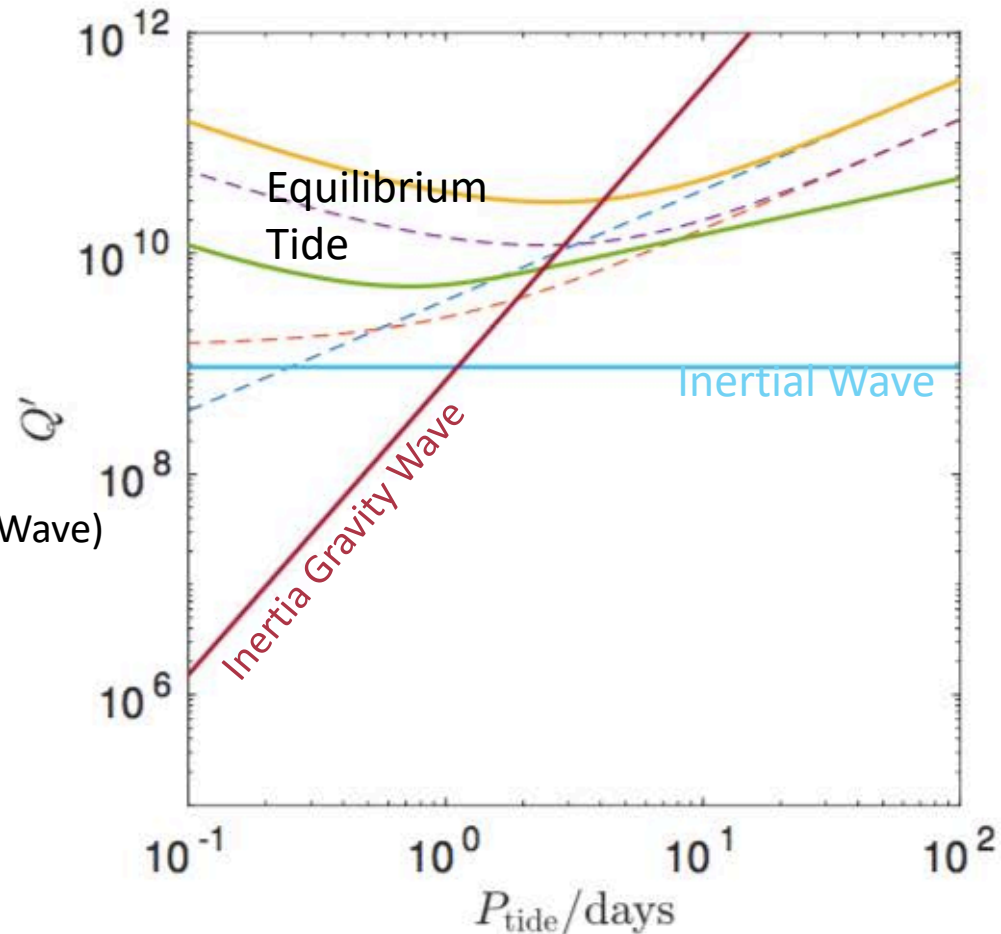
equilibrium tide (convective viscosity)



dynamical tide

(IW: Inertial Wave

IGW: Inertia-Gravity Wave)



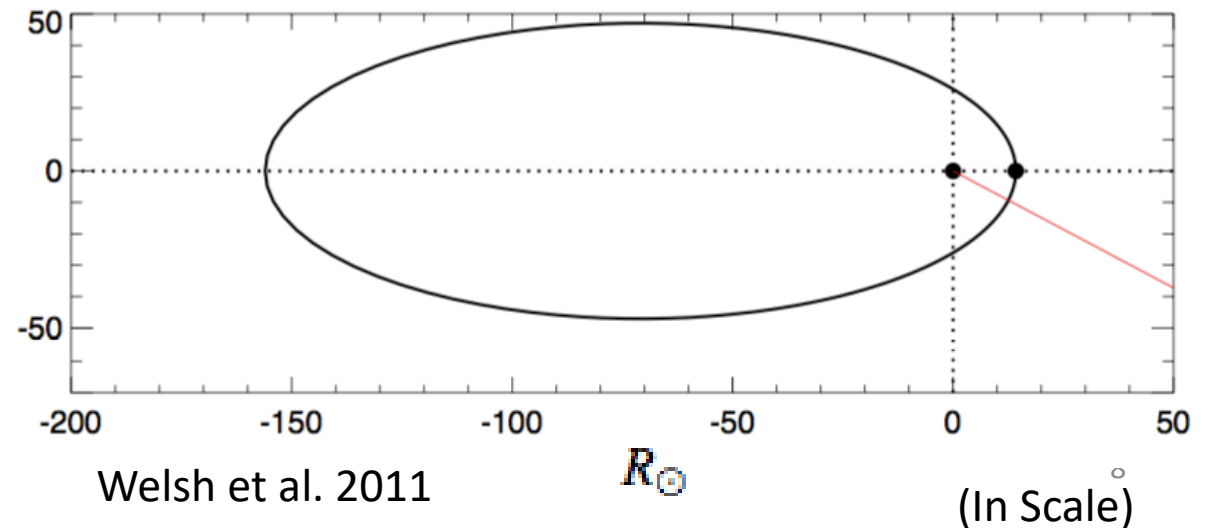
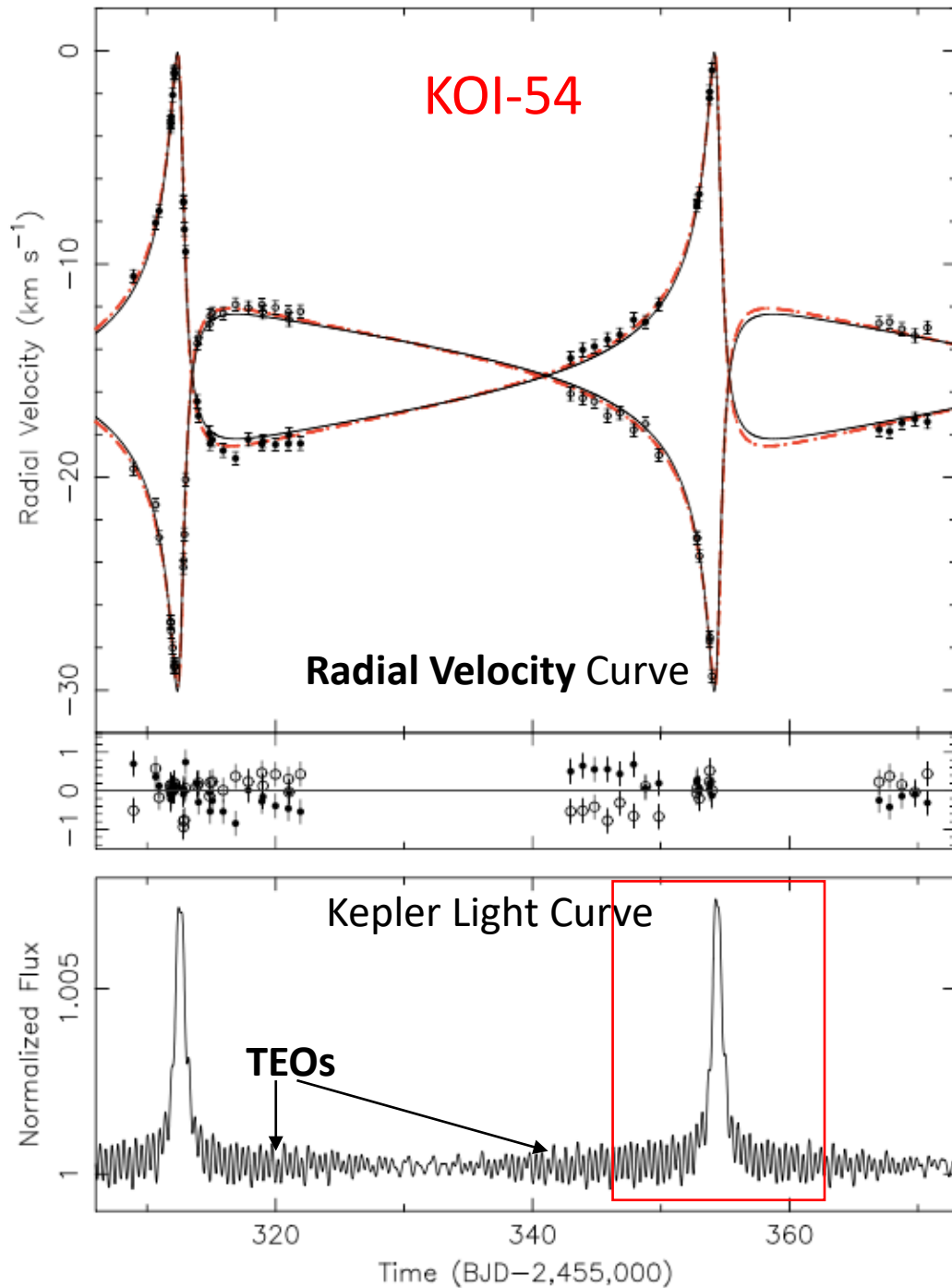
(d) $M/M_{\odot} = 1.4$, age/yr = 1.29×10^9

The Prototype of Heartbeat Stars

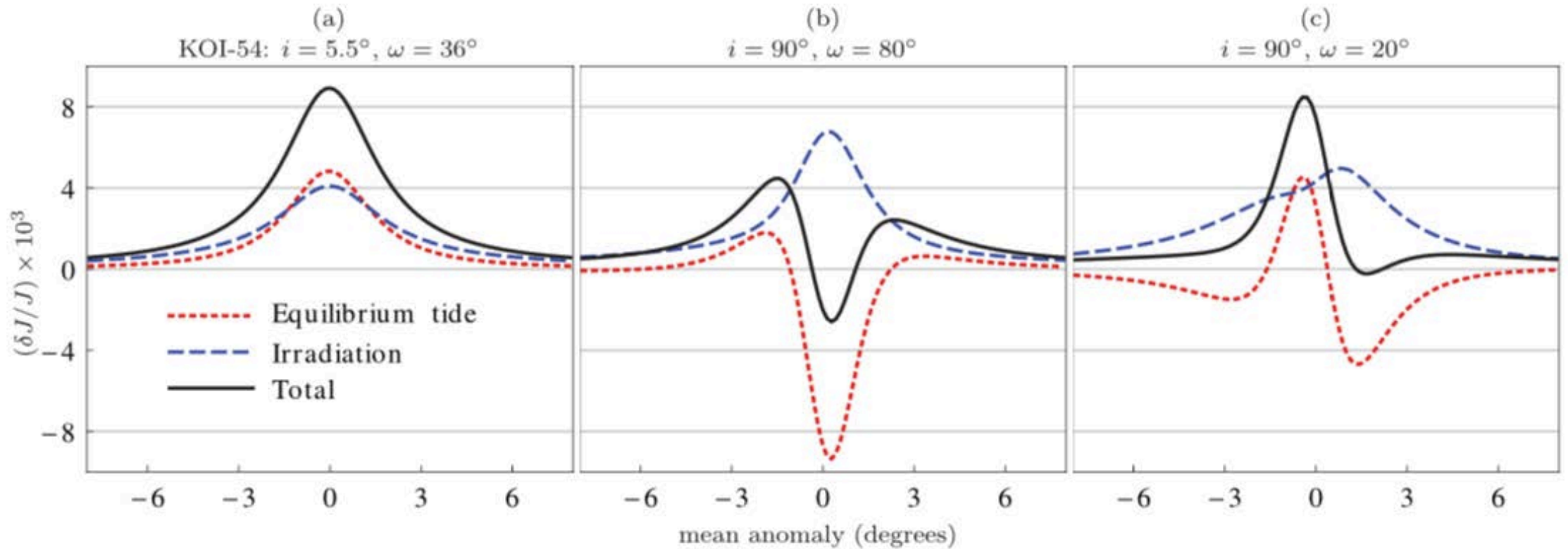
Two A stars ($T_{\text{eff}} \sim 8500\text{K}$)
 $M_1 \sim 2.33$, $M_2 \sim 2.39$ ($R_1 = 2.20$, $R_2 = 2.33$)
 $e = 0.83$, $\omega = 39.5^\circ$, $P = 41.8$ days,

Face-on: inclination = 5.5°

Tidally excited oscillation (TEOs) at exactly
90 and 91 times of orbital frequency



Light Curve of KOI-54 at different i, ω

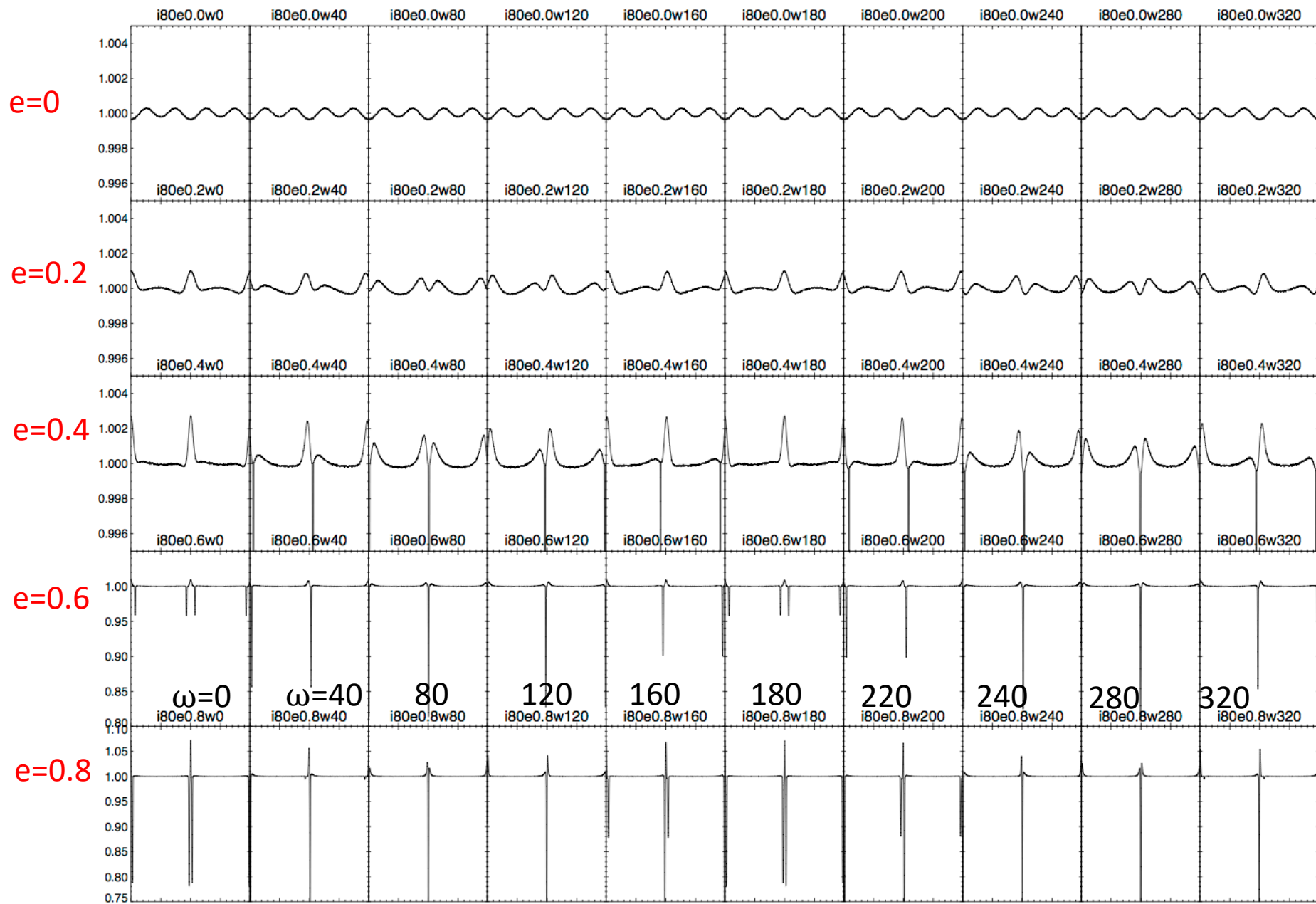


ellipsoidal variation from equilibrium tide
+ mutual irradiation

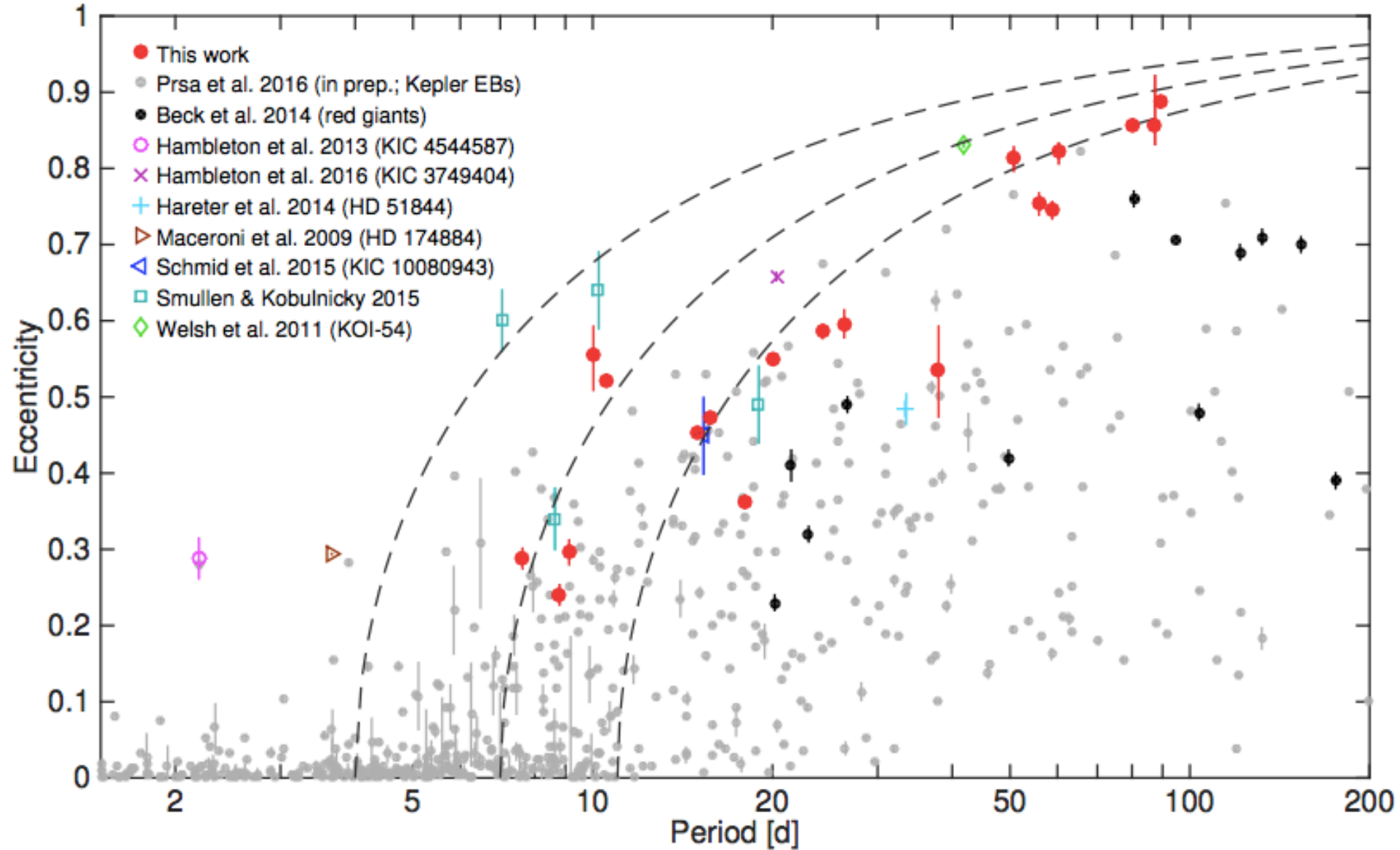
(e, i, ω)

Burkart+12

omega 0 40 80 120 160 180 220 240 280 320 deg



Heartbeats on the Period - e diagram

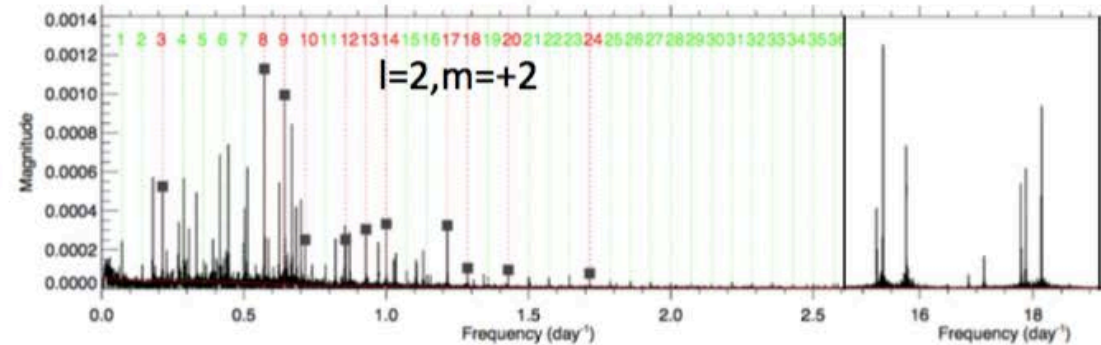
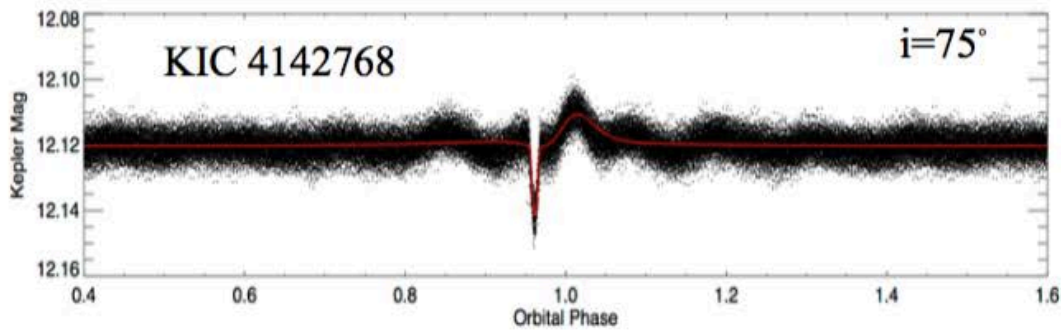
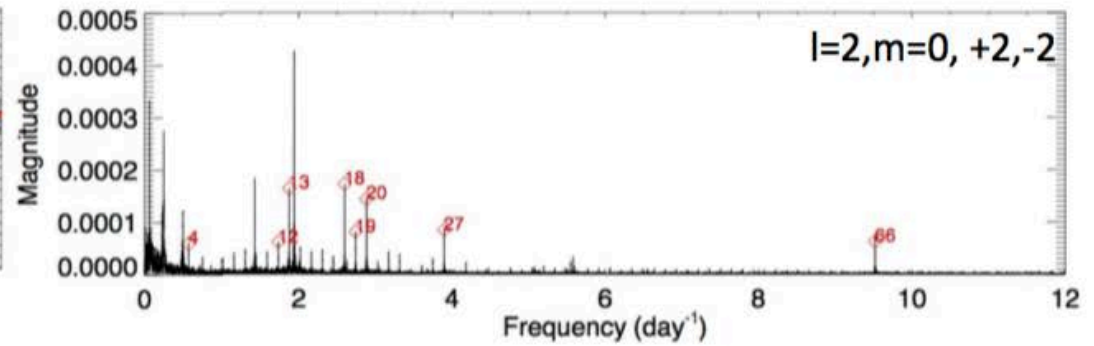
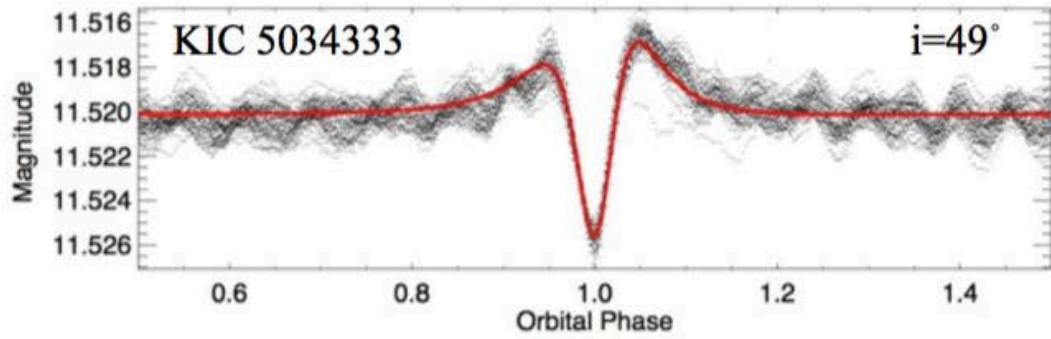
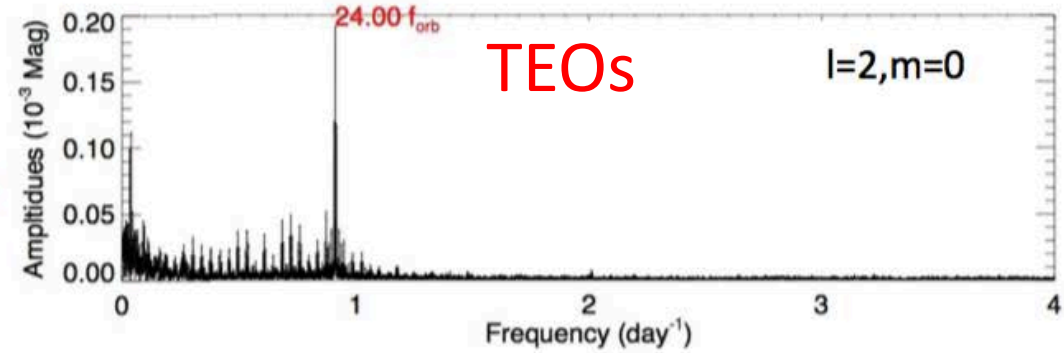
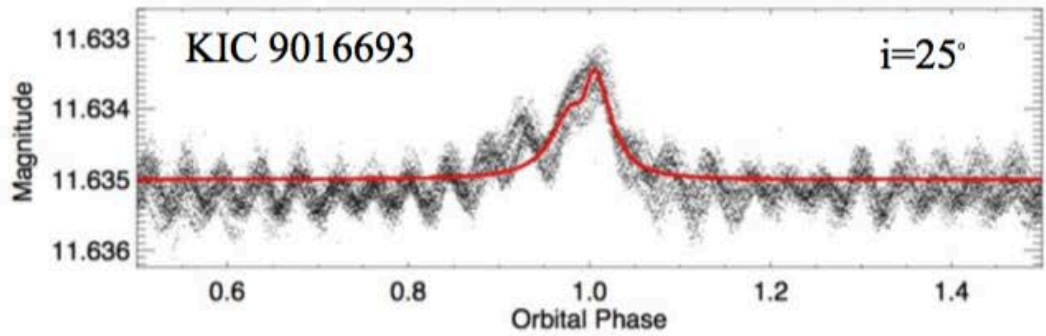


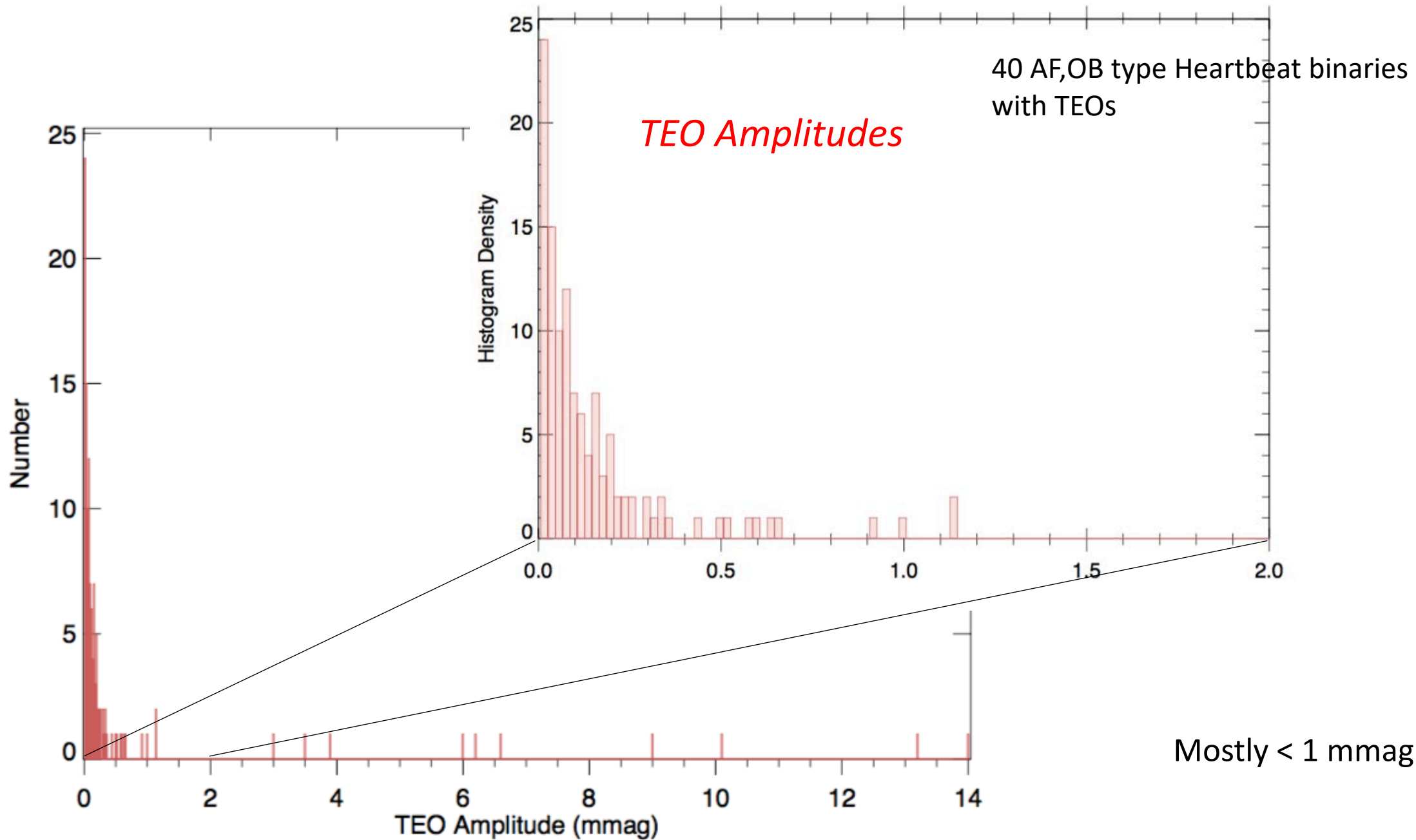
-> Heartbeat stars have $e > \sim 0.2$

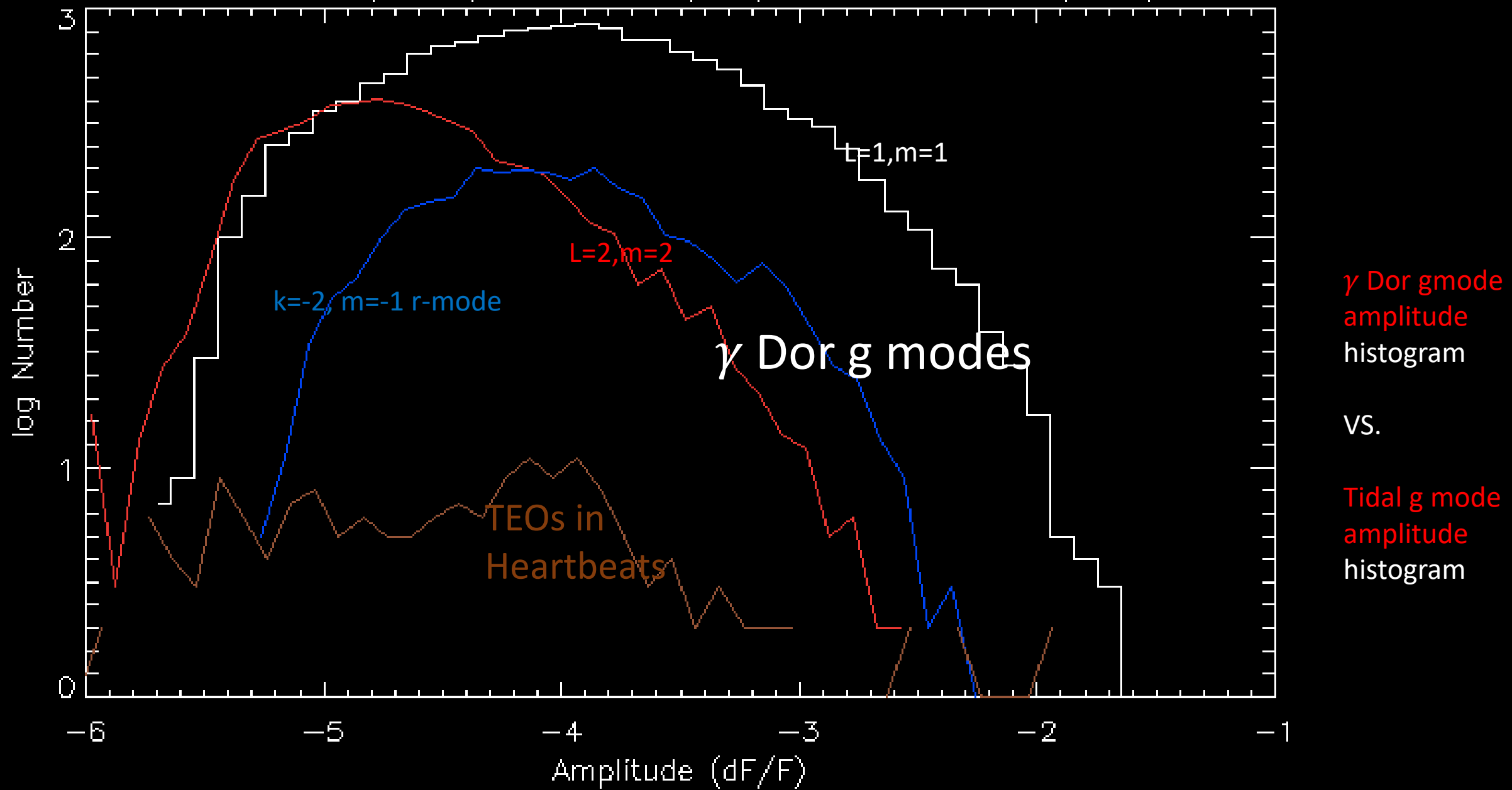
-> Heartbeat stars depict the **upper envelope** of the **Period-ecc.** diagram

Binary Light curve fitting

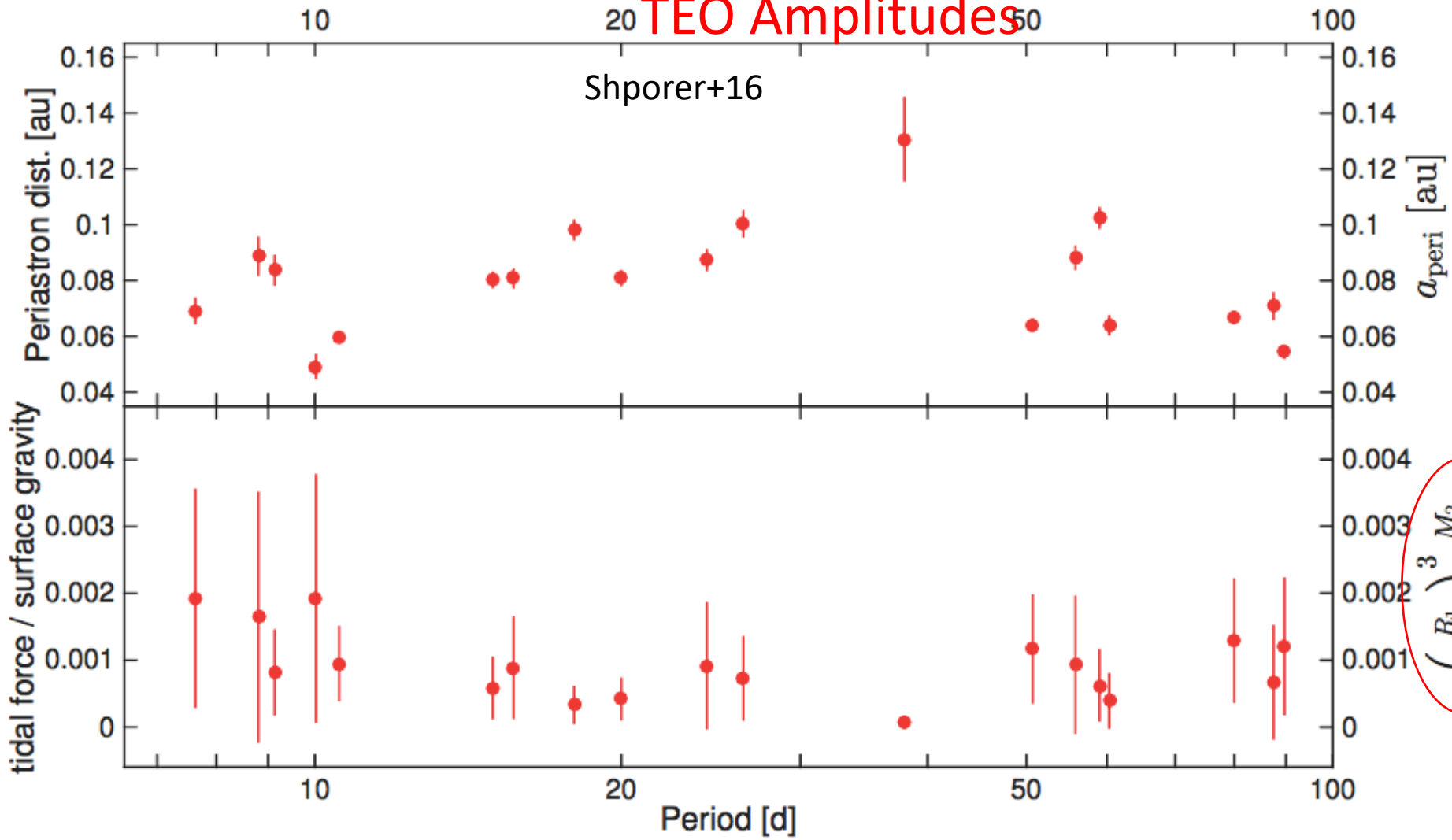
Fourier spectra -> Tidally Excited Oscillations







TEO Amplitudes



Tidal Parameter
determines
the amplitude of tidal strength

$$\frac{\xi}{R_1} \sim \epsilon_l = \left(\frac{M_2}{M_1} \right) \left(\frac{R_1}{D_{\text{peri}}} \right)^{l+1}$$

$$\left(\frac{R_1}{a_{\text{peri}}} \right)^3 \frac{M_2}{M_1}$$

$$D_{\text{peri}} = a(1 - e)$$

Observation:

HBs have tidal parameter $\sim 10^{-3}$,
and $D_{\text{peri}} \sim 0.05 - 0.1$ AU

- In the Solar System:
- 2×10^{-7} (Jupiter, due to Io)
 - 3×10^{-8} (Saturn, due to Titan)
 - 8×10^{-8} (Neptune, due to Triton)

TEO Amplitudes -MD

a) Tidally Excited Oscillation (linear) (~ forced harmonic oscillator)

$$\partial^2 \boldsymbol{\xi} / \partial t^2 + \mathbf{L}(\boldsymbol{\xi}) = \mathbf{f}$$

-> Mode Decomposition (MD)

Tidal response -> Sum of eigenfunctions of free oscillations $\sum_{\alpha} C(t) \boldsymbol{\xi}_{\alpha}(\mathbf{x})$

Use orthogonality condition to obtain eqn. for C(t)

Tidal potential
from companion

M' Companion mass
D distance
 Ω_s spin frequency
 Ω : orbital frequency

$$\dot{c}_{\alpha} + (i\omega_{\alpha} + \gamma_{\alpha})c_{\alpha} = \frac{i}{2\epsilon_{\alpha}} \langle \boldsymbol{\xi}_{\alpha}(\mathbf{r}), -\nabla U \rangle$$

$$= \frac{iGM'W_{lm}Q_{\alpha}}{2\epsilon_{\alpha}D^{l+1}} e^{im\Omega_s t - im\Phi} = \frac{iGM'W_{lm}Q_{\alpha}}{2\epsilon_{\alpha}a^{l+1}} \sum_{N=-\infty}^{\infty} F_{Nm} e^{i(m\Omega_s - N\Omega)t}$$

Non-homog. Solution: $c_{\alpha}(t) = \frac{GM'W_{lm}Q_{\alpha}}{2\epsilon_{\alpha}a^{l+1}} \sum_{N=-\infty}^{\infty} \frac{F_{Nm} e^{-i(N\Omega - m\Omega_s)t}}{(\sigma_{\alpha} - N\Omega) - i\gamma_{\alpha}}$

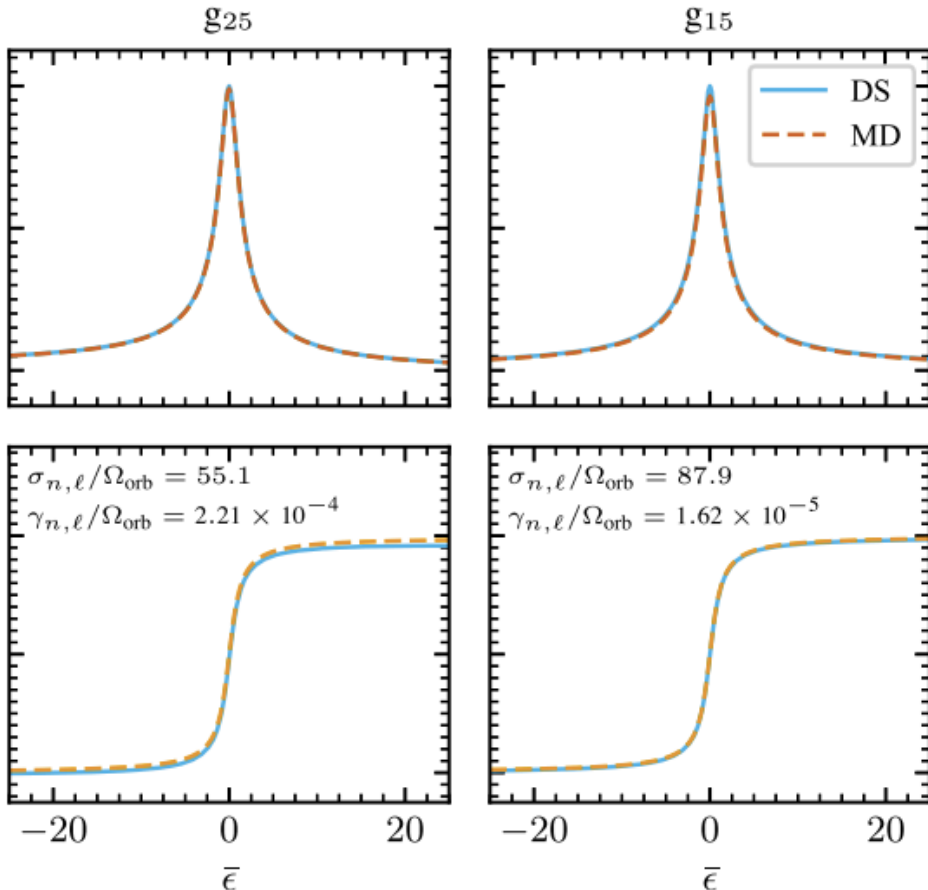
Observationally, tidal excited oscillations -> frequency of forcing = orbital frequency harmonics

GYRE-TIDE

- 1 Direct Solving (DS)
- 2 Mode Decomposition (MD)

(Rotation NOT Included)

Non-adiabatic



Forcing frequency offset from resonance

The screenshot shows the GYRE v7.0 documentation website. At the top is the GYRE logo in orange and blue, with the version number v7.0 below it. A search bar is present. A navigation menu on the left lists: USER GUIDE, Preliminaries, Quick Start, Example Walkthrough, Frontends, Numerical Methods, Interpreting Output Files, Understanding Grids, Working With Tags, Advanced Usage, Non-Adiabatic Oscillations, Tidal Forcing, and Overview. The 'Tidal Forcing' item is highlighted in grey.

Advanced Usage / Tidal Forcing

Edit on GitHub

Tidal Forcing

This section discusses how to evaluate the stellar response (fluid displacements and perturbations) to tidal forcing, using the `gyre_tides` frontend. The response data can be used to calculate the secular rates-of-change of orbital elements, or to synthesize a light curve for a tidally distorted star.

Overview

As discussed in the [Tidal Equations](#) chapter, the tidal gravitational potential (14) of an orbiting companion can be expressed as a superposition of partial potentials of differing harmonic degree ℓ , azimuthal order m and Fourier harmonic k . For each `&tidel` namelist group appearing in its namelist input file, `gyre_tides` solves for the response of the star to these partial potentials with a separate calculation for every combination of (ℓ, m, k) .

Development Team

GYRE remains under active development by the following team:

- Rich Townsend (University of Wisconsin-Madison); project leader
- Warrick Ball (University of Birmingham)
- Zhao Guo (Cambridge University)
- Joel Ong (Yale University)
- Meng Sun (Northwestern University)

(Sun, Townsend & Guo 2023)

Non-adiabatic Calculations

Problem with the Mode Decomposition method

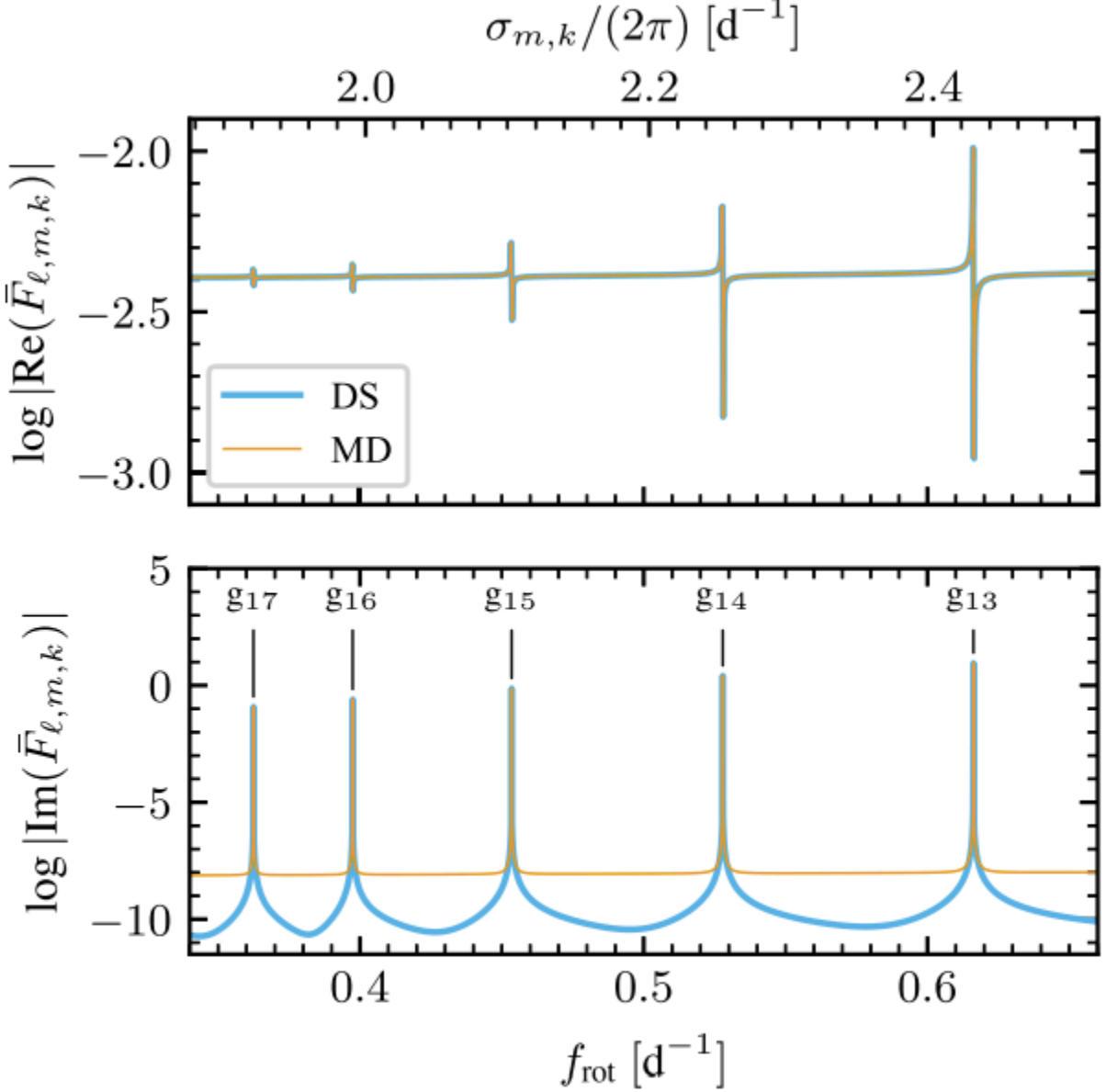


figure 2. Normalized response function $\bar{F}_{\ell,m,k}$ plotted against stellar rotation frequency f_{rot} for the $\{\ell, m, k\} = \{2, -2, 50\}$ partial tide of the KOI-54 primary model. The real (imaginary) part of the function is shown in the upper (lower) panel, and separate curves are plotted for the DS and MD approaches. Peaks

Tidal response

$$\sum_{nlmk} A_{nlmk} (\Delta L/L)_{nlm}$$

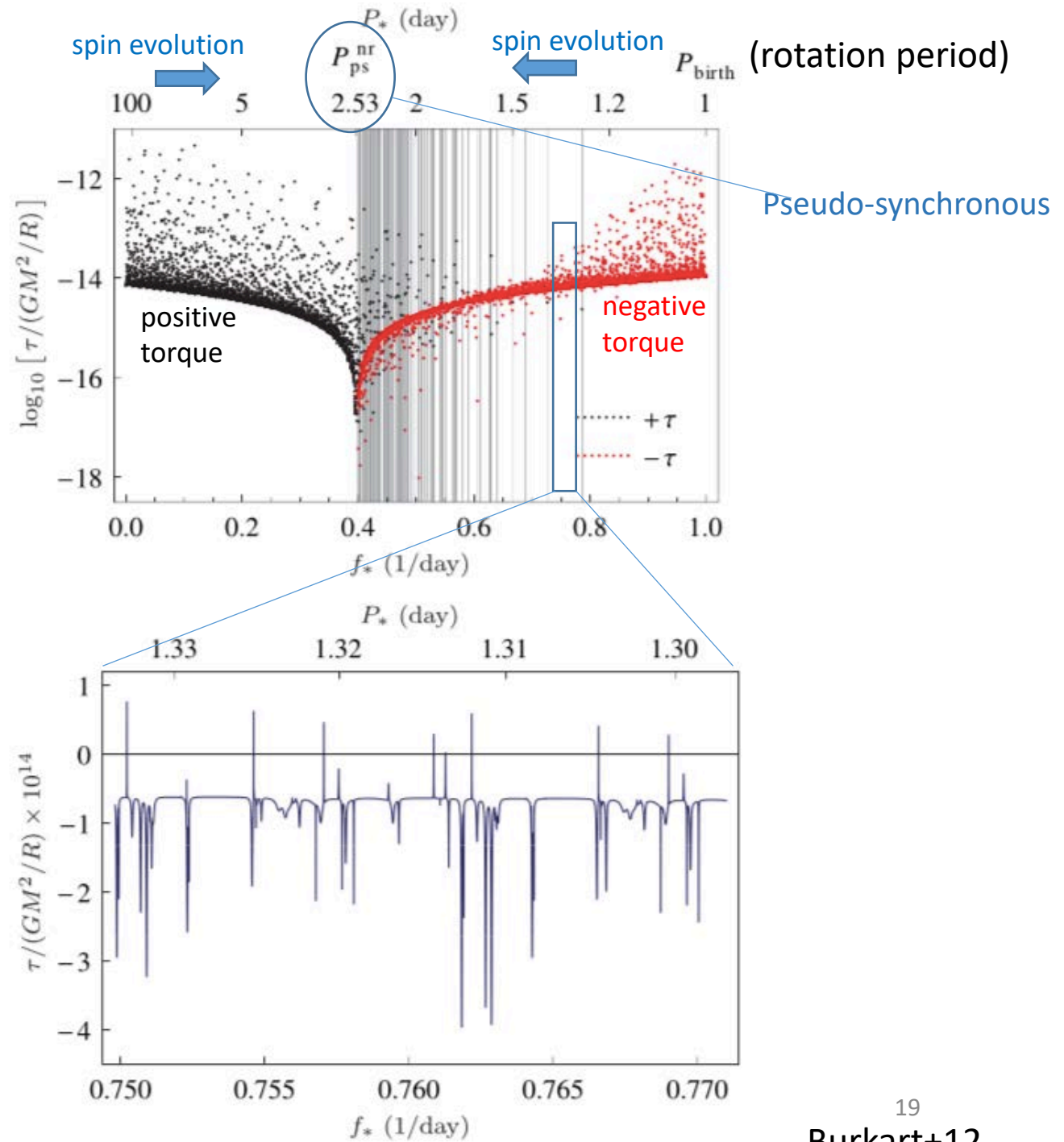
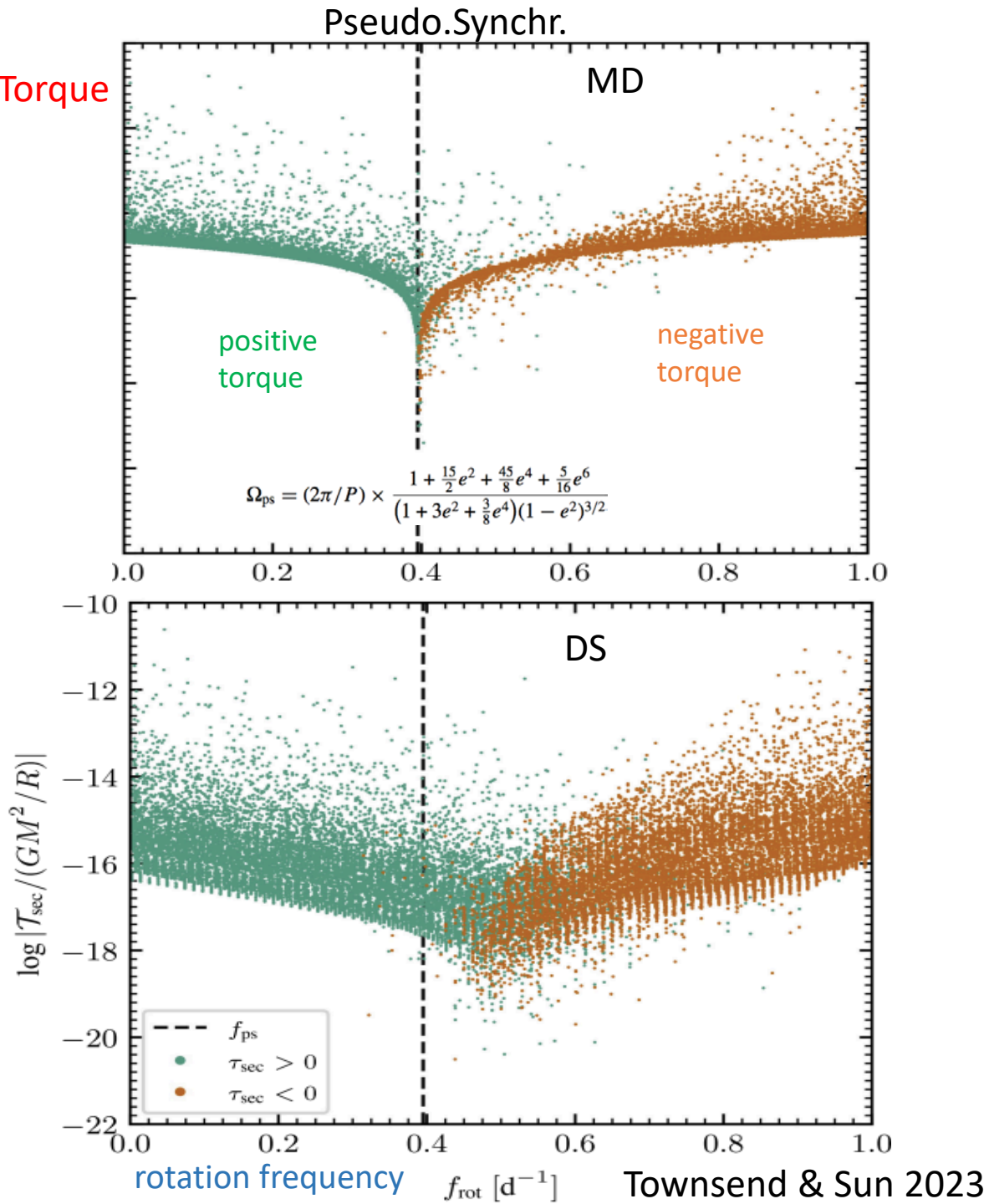
Mode amplitude

$$A_{n,\ell,m,k} = \frac{2\epsilon_T Q_{n,\ell} \bar{c}_{\ell,m,k} \Delta_{n,\ell,m,k}}{E_{n,\ell}}$$

Lorentzian

$$\Delta_{n,\ell,m,k} \equiv \frac{\sigma_{m,k}^2}{(\hat{\sigma}_{n,\ell}^2 - \sigma_{m,k}^2) - 2i\hat{\gamma}_{n,\ell}\sigma_{m,k}}$$

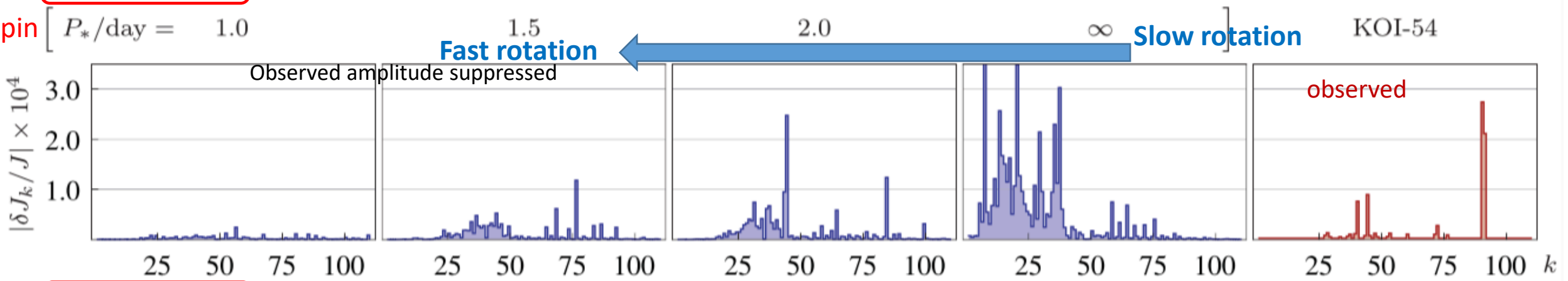
Damping rate



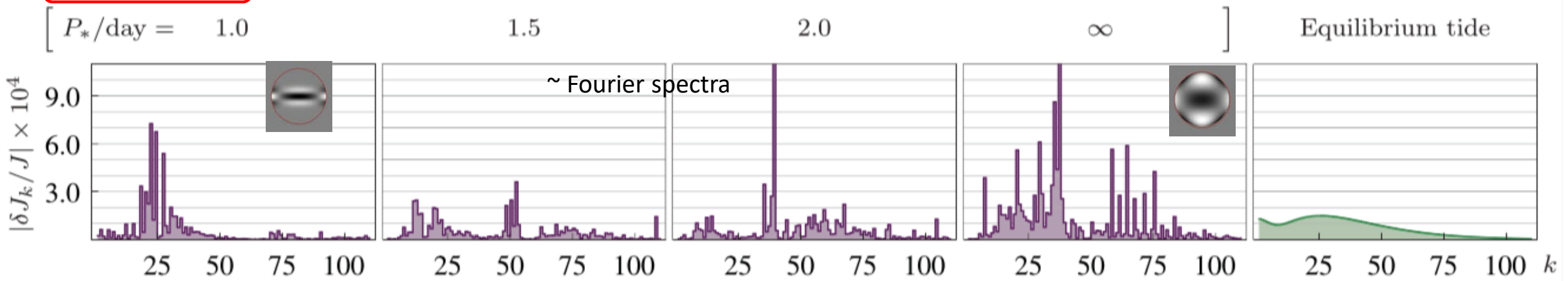
TEO amplitude: The Effect of rotation

Direct Solving + Traditional approximation

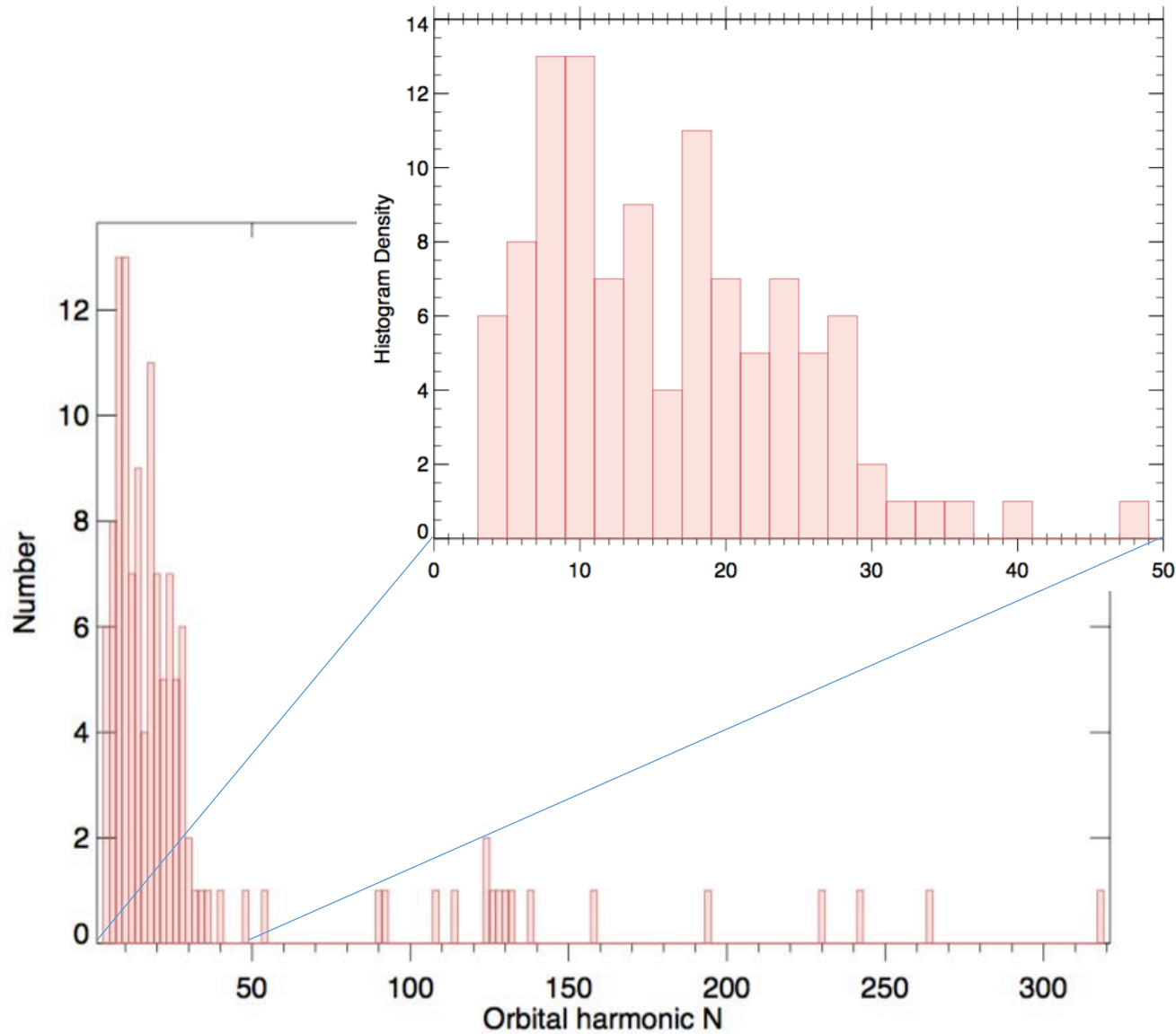
(a) Face on: $i = 5.5^\circ$, $\omega = 36^\circ$ (KOI-54's orientation)



(b) Edge on: $i = 90^\circ$, $\omega = 36^\circ$



Which **orbital harmonics N** do we expect?

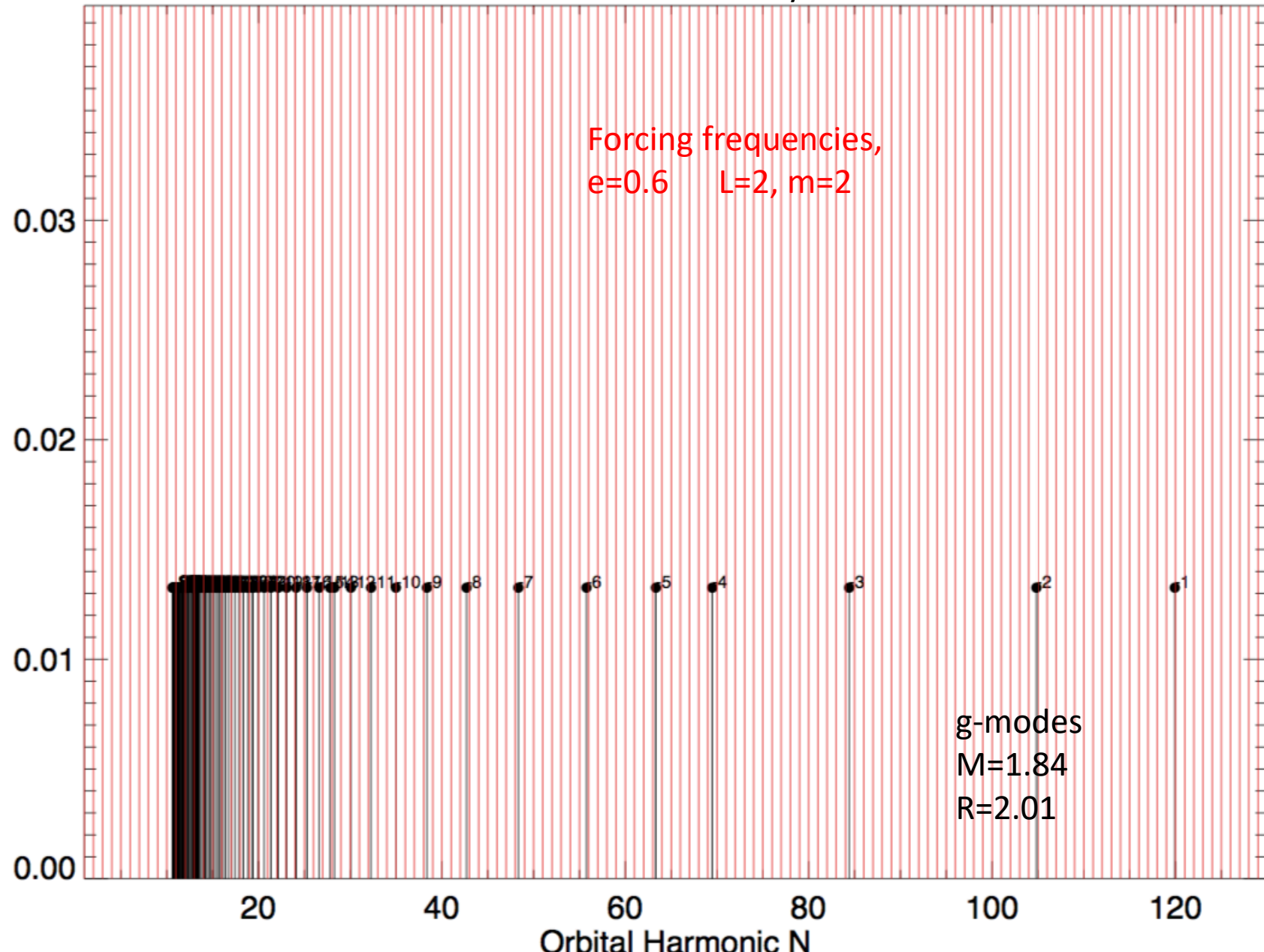


TEO orbital-harmonic number

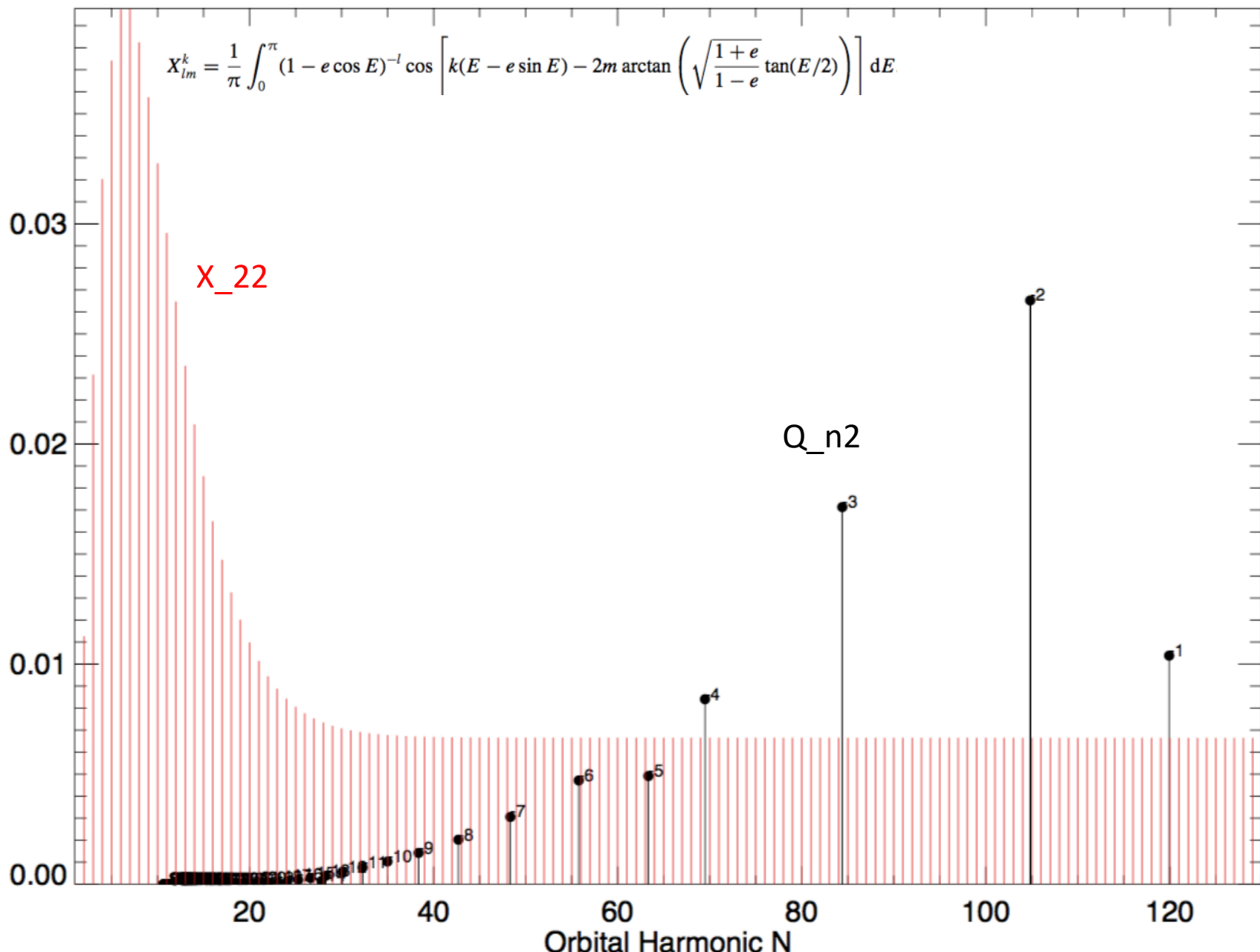
N can reach \rightarrow 300 (KIC8164262)

Mostly from 4 to 40

Which **orbital harmonics N** are tidally excited?



Forcing frequencies
=
Orbital harmonic $N \cdot \text{forb}$

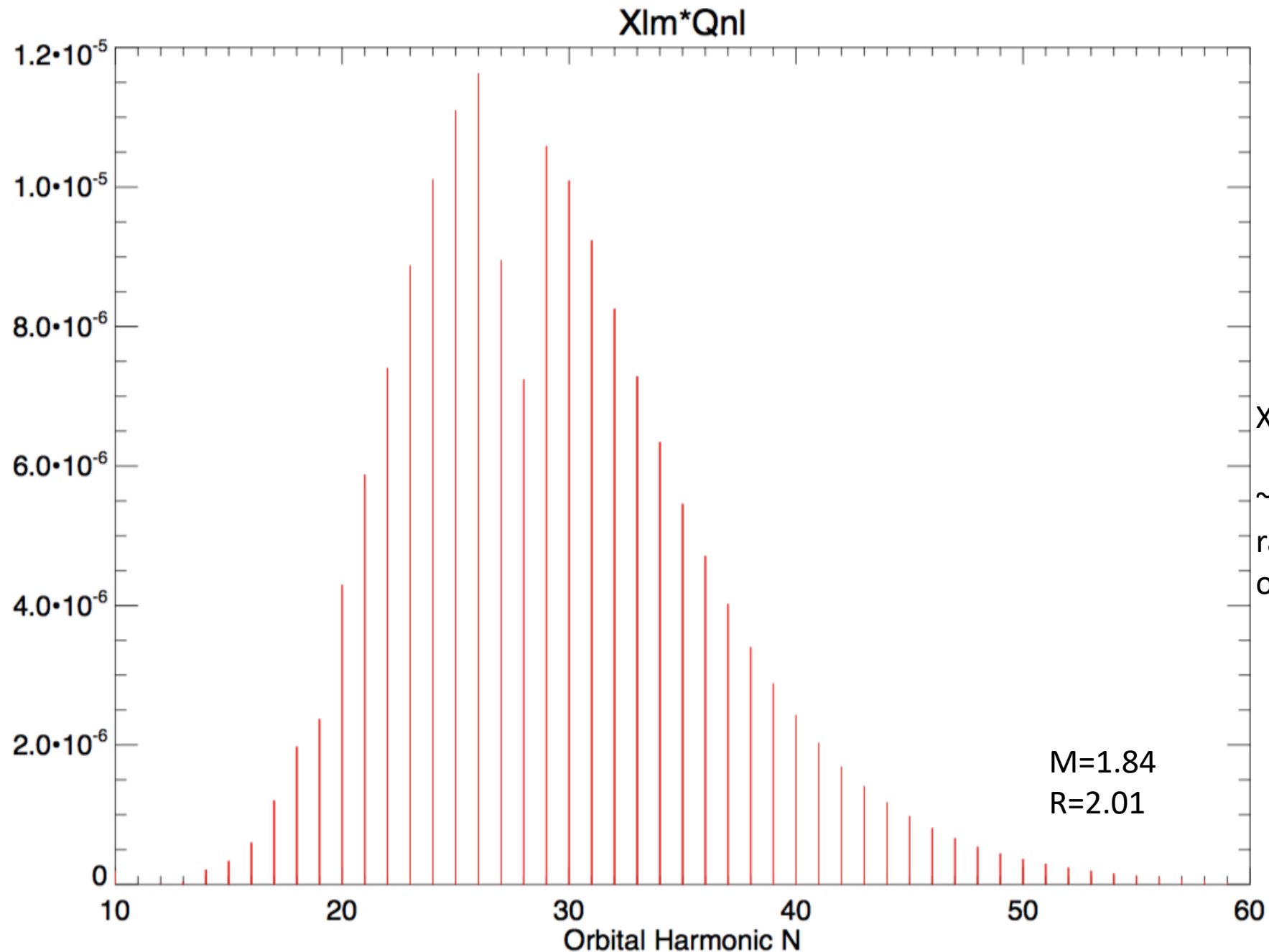


G-mode do not couple with the tidal potential equally

weights:

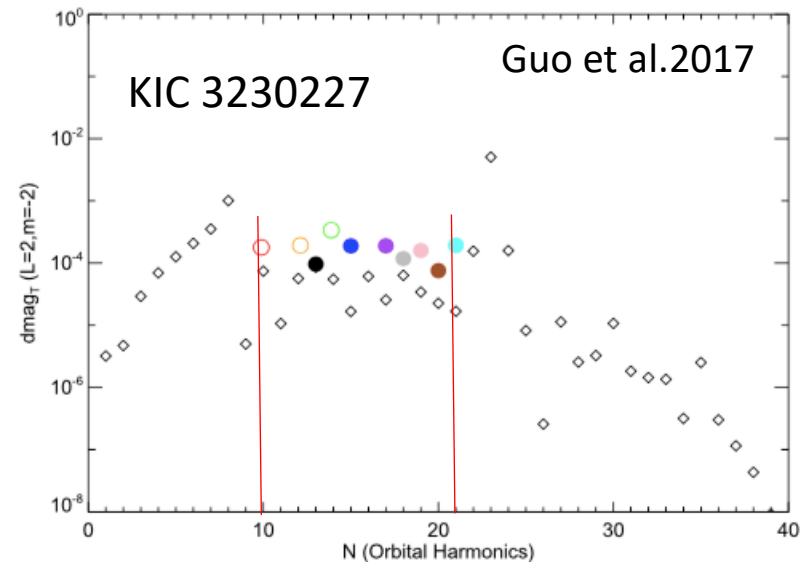
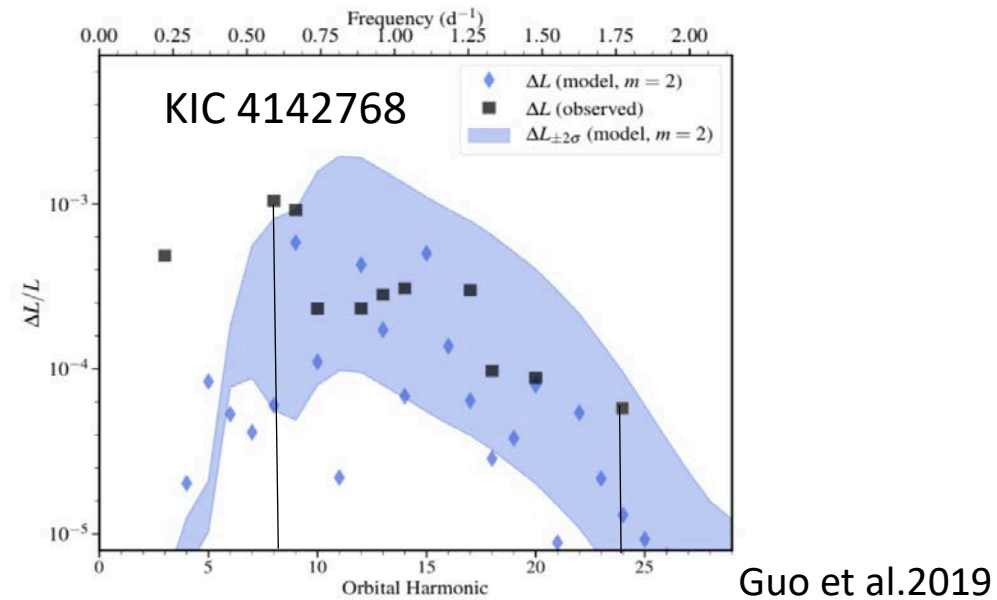
Tidal overlap integral
Q_n2

$$= \frac{1}{MR^\ell} \int d^3x \rho \xi_a^* \cdot \nabla (r^\ell Y_{\ell m})$$

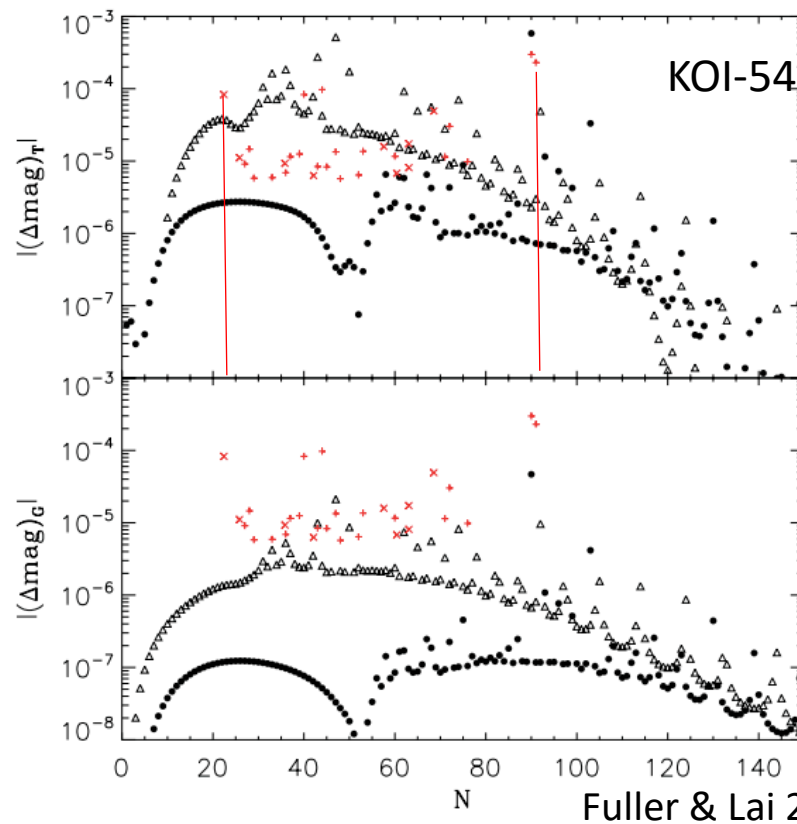
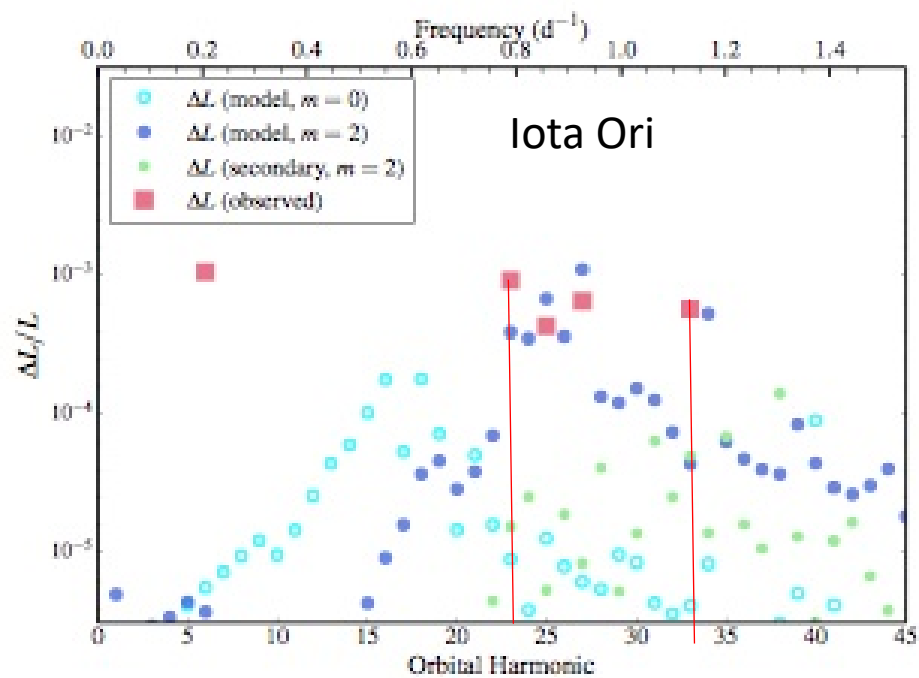


X_{lm} * Q_{nl}
~ roughly determines the
range of the excited
orbital harmonics N

M=1.84
R=2.01



TEO Amplitude Modeling



$$A_{nlmk} = \frac{2 \varepsilon_l Q_{nl} \tilde{X}_{lm}^k W_{lm} \Delta_{nlmk}}{E_{nl}}$$

TEO Phases

TEO phases

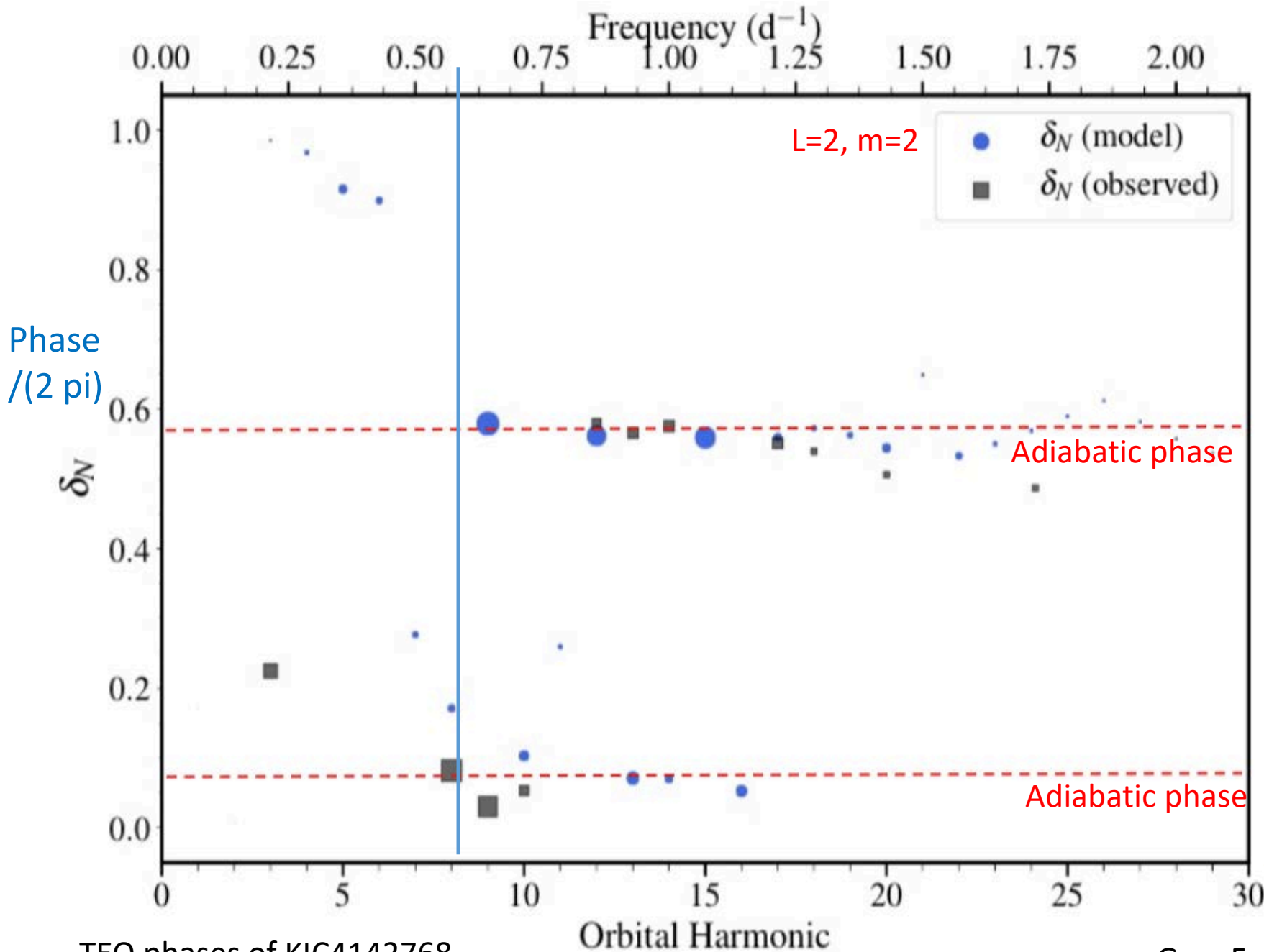
-> observer's coordinates
 -> ω (argument Of periastron)

$N < 8 f_{\text{orb}}$
 Strong non-ad. Effect

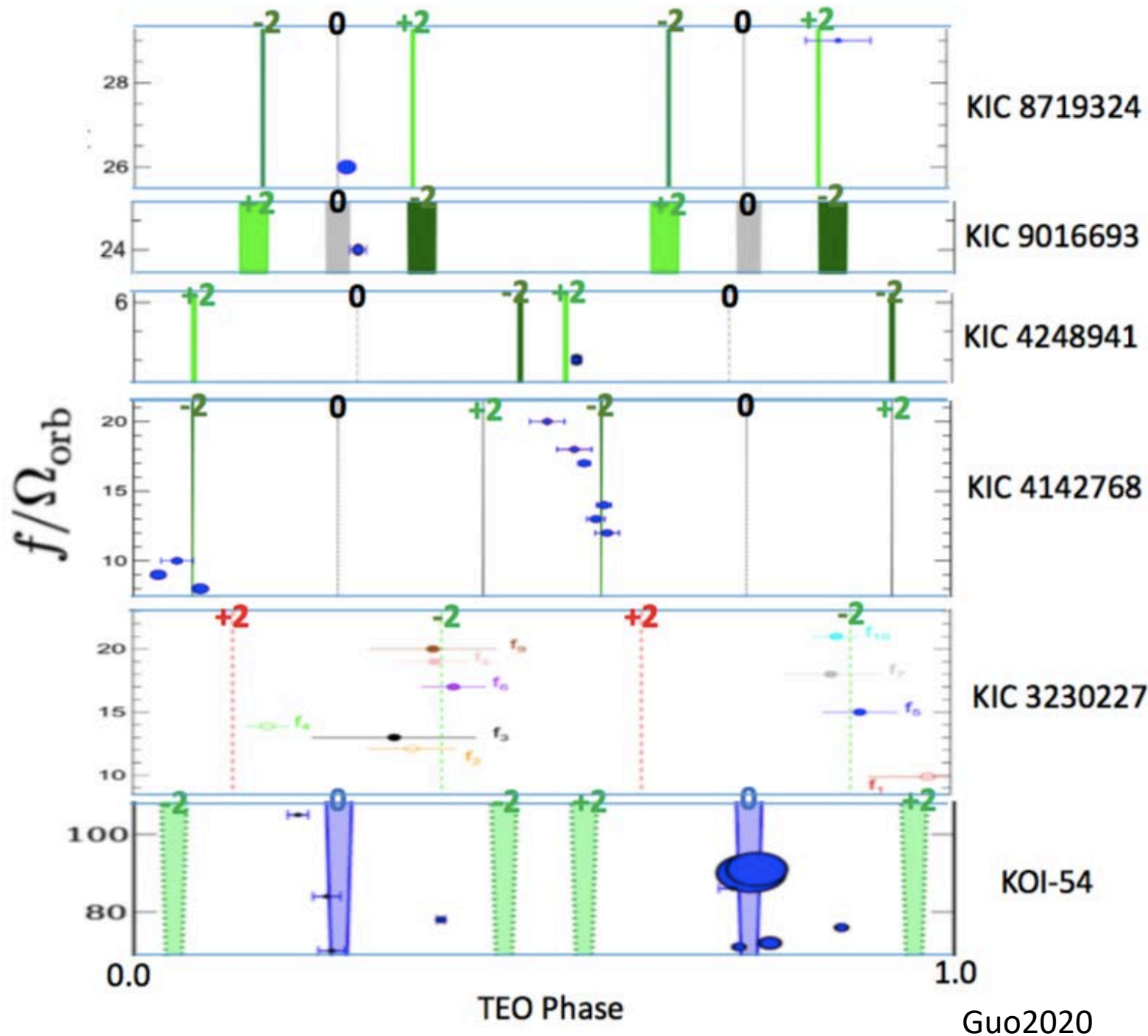
Most observed TEO phases agree with adiabatic phases, \sim Scatter around the adiabatic predictions

$$\phi_{l=2,m} = \begin{cases} 0.25 + m\phi_0 & \text{if } m = 2 \text{ or } -2 \\ 0.25 & \text{if } m = 0 \end{cases} \quad (3)$$

where $\phi_0 = 0.25 - \omega_p/(2\pi)$ and m is the mode azimuthal number.



TEO phases



Observed TEO phases agree with adiabatic phases, Scatter around the adiabatic predictions

TEO phases:

Burkart12; Oleary & Burkart14
Jayasinghe20;
Guo+20; Li+24

-> Can be used to identify spin-orbit misalignment

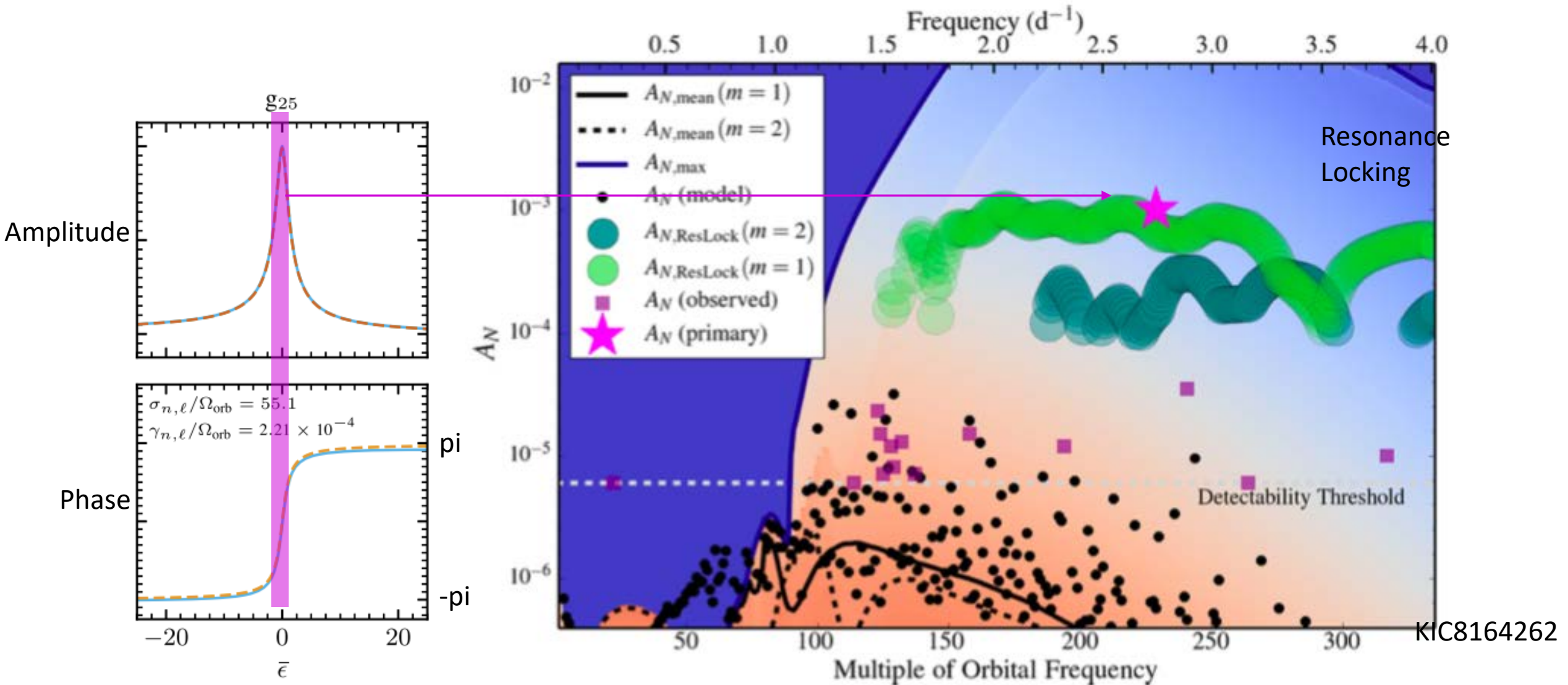
TEOs in Resonance Locking

mode freq.= forcing freq.

If the mode is locked in resonance, then this condition does not change with time, i.e.

$$\dot{\sigma}_\alpha \simeq \dot{\sigma}_N = N\dot{\Omega},$$

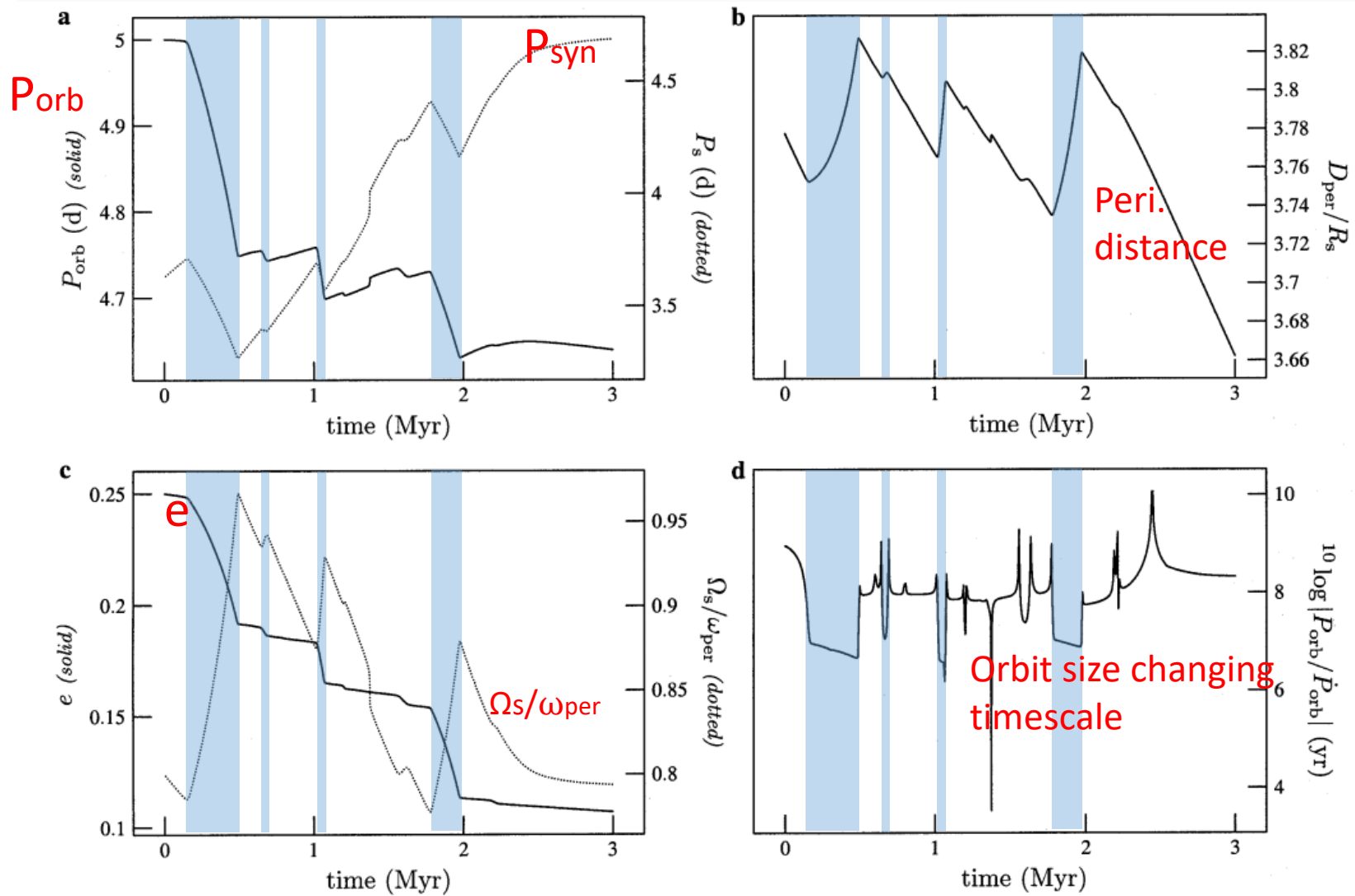
TEOs in Resonance Locking



KIC8164262

Fuller 17

TEOs in Resonance Locking show larger amplitudes and more random phases, limited range of N

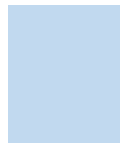


10M + 1.4M binary

$e = 0.25$

$P_{\text{orb}} = 5 \text{ days}$

Fast evolution
of P, e
during
Resonance
Locking



Resonance
Locking

$\Omega_s/\omega_{\text{per}} = 1$
: Pseudo-syn.
rotation

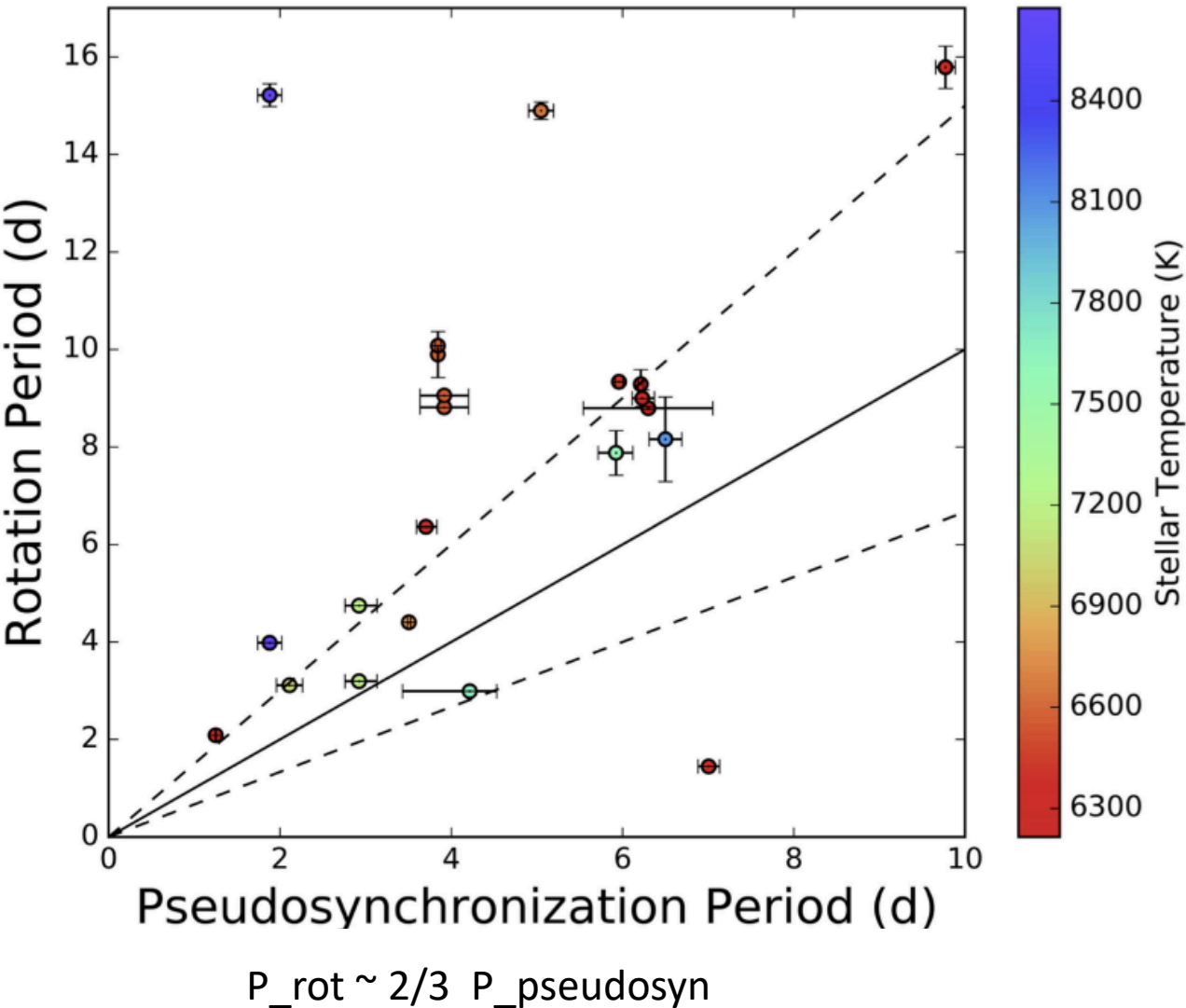
$\Omega_s/\omega_{\text{per}} < 1$
: sub-Pseudo-syn.
rotation

Fig. 3a–f. Orbital evolution of a system with initial period $P_{\text{orb}} = 5$ days and eccentricity $e = 0.25$. Initially the star rotates sub-synchronously ($\Omega_s/\omega_{\text{per}} = 0.8$), but resonance locking of the $n = 7$ harmonic of the tidal forcing quickly forces the star to spin near (pseudo) corotation. During the subsequent tidal evolution of the system, a few more cases of resonance locking (now with the $n = 6$ harmonic) occur, recognizable by the more or less square shape of the dips in panel **d**, which shows the timescale for the orbital decay. **a** Orbital and stellar rotation period, **b** ratio of periastron distance to stellar radius, **c** eccentricity and ratio of stellar rotation frequency to orbital frequency at periastron, **d** timescale for the change of the size of the orbit, **e** and **f** schematic representation of the frequency distribution of forcing harmonics and stellar resonance frequencies at $t = 0$ (panel **e**) and $t = t_{\text{max}}$ (panel **f**).

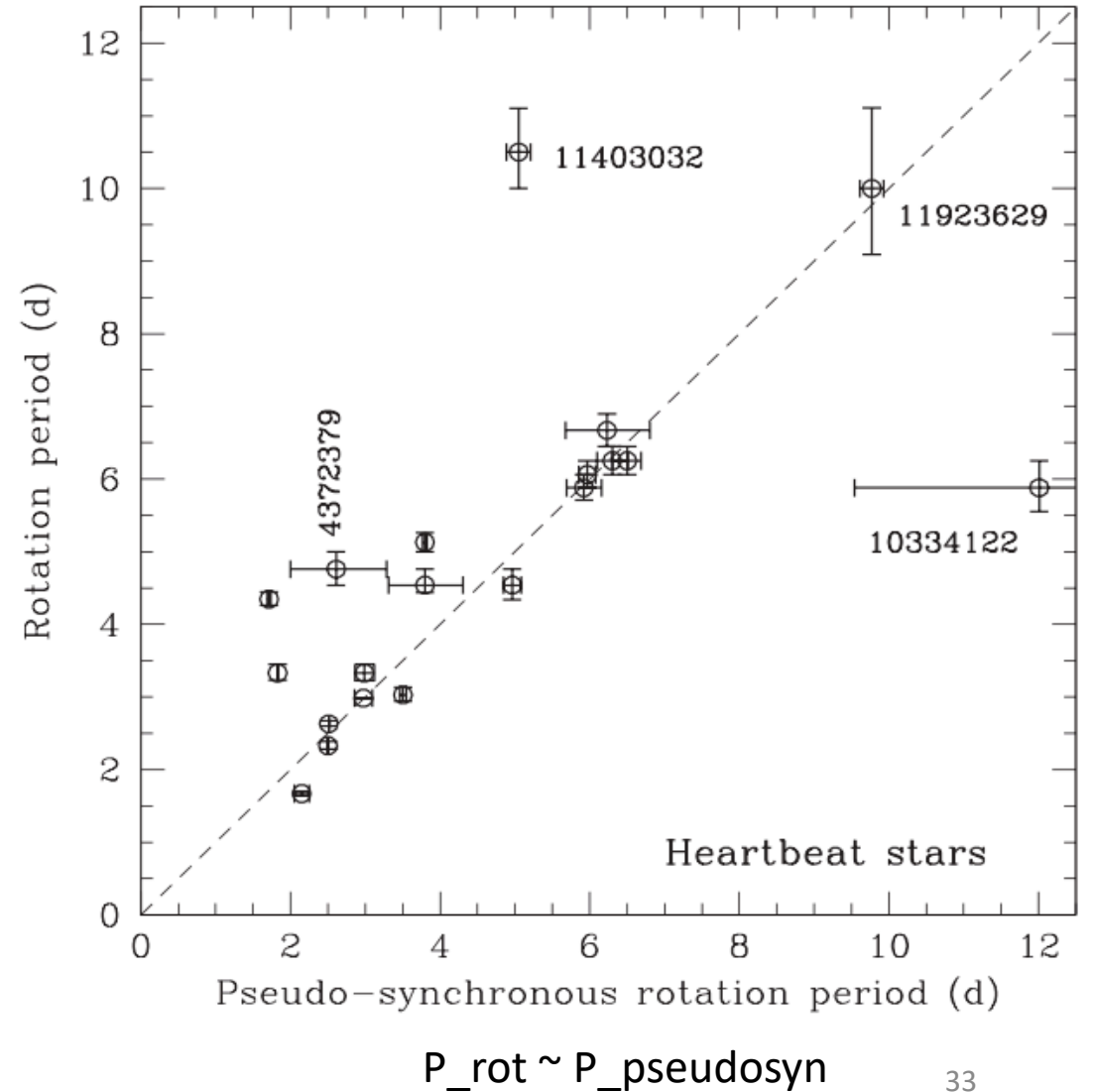
Stellar Spins in HBs

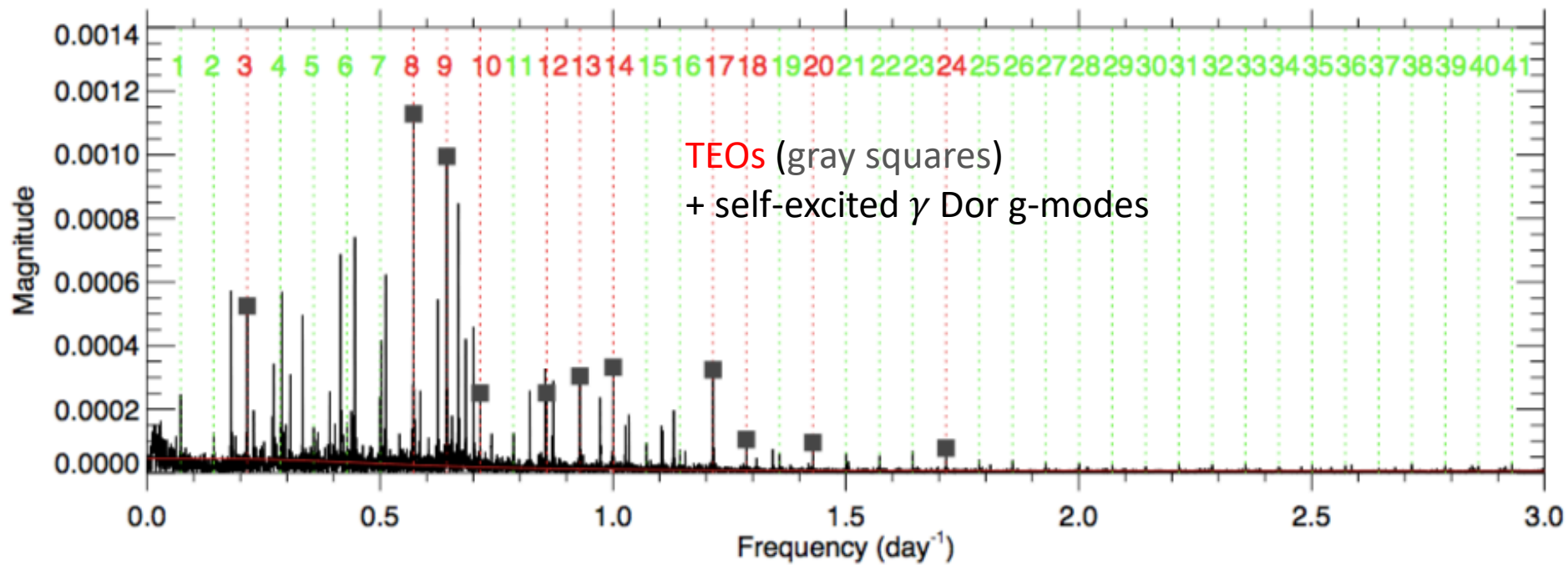
Stellar Spin in Heartbeat Binaries: 2:3 or 1:1?

Zimmerman+17



Saio & Kurtz 2022 (r-mode fitting)





KIC 4142768
two A-star
~ M=2.05

P=14, e=0.6

TEOs

+ γ Dor oscillations

+ δ Sct oscillations

**Very slow rotation
In KIC 4142768**

$P_{\text{core}} \sim 1000 \pm 300$ days !!

$v_{\text{ini}} = 7 \text{ km/s}$

1) Fuller 2021 : “Inverse tide”

2) Triples

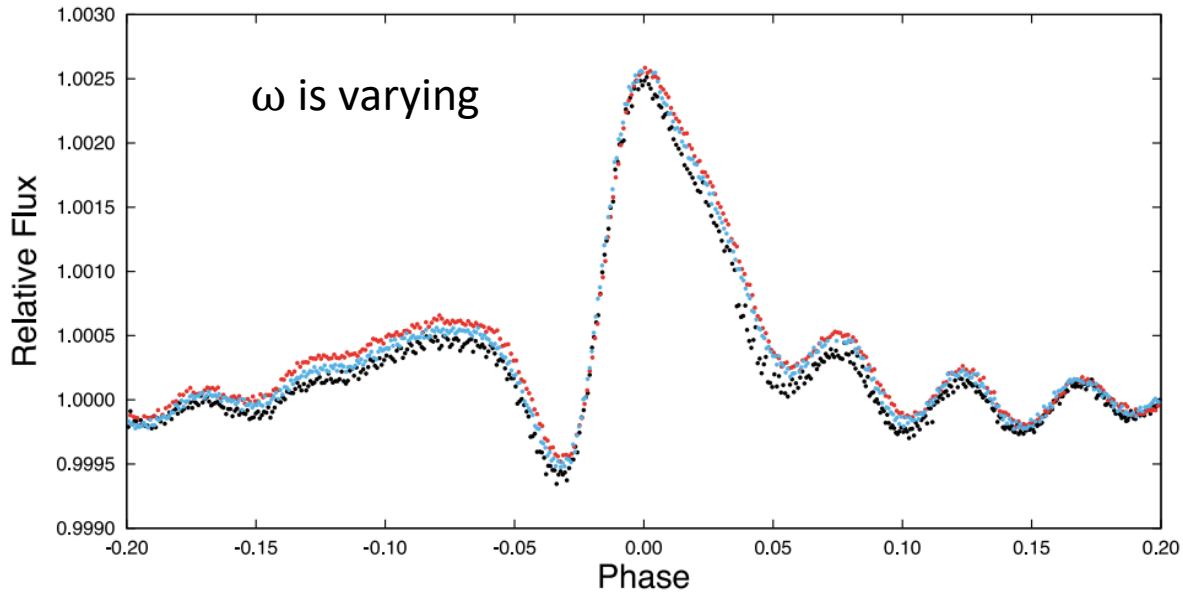
-> Energy can be transferred from the self-excited γ Dor g-modes to the orbit

A new equilibrium state:
Cassini State 2

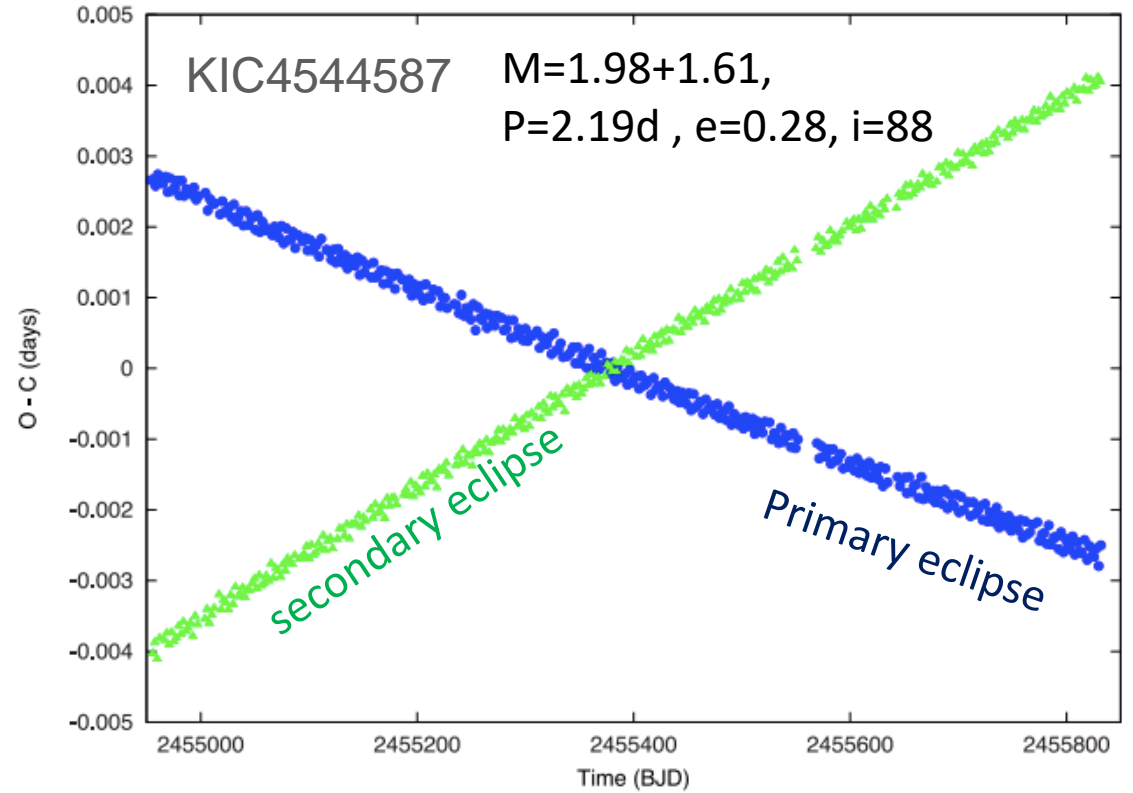
-> a negative tidal Q

*Spin-Orbit Misalignment & Apsidal Motion
in HBs*

HBs: Apsidal motion



Hambleton+ 2016
KIC3749404, 3rd star



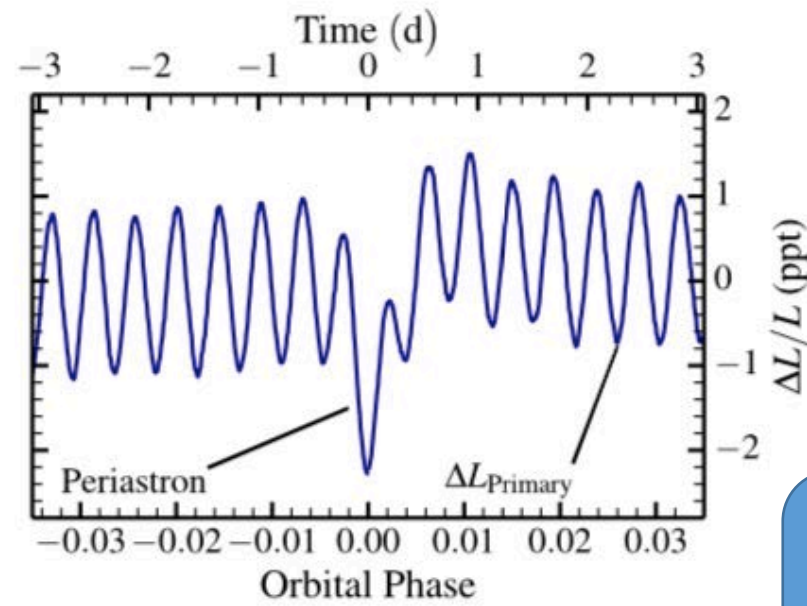
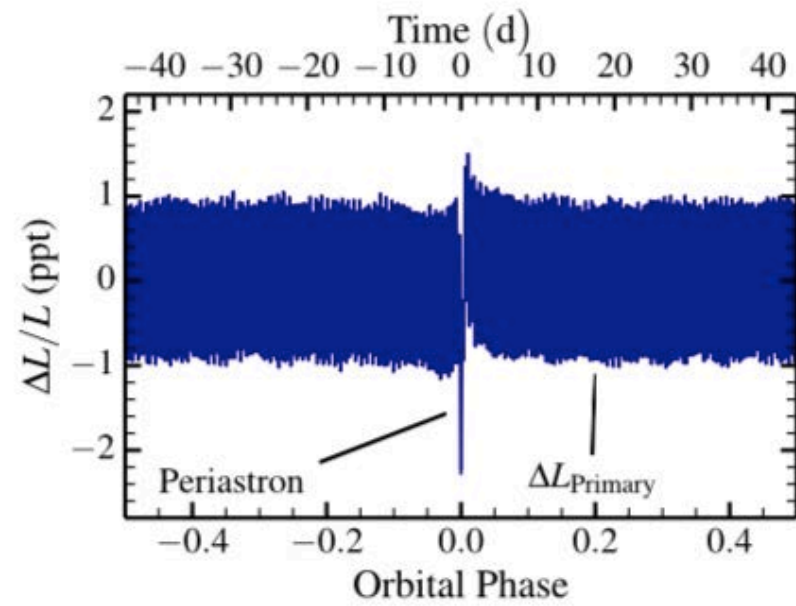
Hambleton+13; Ou+21,

The opposing direction

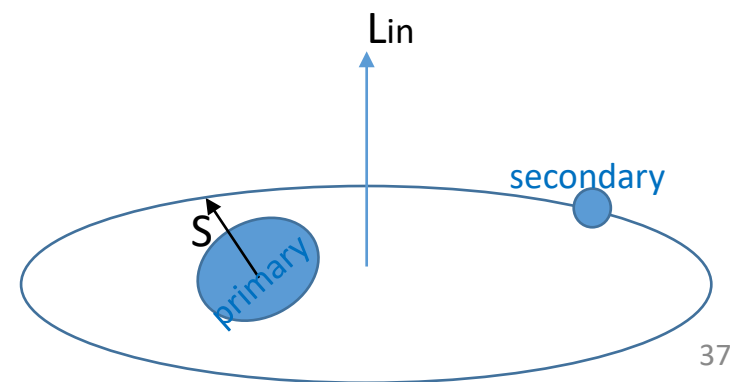
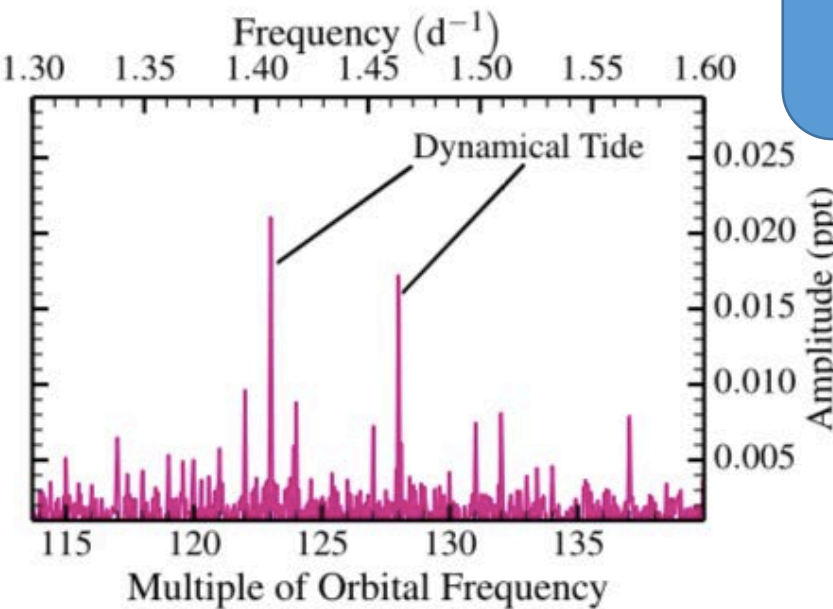
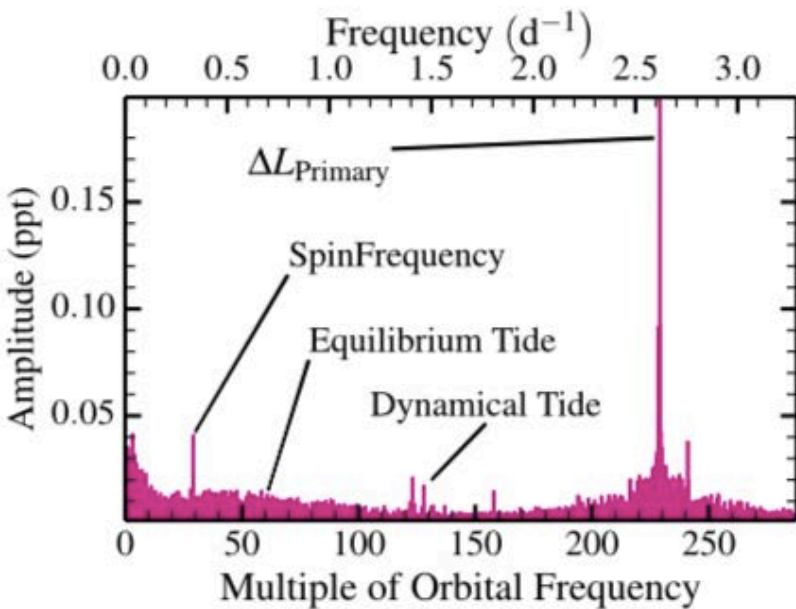
-> classical apsidal motion signature, $P_{ap} \sim 182$ yr

-> Newtonian + GR + Dynamical tide

-> 3rd star 36



Spin-orbit misalignment in KIC 8164262



Masses: 1.70 + 0.36
e=0.89, P= 87.45d,

Hambleton+18
Fuller17

Orbital inclination ~ 65deg, Spin incl.=35deg
-> **Misaligned!**

Very slow stellar rotation/high obliquity
in Heartbeat Binaries

Slow Stellar Spin in Cassini State 2

Explanation: triples

Stars in triple systems can become caught in a **Cassini State (CS2)**
(a **high spin-orbit obliquity** and **slow rotation** of one or both stars).

For main-sequence stars, an inner binary period of $P_{in} \sim 1-10$ d with tertiary periods of $P_{out} \sim 10-10^5$ d.

Such systems would stand out as having **very long rotation periods $P_s \sim 10-10^3$ d**

in contrast to the expectation of tidal synchronization at short orbital periods

Spin Evolution in a triple system

We follow Su & Lai (2021), and assume that the spin AM, S , is much smaller than the orbital AM, L . The spin-orbit evolution equations are given by

$$\frac{d\hat{s}}{dt} = \alpha \cos \theta \hat{s} \times \hat{L}_{\text{in}} + \frac{1}{t_{\text{al}}} \hat{s} \times (\hat{L}_{\text{in}} \times \hat{s}), \quad (1)$$

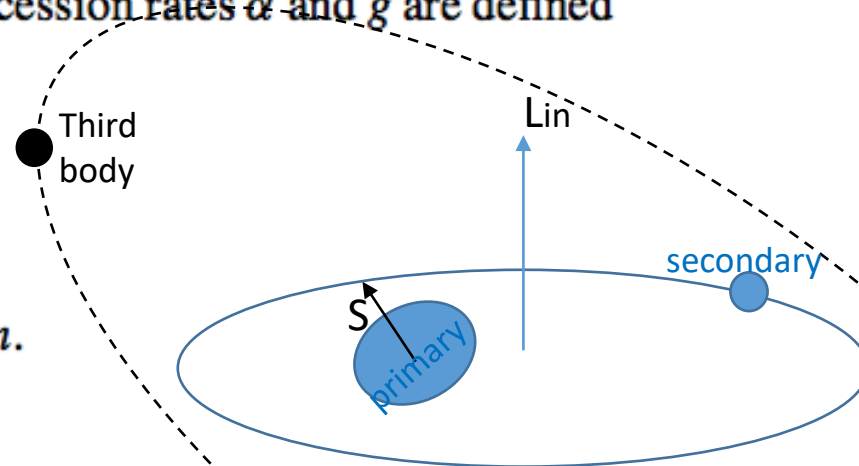
$$\frac{d\hat{L}_{\text{in}}}{dt} = \omega_{\text{lp}} \cos I \hat{L}_{\text{in}} \times \hat{L}_{\text{out}} = -g \hat{L}_{\text{in}} \times \hat{L}_{\text{out}}, \quad (2)$$

where θ is the obliquity, and the precession rates α and g are defined via

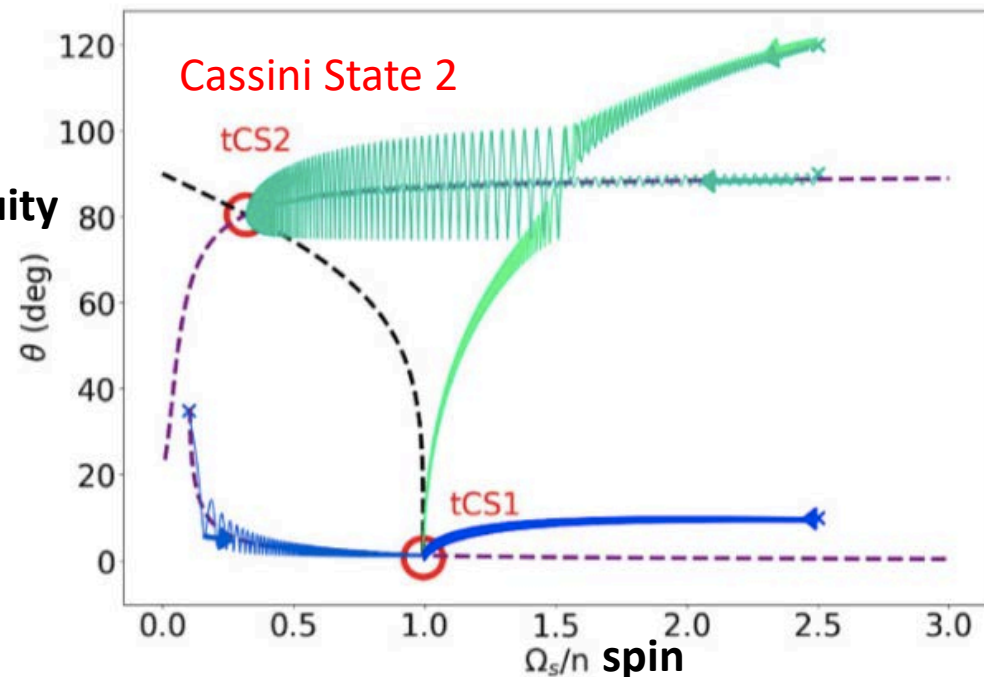
$$\alpha = \frac{k_2 M_2}{2k M_1} \left(\frac{R}{a_{\text{in}}} \right)^3 \Omega_s,$$

$$g = -\frac{3M_{\text{out}}}{4(M_1 + M_2)} \left(\frac{a_{\text{in}}}{a_{\text{out}}} \right)^3 \cos I n.$$

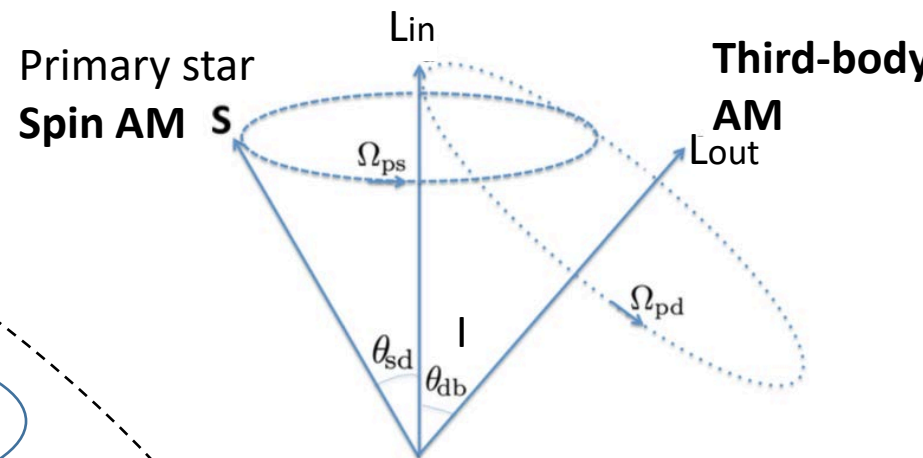
Felce & Fuller 23



obliquity



secondary star
Orbital AM inner binary



I : orbital inclination

θ : obliquity

Constraints on the orbit of the third Star

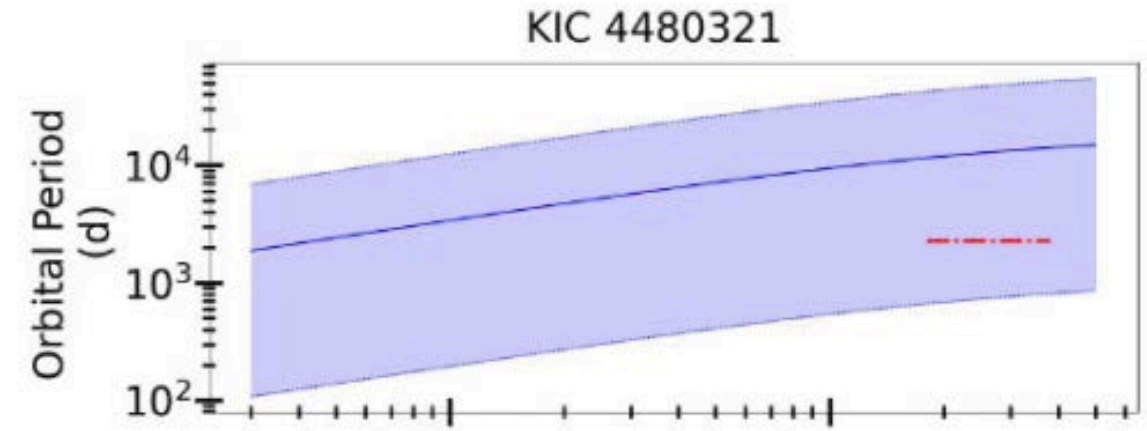
Assuming Cassini State 2:

$$a_{\text{out}} = \left(\frac{3k}{2k_2} \frac{M_{\text{out}}M_1}{M_2(M_1 + M_2)} \frac{a^6 \cos I}{\eta_{\text{sync}}} \right)^{1/3}. \quad (46)$$

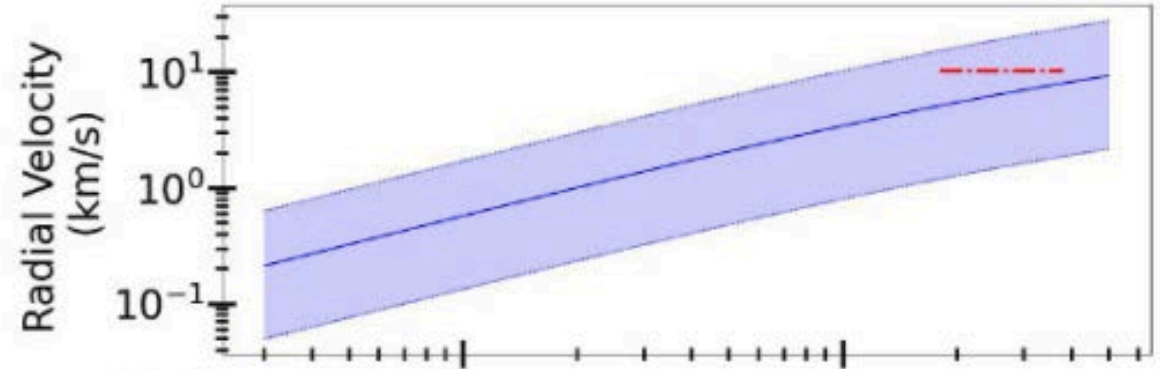
The projected radial velocity of the inner binary about the tertiary system's centre of mass is

$$K_{12} = m_{\text{out}} \sin i \sqrt{\frac{G}{a_{\text{out}}(m_{\text{out}} + M_1 + M_2)}}. \quad (47)$$

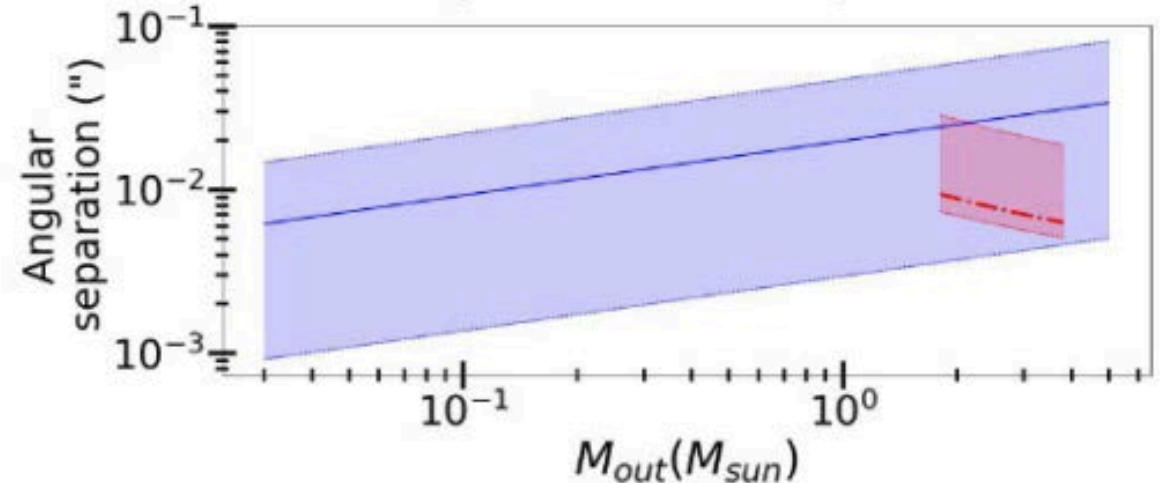
P_{orb}



K



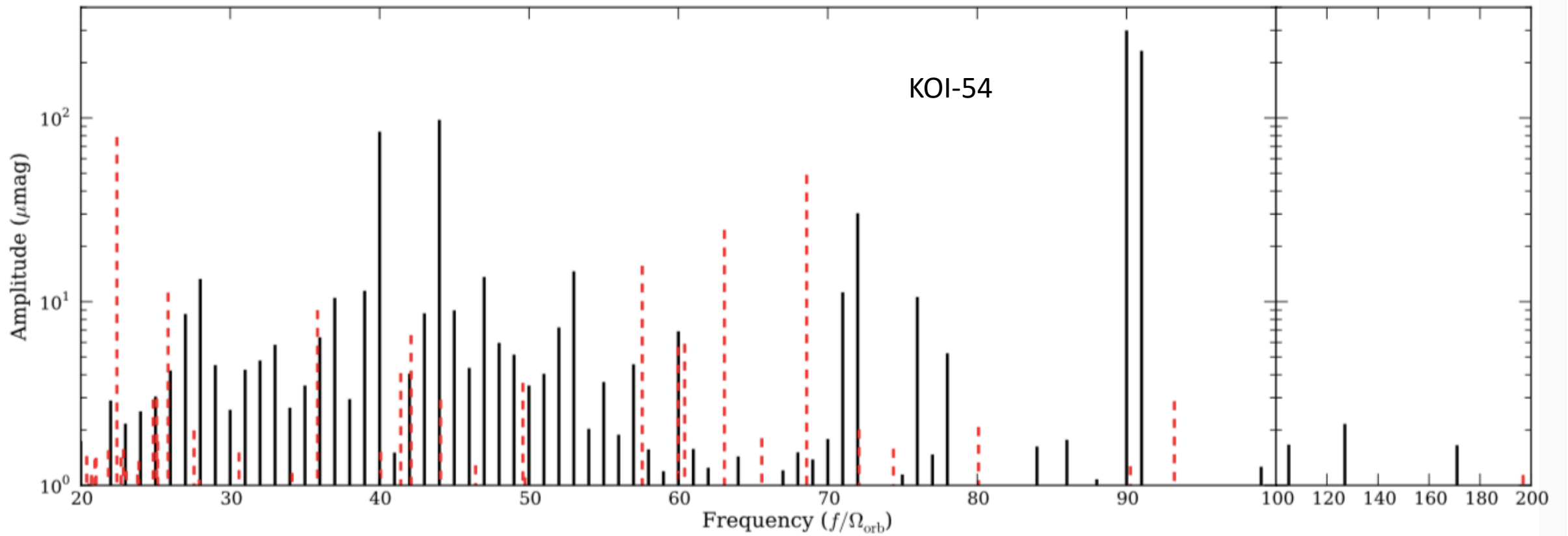
θ



Tertiary mass

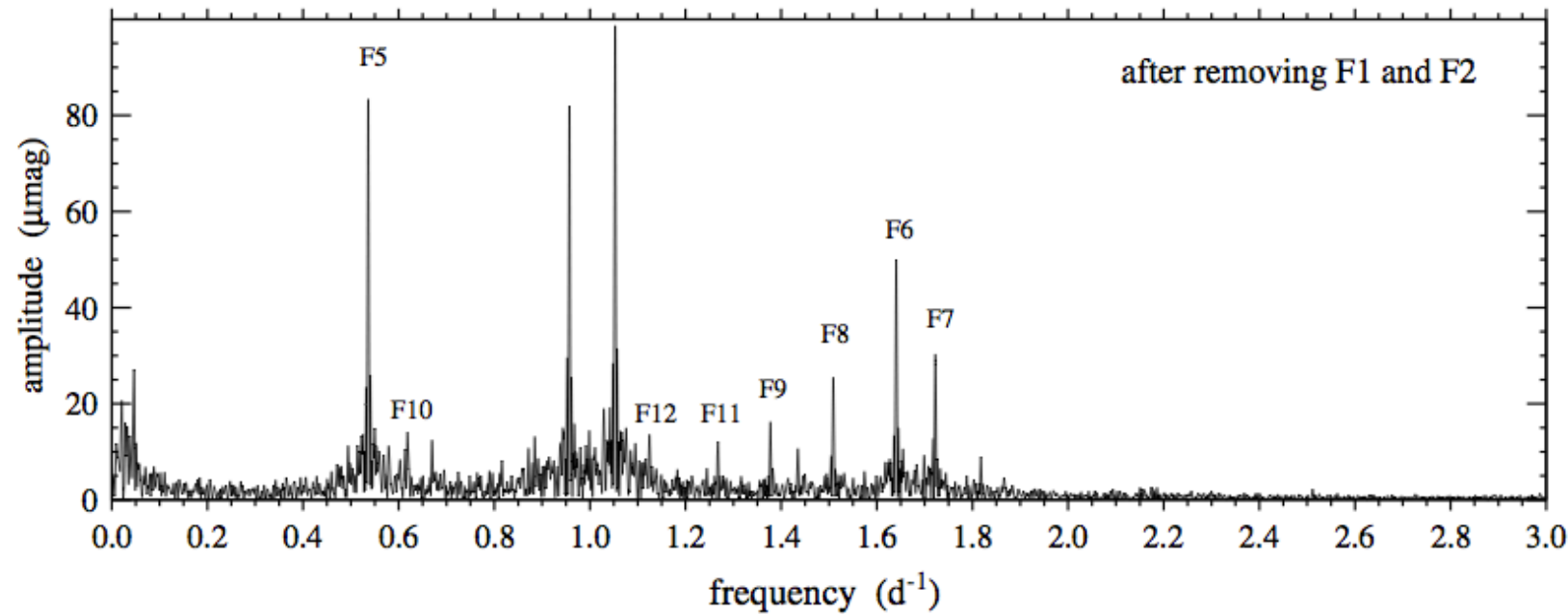
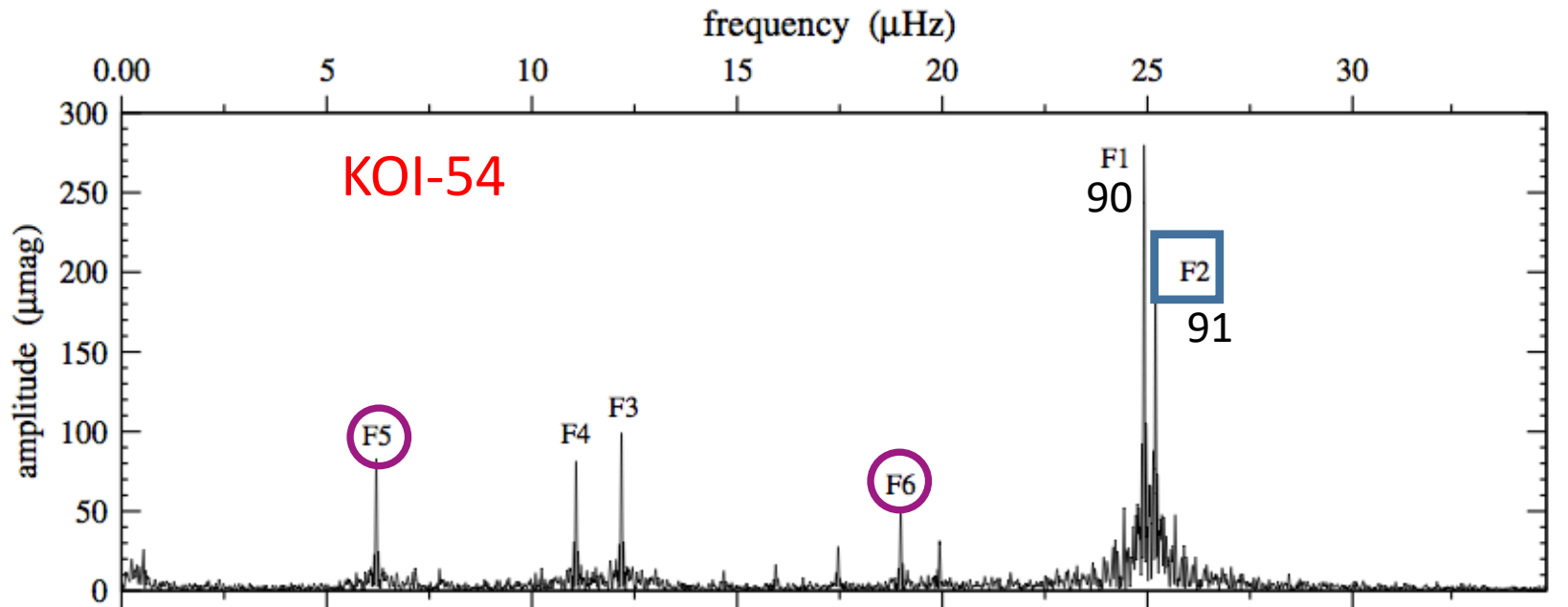
(Weakly) Nonlinear TEOs

$$\dot{q}_a + i\omega_a q_a = \underbrace{-\gamma_a q_a}_{\text{damping}} + \underbrace{i\omega_a U_a(t)}_{\text{linear tide}} + \underbrace{i\omega_a \sum_b U_{ab}^*(t) q_b^*}_{\text{non-linear tide} \sim \text{cancel with equil. tide}}$$
$$+ \underbrace{i\omega_a \sum_{bc} \kappa_{abc}^* q_b^* q_c^*}_{\text{three-mode coupling}} + \text{High Order Terms}$$



Oleary & Burkart 14

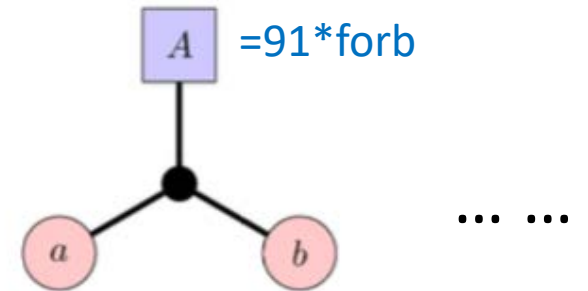
Black: orbital harmonic TEOs (excited by linear dynamical tide)
Red: non-orbital harmonic TEOs (non-linear mode coupling)



Parent mode = 91 orbital harmonic
linearly driven by tide, it has

Four pairs of daughter modes:

$$f_A = f_a + f_b \quad \text{resonance condition}$$



F2 (=91*forb)

$$= F5 + F6$$

$$= F8 + F100$$

$$= F25 + F76$$

$$= F39 + F42$$

f_A has larger amplitude,
than daughter modes
(f_a, f_b) = (F5, F6)

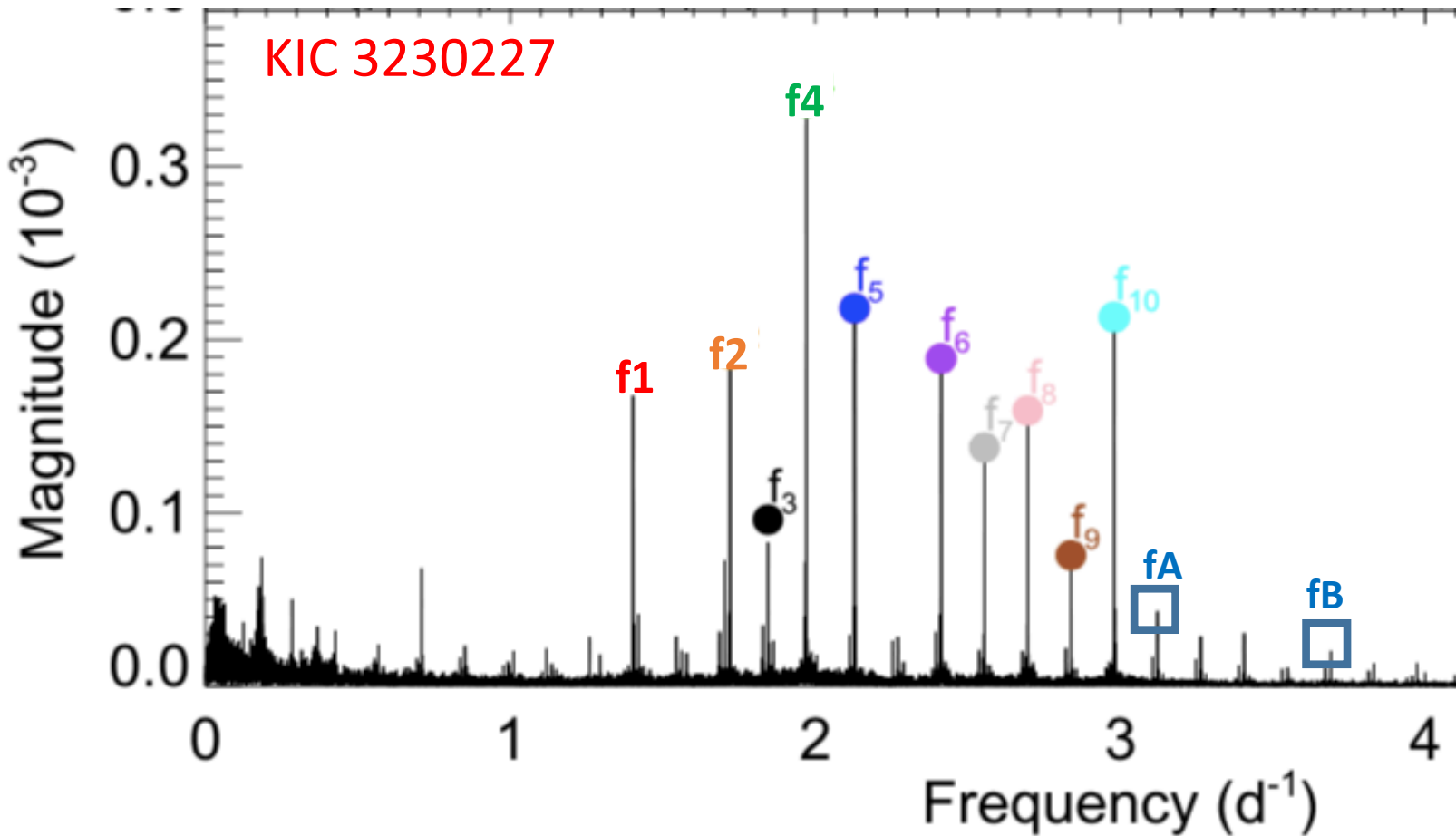
F5 22.419 **F25 60.419** **F39 41.417**

F6 68.582 **F76 30.587** **F42 49.589**

F8 63.076

F100 27.929

KIC 3230227

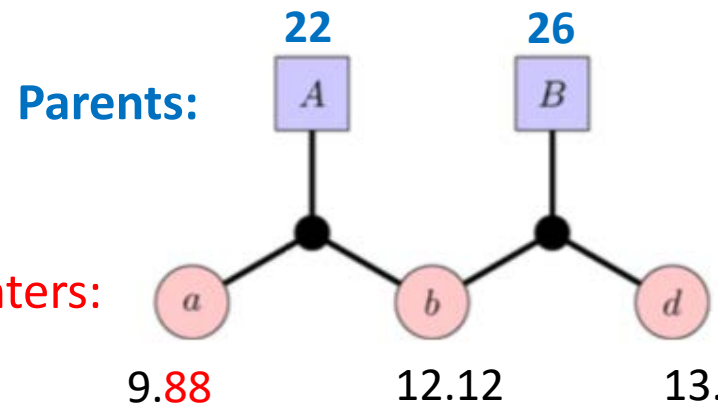


| | Frequency | N forb |
|-----------------|-------------------------|-----------------|
| fa f_1 | 1.40214 ± 0.00002 | $9.88 f_{orb}$ |
| fb f_2 | 1.71988 ± 0.00002 | $12.12 f_{orb}$ |
| f_3 | 1.84482 ± 0.00002 | $13 f_{orb}$ |
| fd f_4 | 1.969765 ± 0.000008 | $13.88 f_{orb}$ |
| f_5 | 2.12855 ± 0.00001 | $15 f_{orb}$ |
| f_6 | 2.41235 ± 0.00001 | $17 f_{orb}$ |
| f_7 | 2.55425 ± 0.00002 | $18 f_{orb}$ |
| f_8 | 2.69615 ± 0.00002 | $19 f_{orb}$ |
| f_9 | 2.83805 ± 0.00002 | $20 f_{orb}$ |
| f_{10} | 2.979948 ± 0.000008 | $21 f_{orb}$ |

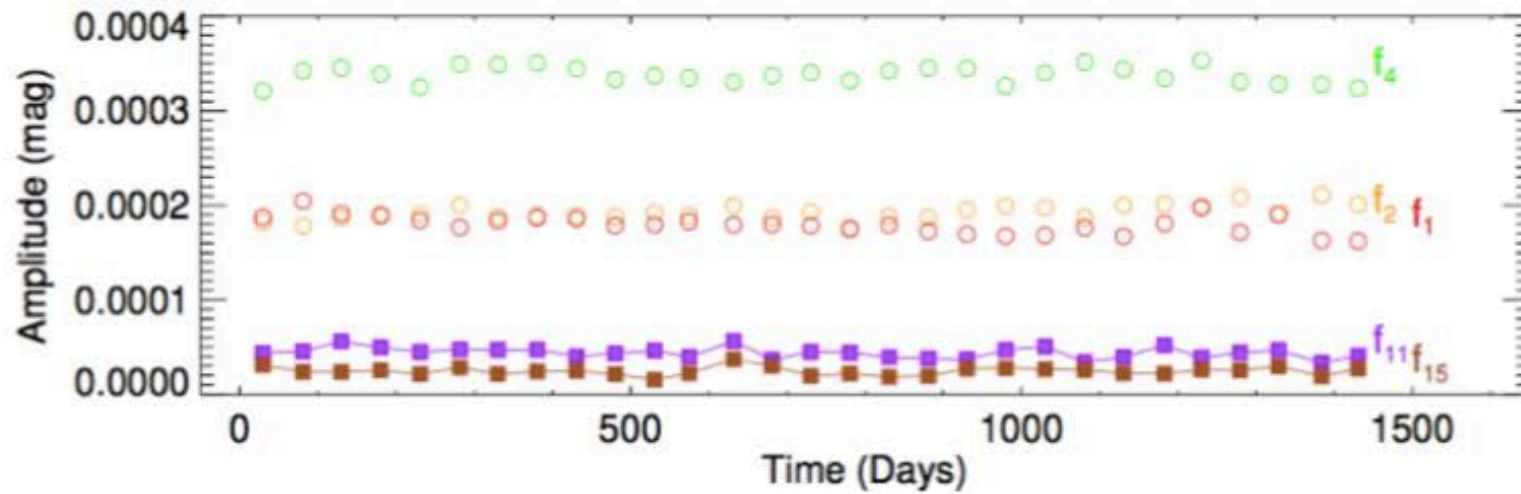
fA = 22 forb
fB = 26 forb

Selection Rules:
fA = f1 + f2
fB = f4 + f2

$m_A = m_a + m_b,$
 $\text{mod}(l_A + l_a + l_b, 2) = 0,$
 and
 $|l_a - l_b| \leq l_A < l_a + l_b.$



Amplitude



Mode coupling in
a steady state or
limit cycle ?

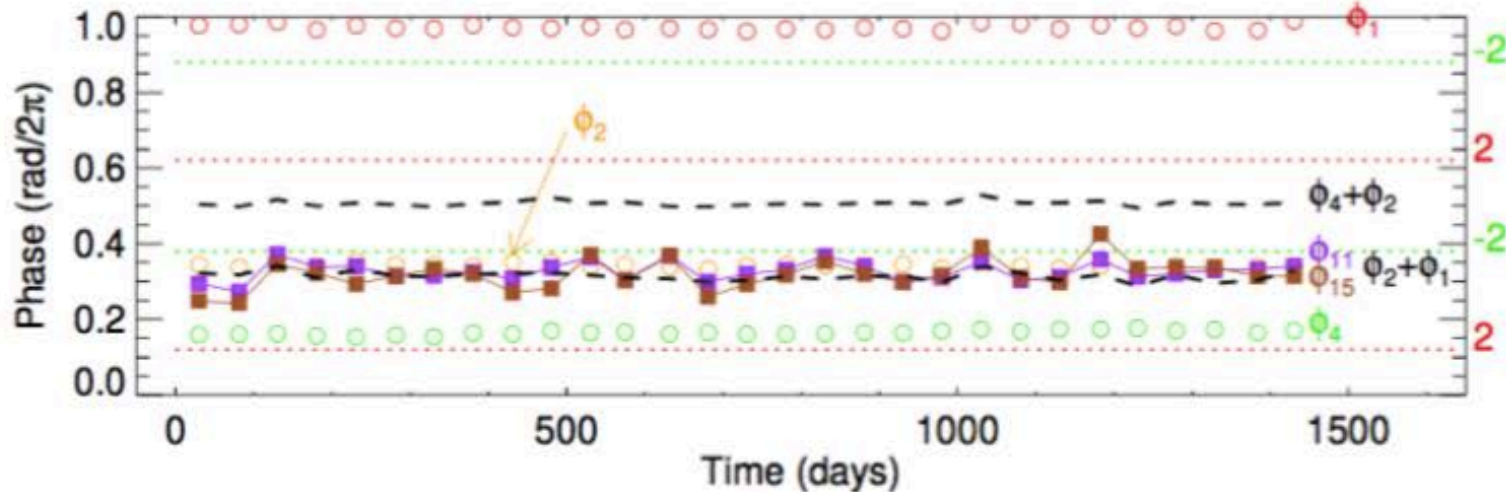
Two parent modes:
F15, F11

3 Daughter modes:
f1, f2, f4

Amplitude/Phase
Stable over 4 yrs

Temporal
variations
(4yr Kepler+
1yr TESS)

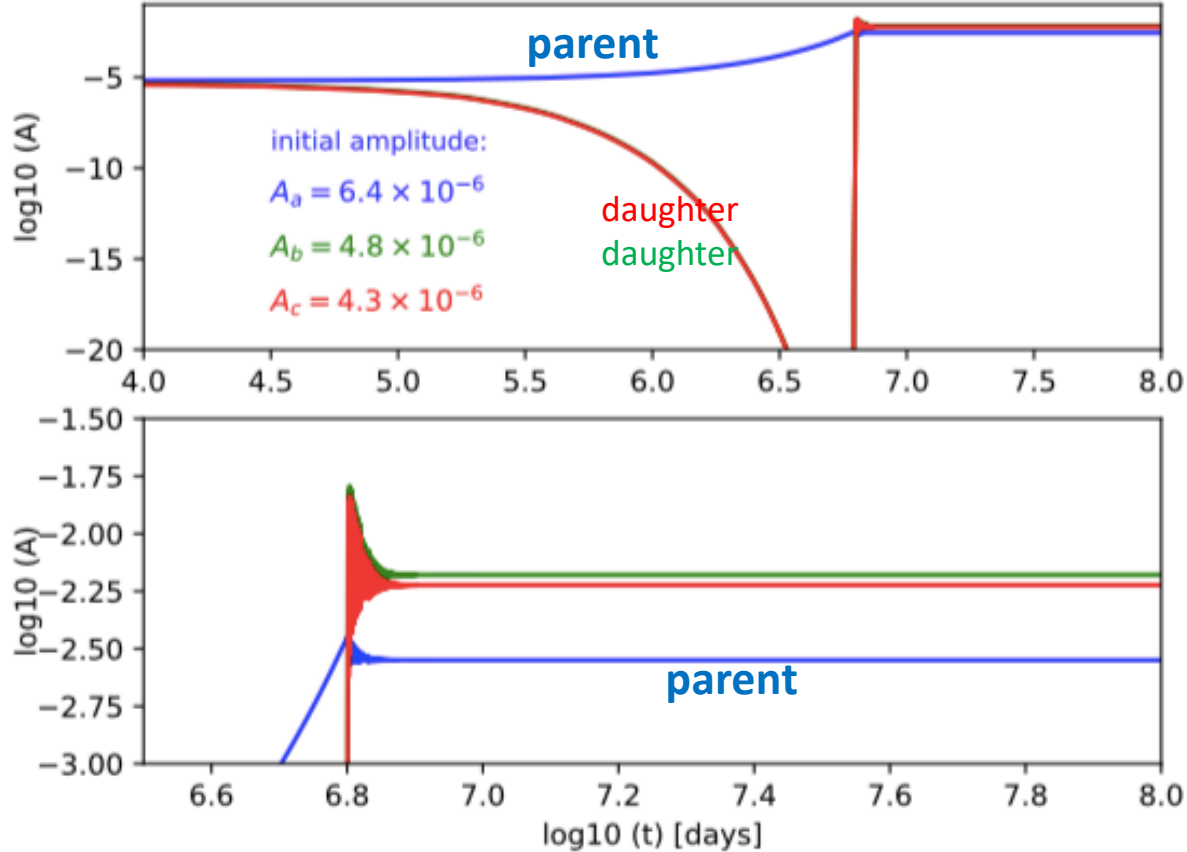
Phase



Three-mode Coupling: (a, b, c) = (f₁₁, f₂, f₁)

Parent mode self-excited

$$\gamma_a = -10^{-6}, \gamma_b = 10^{-5}, \gamma_c = 10^{-5}, \kappa = 10$$



Parent mode driven by tide

$$\gamma_a = 10^{-5}, \gamma_b = 10^{-5}, \gamma_c = 10^{-5}, \kappa = 10$$

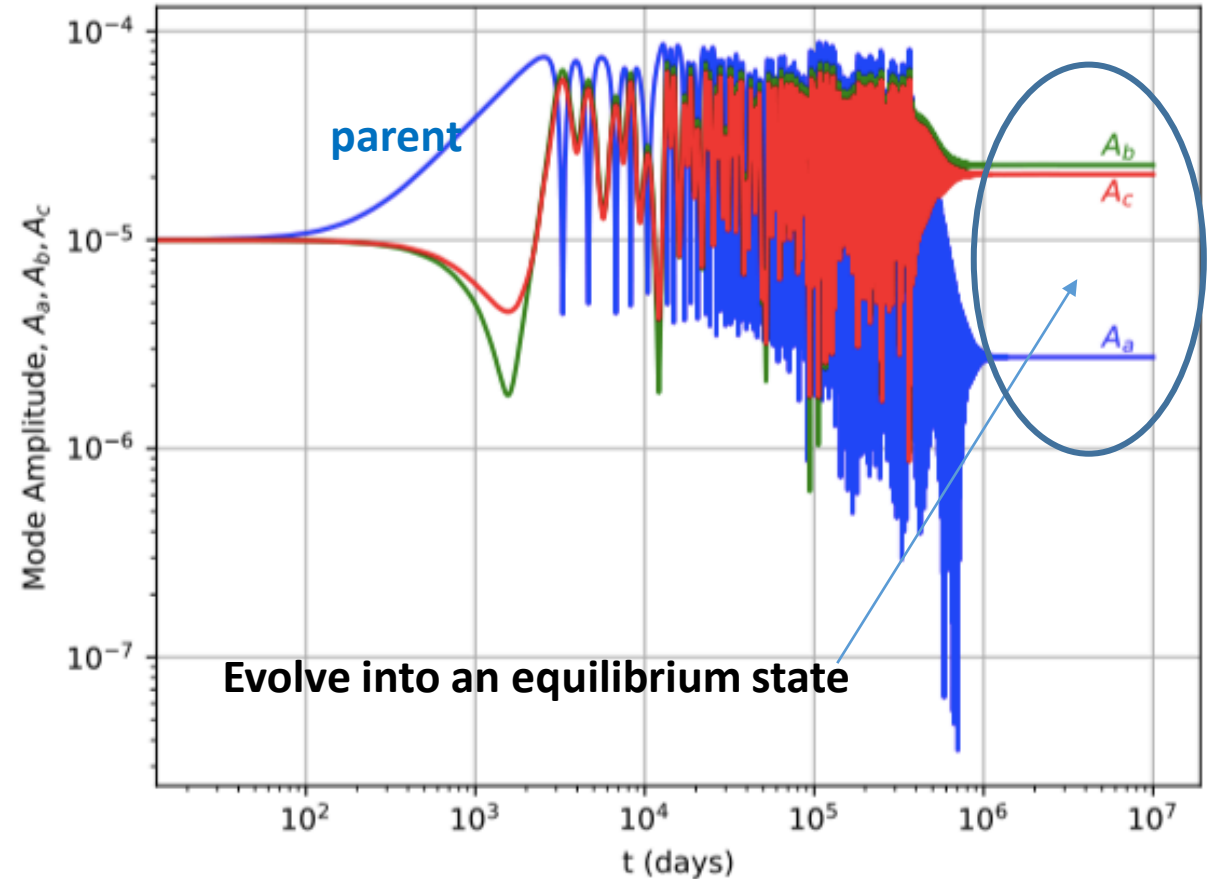
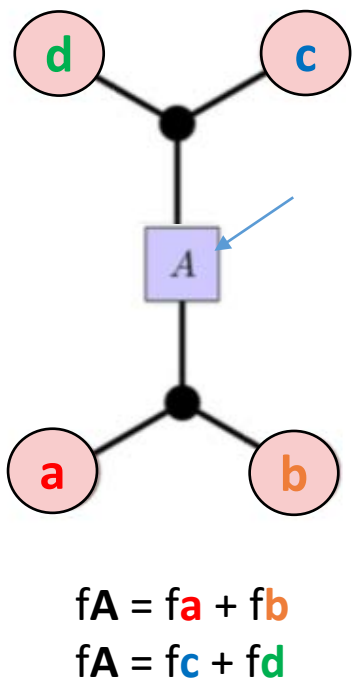
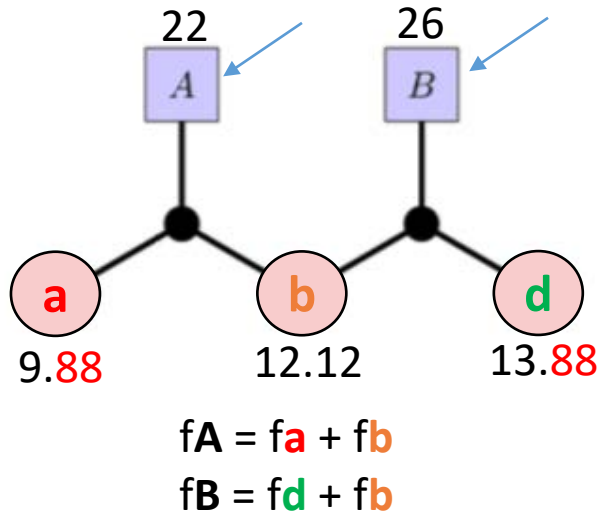
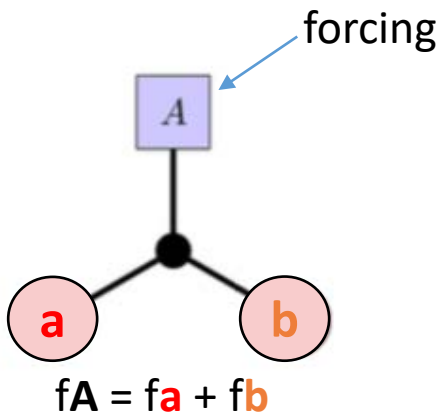
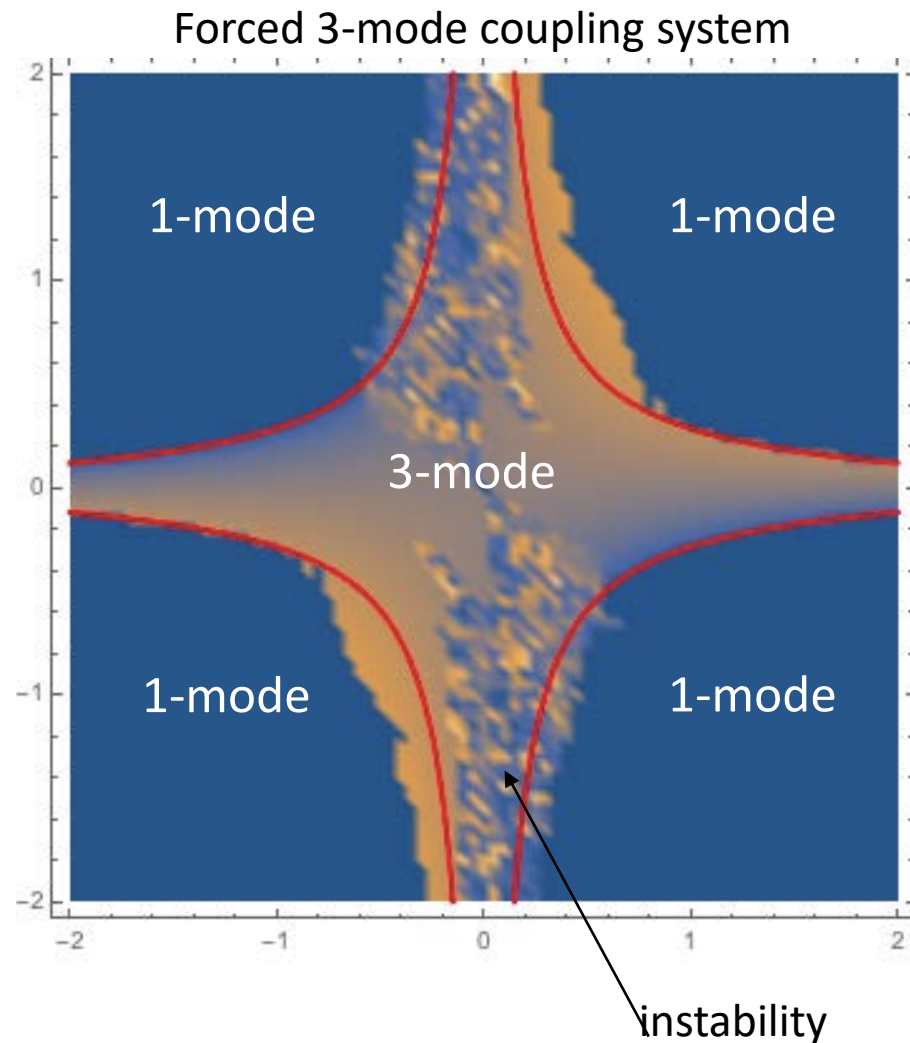
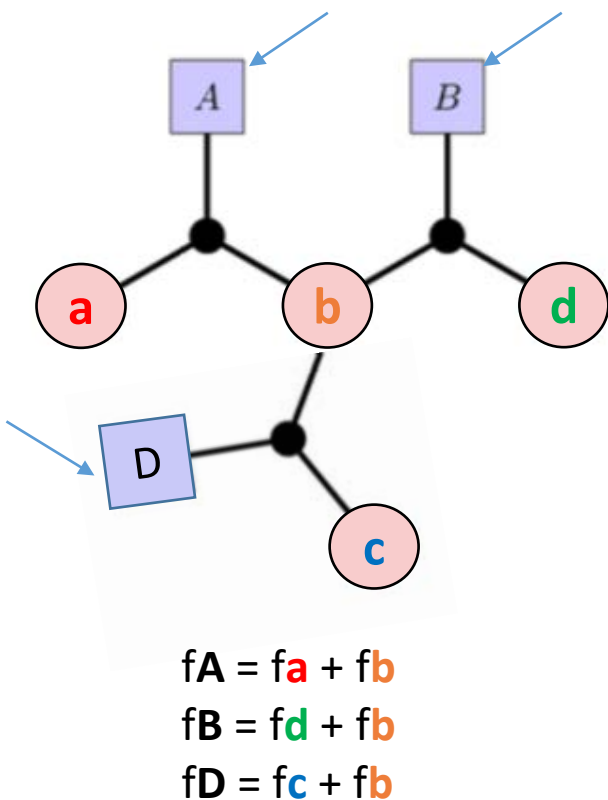


Figure 3. Mode amplitude evolution of a three-mode system representing the observed g-mode triplet in KIC 3230227: $(f_a, f_b, f_c) = (f_{11}, f_2, f_1)$. Left: Parent mode is self-excited, with $\gamma_a < 0$; Right: Parent mode is stable ($\gamma_a > 0$), but driven by a tidal term $U_a e^{-i\omega t}$, with ω being an orbital harmonic ($=22f_{\text{orb}}$). In both cases, the three-mode system settles into an equilibrium state.



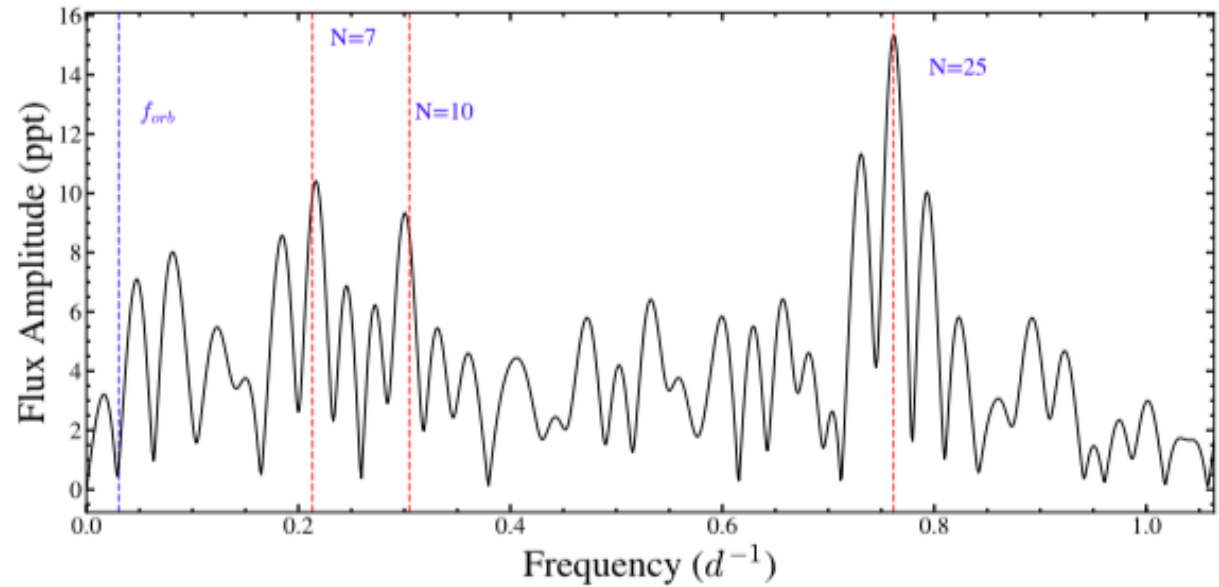
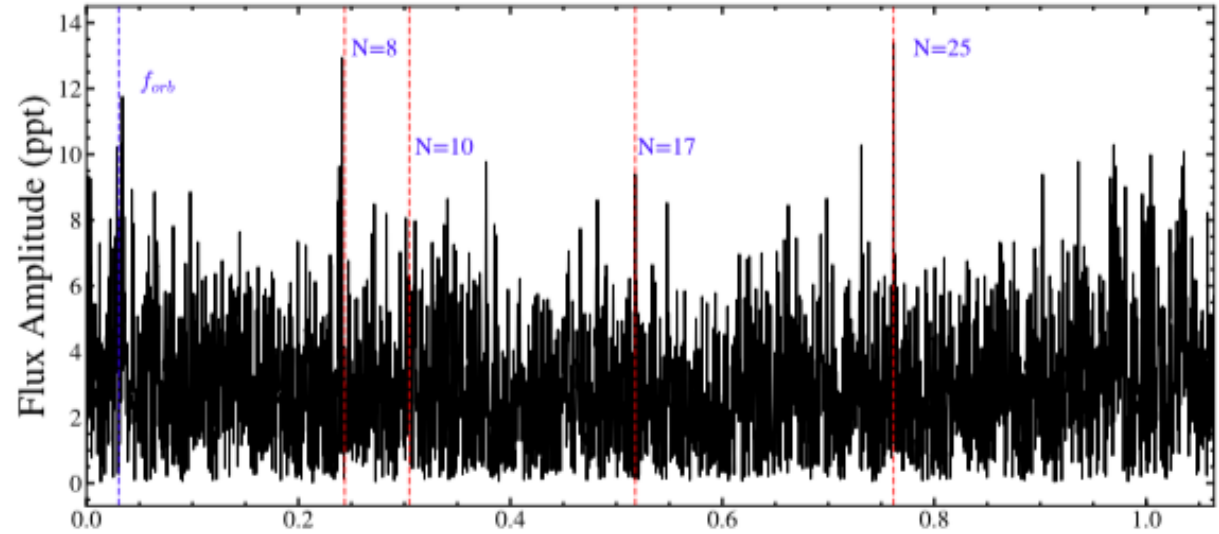
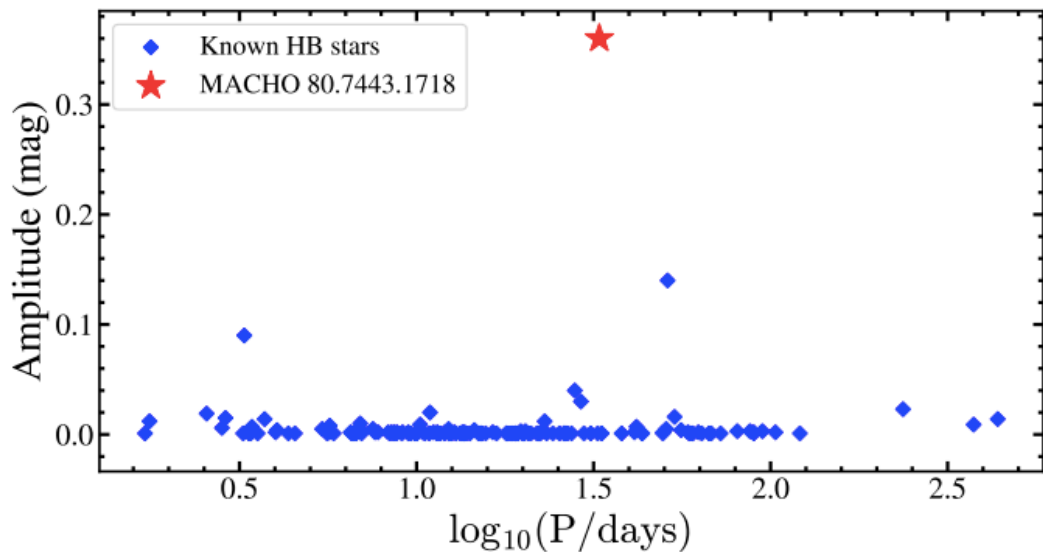
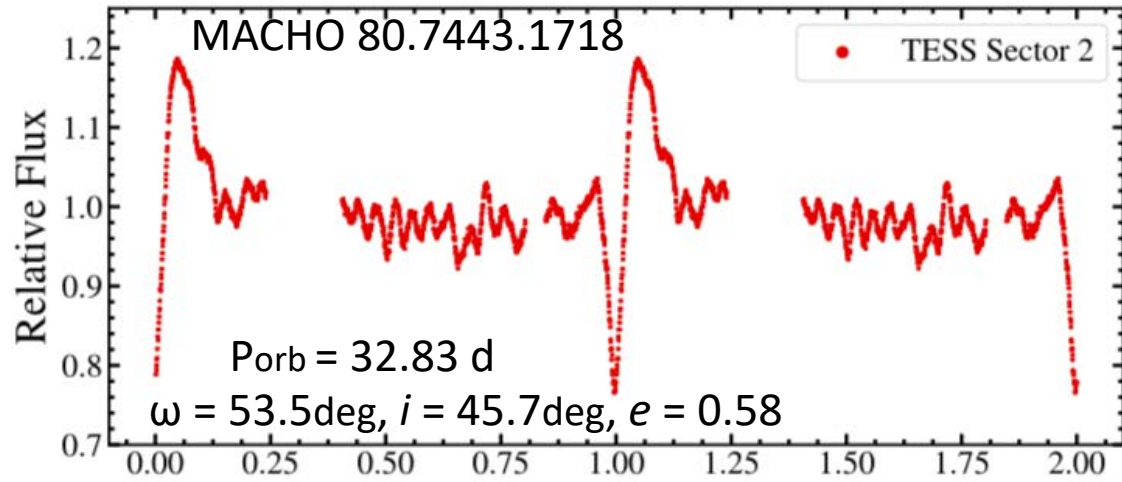
2nd-order
coupling



Limit cycle, Period doubling,
chaotic;
Hysteresis

Notable HB systems

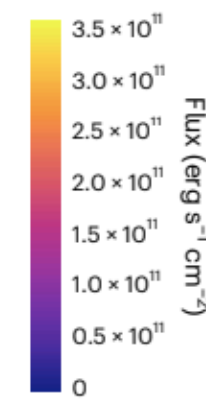
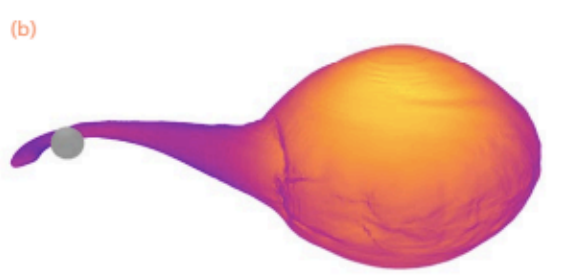
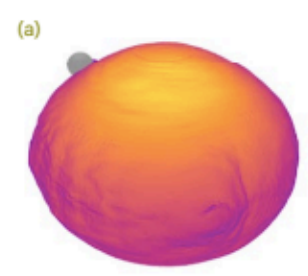
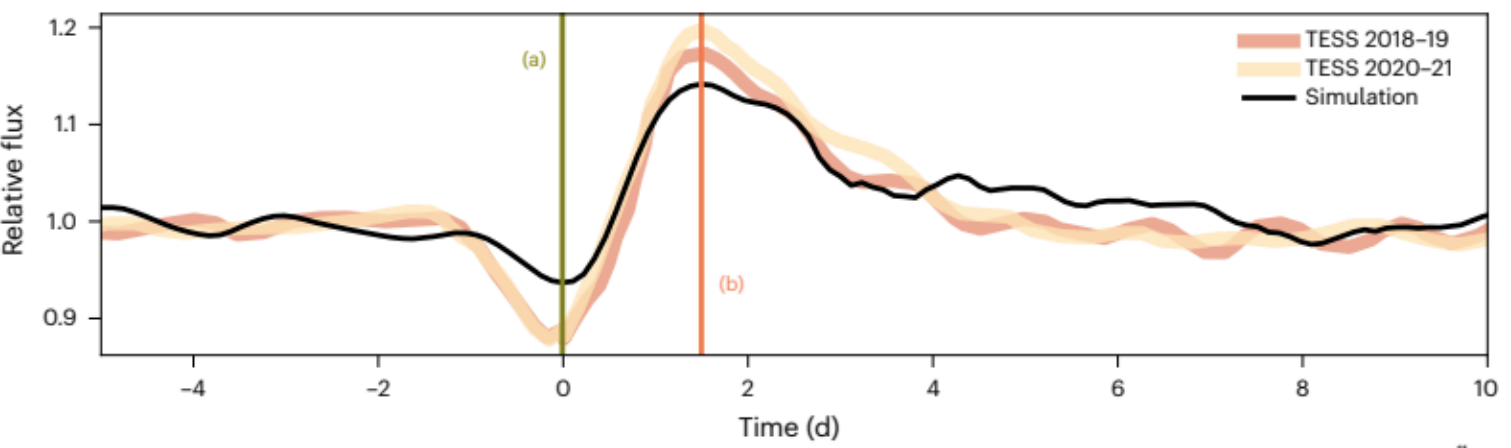
Heartbeat Star with the largest TEO amplitude



$M \sim 35$, $R \sim 24$ (solar)

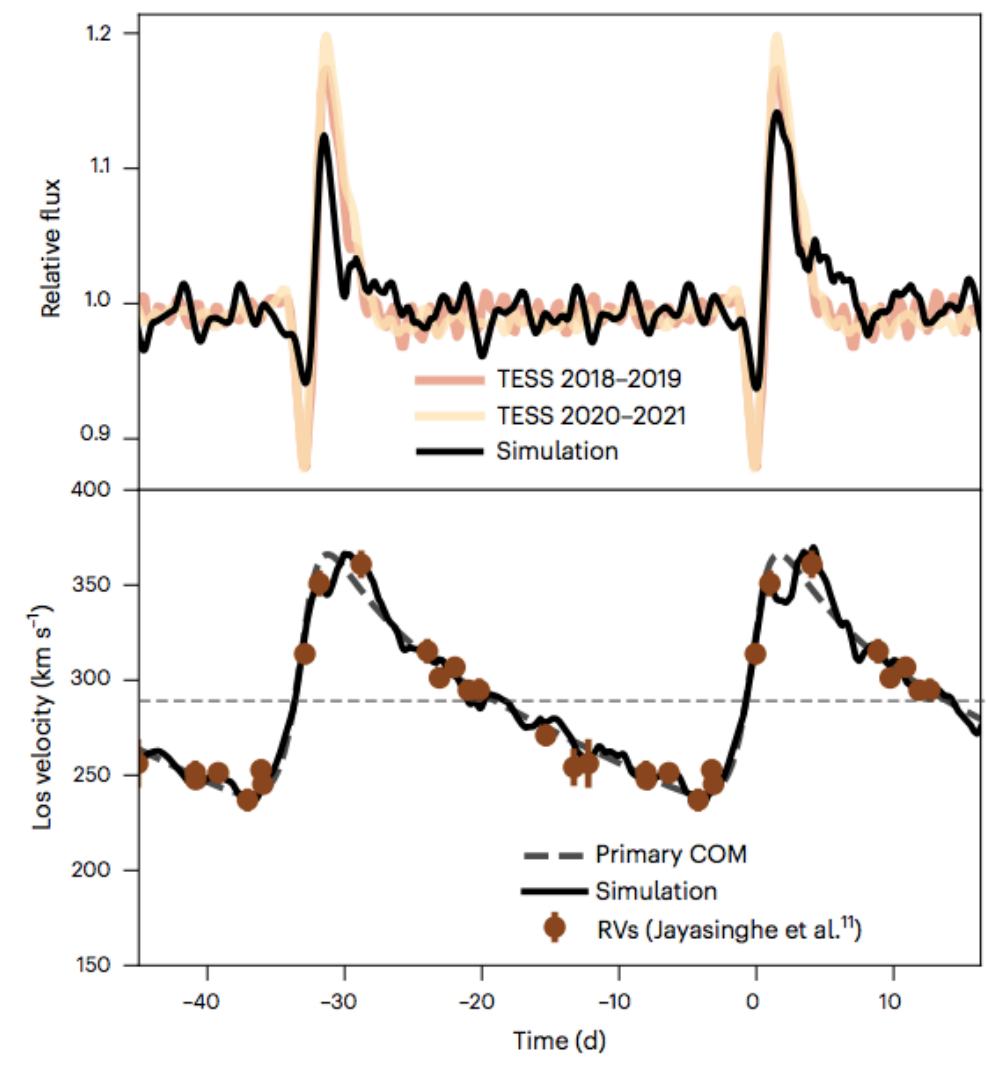
Jayasinghe+ 2019

From Heartbeat to Heartbreak?



Macleod & Loeb 23

Jayasinghe +19,21; Kolaczek-Szymanski+22,24; Macleod 23;



Time (d)

$M_1 \sim 35$, $R_1 \sim 24$ (solar units)

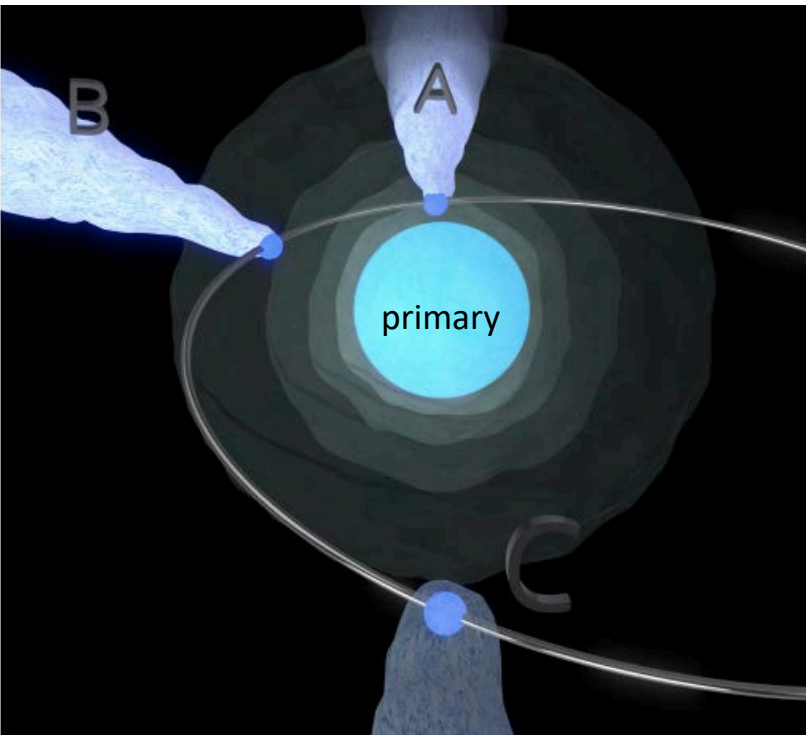


Fig. 12. Artistic representation of the orbit (silver ellipse) of the secondary component of ExtEV (dark blue circle) around the primary component (light blue circle near the center). The size of the orbit and both components are to scale. The orientation of the orbit corresponds to the view of the ExtEV in the sky (i.e., as seen by the observer on Earth). The primary component is the source of a slow but dense stellar wind (semi-transparent spheres, concentric with the primary component), which, after colliding with the surface of the wind of the secondary component, forms a WWC cone, a turbulent structure, with its apex located near the secondary component. Three orbital phases are labeled, corresponding to: the superior conjunction (A), periastron passage (B), and inferior conjunction (C). More details can be found in the main text.

Kolaczek-Szymanski+ 23

No wave breaking!

large amplitude explained by
Wind-Wind Collision cone (WWC)

Summary

Tidal Dissipation: Equilibrium tide vs. **Dynamical Tide**
(turbulent viscosity) (Inertial Wave, Internal Gravity Wave)

IGW -> Normal modes -> TEOs

TEO Amplitude, Phases, Orbital-harmonic Number

mode-decomposition vs direct solving

The effect of rotation

Resonance locking

Non-adiabaticity: GYRE-tide (pseudo-syn. blurring)

Stellar spins in HBs

very slow rotators/misaligned systems -> Triple -> Cassini State 2

HB apsidal motion/orbital decay ->Triple

Nonlinear TEOs

seismology with daughter modes only

coupling networks

Thank you very much for your attention!

Notable HB Systems (WWC cone ~ resemble heartbeat feature)

